Mendeléeff, Bode, and precognition

Perhaps it is something about human psychology that if a theoretical model for some problem made predictions, and better yet, sensational or bizarre predictions, which later come out to be true, then we take the model more seriously. Should we? After all, perhaps other models would have made the same or similar predictions. Today, my intention isn't to launch a debate on that litigious issue, but to retell two stories of romantic nostalgia that I have myself found fascinating.

It was 1869. An otherwise sane and supremely distinguished professor of chemistry at St. Petersburg started a bonanza of chemical predictions, predictions of discovering hitherto unknown substances on the face of the earth. So vivid were his predictions as to the appearance and properties of these ghostly substances, that apparently he was considered either a prophet or perhaps slightly out of his mind. It had then been only about thirty years that numerical values of the atomic weights of the known elements had started to get known. For more than twenty years, Dmitri Mendeléeff just played with those limited set of numbers, and had painstakingly arranged and rearranged the elements with their atomic weights. He found an incredible pattern in the weights of elements which were chemically equivalent or related. When he found gaps in the atomic weights of groups of equivalent elements, he predicted an undiscovered element that will fill that hole. Coincidence? Some obviously thought so: at a meeting of the English Chemical Society, the question was asked if he had considered arranging the elements according to the first letter of their names and look for patterns. Ah, how subtle the sarcasm. Six years later, Boisbaudran discovered one of the predicted substances in the deep mines of Pyrenees, and every one of its properties matched Mendeléeff's predictions; eka-aluminum was found on earth! One after another, came the vindications. Mendeléeff, now we know, was fearlessly hounding the scent of the periodic law, today an established truth. Some of his predecessors had similar instincts, notably Lothar Meyer, Dumas and Strecker; but none had the dogged determination and an ability to ignore ridicule as did Mendeléeff. A numerical law with very limited data boldly envisioned by an individual was eventually accepted as a remarkable scientific fact.

Mendeléeff had carried with him other striking scientific ideas throughout his

life, such as *a substance which pervades the whole universe*. He had, apparently, a uniquely ingenious theory on the origin of the fantastic oil deposits in Baku. He was summoned to Pennsylvania to help produce oil on a commercial scale for the first time. He was twice denied the Nobel prize, the dark story of which is now well known.

The other story I want to retell is that of Johann Bode, the German astronomer who too noticed remarkable regularities in the planetary spacings in our solar system and put forth a mathematical formula for the locations. Bode's formula says that the distances of the planets from the sun in their natural order follow the formula $\alpha, \alpha + \beta, \alpha + 2\beta, \alpha + 4\beta, \alpha + 8\beta, \cdots$, with $\alpha = .4, \beta = .3$. The predicted positions of the planets by Bode's formula are $.4, .7, 1, 1.6, 2.8, 5.2, 10, 19.6, 38.8, 77.2$, etc. The actual positions are .39 (Mercury), .72 (Venus), 1 (Earth), 1.52 (Mars), 2.77 (Asteroid belt), 5.20 (Jupiter), 9.54 (Saturn), 19.18 (Uranus), 30.06 (Neptune), 39.79 (Pluto). The good fit for Mercury, Venus, and Earth is not relevant; Bode found his formula by using the positions of those three planets. But it is impressive that the formula fits Jupiter exactly, even though Jupiter was an extrapolation. In 1772, when Bode gave his formula, the existence of the asteroids, Uranus, Neptune, and Pluto was not known. In 1781, William Herschel, a musician turned astronomer, discovered a faint object moving very slowly around the sun and used Kepler's third law to find its orbit. Uranus was discovered at a mean distance of 19.2, the exact hypothetical position predicted by Bode's formula. King George III was ruling England then, and Herschel named the planet "Georginum Sidus." Herschel was made a Fellow of the Royal Society, and given a King's medal. Public outrage at naming the planet after the King and the awards was so high that the name didn't last long; it was renamed Uranus. And the coincidences continue. On the night of January 1, 1800, Giuseppe Piazzi found a new object between Mars and Jupiter and then lost it. Heinrich Olbers happened to know of the newly found least squares method and used it to predict the position of the lost object. And, there it was, when the winter cleared, the asteroid belt, at almost exactly the Bode predicted distance. But, of course, Bode doesn't predict any planet at Neptune's current position, which is an inconsistency in Bode's law.

There is mixed reaction among the few I have talked to about whether there could be some truth in Bode's law. I know of three wonderful writings, by Jack Good (1969), Brad Efron (1971), and Persi Diaconis (1978). They provide different perspectives on whether Bode's law is a coincidence or not. In 2003, Jim Zidek and I enquired if short sequences of numbers usually follow Bode's law with a suitable α and β. Some short sequences that didn't behave erratically did give a terrific fit; two that we tried didn't. The early primes, because there are too many early twin primes; and the gestation periods of ten small, medium, and huge mammals. When we excluded the whale and the elephant, the fit was wonderful.

But I must suggest abundant caution in accepting numerical laws, For here in vol. 12 of the Math Gazette, (Bell and Hyman, 1924), I find Mrs. La Touche complaining, "There is no greater mistake than to call arithmetic an exact science. There are aberrations discernible to minds noble like mine. For example, if you add a sum from the bottom up, and then again from the top down, the results are always different!"