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# A note on Morris' bound for search designs

Lih-Yuan Deng[a], Dennis K.J. Lin[b,*]

[a] Department of Mathematical Sciences, The University of Memphis, Memphis, TN 38152, USA

[b] Department of Management Science and Information Systems, 314 Beam Building, Pennsylvania State University, University Park, PA 16802-1913, USA

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**Abstract**

A lower bound is given for the number of experimental runs required in search designs for two-level orthogonal array of strength one. 

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One important class of screening designs is the search design first proposed by Srivastava (1975). Morris (1984) obtained an upper bound for search design for the number of factors under investigation. Namely, $(q-1)2^{n-q+1} \geqslant v$, where the search design matrix has dimension $n \times v$ and each subset of $q$ columns are mutually linearly independent. Such a bound is valid for any two-level design. However, two-level designs for practical use are more likely to be orthogonal array of strength one. A two-level design is called *orthogonal array of strength one* if there are equal numbers of $+1$ and $-1$ in each column of the design matrix. Morris' bound can be further improved if a two-level orthogonal array of strength one is used.

For each column in the design matrix, there are $n/2$ elements of $+1$ and $n/2$ elements of $-1$. Suppose there are $p$ elements of $-1$ in the last $q-2$ positions ($p=0,\ldots,q-2$). Then only $\binom{n-q+1}{n/2-p}$ subsets are possible. Furthermore, only $\binom{q-2}{p}$ columns are possible in each subset and these columns have at most $q-1$ dimensions. Therefore, for an orthogonal array of strength one, we have the following result.

**Theorem 1.** *For any $n \times v$ design matrix consisting only of elements that are $+1$ and $-1$ with equal numbers of $+1$ and $-1$ in each column, if each set of $q$ columns are mutually linearly independent, then* $\sum_{p=0}^{q-2} \binom{n-q+1}{n/2-p} \min(\binom{q-2}{p}, q-1) \geqslant v$.

* Corresponding author. Tel.: (814) 865-0377; fax: (814) 863-2381; e-mail: DKL5@PSU.EDU.


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Table 1
Comparison on the upper bounds

| $q$ | Method | $n=4$ | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | Morris | 8 | 32 | 128 | 512 | 2048 | 8192 | 32 768 | 131 072 | 524 288 |
| | Present work | 3 | 10 | 35 | 126 | 462 | 1716 | 6435 | 24 310 | 92 378 |
| $n/2$ | Morris | 8 | 32 | 96 | 256 | 640 | 1536 | 3584 | 8192 | 18 432 |
| | Present work | 3 | 10 | 35 | 126 | 427 | 1212 | 3117 | 7546 | 17 567 |

A comparison with Morris' bound is given in Table 1 for various situations. It is clear that the new bound is consistently sharper than Morris' bound for $q<n/2$. This is particularly true when $q$ is relatively smaller, as expected in the search design context. In fact, it can be shown that

$$2^{n-q+1} = \sum_{p=0}^{n-q+1} \binom{n-q+1}{p} \geqslant \sum_{p=0}^{q-2} \binom{n-q+1}{p} \geqslant \sum_{p=0}^{q-2} \binom{n-q+1}{n/2-p}$$

and

$$(q-1) \geqslant \min\left(\binom{q-2}{p}, q-1\right).$$

Thus, Morris' bound is always larger than the new upper bound given in Theorem 1, when $q<n/2$. When $n$ is odd, a perfect balance is impossible. We thus consider the *near balance* property which can be defined as the numbers of $+1$ and $-1$ elements in each column differ by 1. In this case, a straightforward computation following Theorem 1 can be presented as the following theorem.

**Theorem 2.** *For any $n\times v$ design matrix consisting only of elements that are $+1$ and $-1$ with $n$ odd and the near balance property, if each set of $q$ columns are mutually linearly independent, then* $\sum_{p=0}^{q-2} \binom{n-q+1}{\frac{n-1}{2}-p} \min(\binom{q-2}{p}, q-1) \geqslant v$.

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