

A Strange Mathematical Formula with a Hilarious Philosophical Implication

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ABSTRACT Let V_n denote the volume of an n -dimensional unit sphere, $n \geq 1$. Suppose the dimension N is chosen randomly, according to a Geometric distribution with parameter p . With ϕ denoting the standard normal density, Φ the standard normal CDF, $q = 1 - p$, and $z = q\sqrt{2\pi}$, we prove the peculiar identity

$$E(V_N) = \frac{p}{q} \left[\frac{\phi(0)\Phi(z)}{\Phi(0)\phi(z)} - 1 \right],$$

which is uniquely maximized at $p = .247519$.

Thus, suppose an entity, that we call God, chooses the dimension to live in randomly, and then lives in a unit sphere in His chosen dimension. Then, to maximize His expected living space, He should choose to live in a space of expected dimension $\frac{1}{p} = 4.04 \approx 4$, i.e., four dimensions would be the best choice.