Solutions to Homework #2
Due on Wednesday, September 12, 2007

Exercise 1.3

19.a).

\[
\begin{align*}
R\text{Gui Code:} \\
> & \quad \text{plot(c(4,6), c(0.5,0.5), type = "o", xlab = "x", ylab = "Density", xlim = c(3,7), ylim = c(0,1), col = "red", lwd = 2, lend = 1)} \\
> & \quad \text{lines (c(3,4), c(0.5,0.5), lty = "dashed")} \\
> & \quad \text{polygon (c(4,6,6,4), c(0,0.5,0.5), col = "gray", border = FALSE)} \\
> & \quad \text{lines (c(4,4), c(0,0.5), lty = "dashed")} \\
> & \quad \text{lines (c(6,6), c(0,0.5), lty = "dashed")}
\end{align*}
\]

Area under the curve \( = \text{height} \times \text{base} \)
\[
= \frac{1}{2} \times (6 - 4) \\
= 1
\]

Hence proved!

b). Let \( T \) denote the time (minutes) taken by a clerk to process a certain application form.

\[
P(4.5 < T \leq 5.5) = \frac{5.5 - 4.5}{6 - 4} = 0.5
\]

\[
P(T \geq 4.5) = 1 - P(T < 4.5) \\
= 1 - \frac{4.5 - 4}{6 - 4} \\
= 0.75
\]
c). Median = \( \frac{4 + 6}{2} = 5 \)

The median of this distribution is 5 because exactly half the area under this density sits over the interval [4, 5].

d). \( \frac{10}{100} \times (6 - 4) = 0.2 \)
\[ 4 + 0.2 = 4.2 \]

The value that separates the best 10% from the remaining 90% is 4.20.

25.a). \( f(x) = \begin{cases} 
  c(1-(x-3)^2) & 2 < x < 4 \\
  0 & \text{otherwise}
\end{cases} \)

\[ 1 = \int_{2}^{4} c \left[ 1-(x-3)^2 \right] \, dx \]

\[ 1 = c \left[ \frac{(x-3)^3}{3} \right]_{2}^{4} \]

\[ c = \frac{3}{4} \]

b). Let \( W \) denote the tracking weight of a stereo cartridge set.

\[ P \left( W > 3 \right) = 1 - F(3) \]

\[ = 1 - \int_{2}^{3} \left[ 1-(w-3)^2 \right] \, dw \]

\[ = 1 - \frac{3}{4} \left[ w - \frac{(w-3)^3}{3} \right]_{2}^{3} \]

\[ = \frac{1}{2} \]

25.3.9

25.3.9

- 2/4 -
Exercise 1.4

30.a). Let $Z$ denote the standard-normally distributed random variable.

\[
P (Z \leq 2.15) = \text{area to the left of 2.15} = 0.9842
\]

\[
P (Z < 2.15) = \text{area to the left of 2.15} = 0.9842
\]

b). \[
P (Z > 1.50) = 1 - P (Z \leq 1.50)
\]
\[
= 1 - \text{area to the left of 1.50}
\]
\[
= 1 - 0.9332
\]
\[
= 0.0668
\]

\[
P (Z > -2) = 1 - P (Z \leq -2)
\]
\[
= 1 - \text{area to the left of -2}
\]
\[
= 1 - 0.0228
\]
\[
= 0.9772
\]

c). \[
P (-1.23 < Z \leq 2.85) = P (Z \leq 2.85) - P (Z \leq -1.23)
\]
\[
= \text{area to the left of 2.85} - \text{area to the left of -1.23}
\]
\[
= 0.9978 - 0.1093
\]
\[
= 0.8885
\]

d). \[
P (Z > 5) = 1 - P (Z \leq 5)
\]
\[
= 1 - \text{area to the left of 5}
\]
\[
= 1 - 1
\]
\[
= 0
\]

\[
P (Z > -5) = 1 - P (Z \leq -5)
\]
\[
= 1 - \text{area to the left of -5}
\]
\[
= 1 - 0
\]
\[
= 1
\]

e). \[
P (|Z| < 2.50) = P (-2.50 < Z < 2.50)
\]
\[
= \text{area to the left of 2.50} - \text{area to the left of -2.50}
\]
\[
= 0.9938 - 0.0062
\]
\[
= 0.9876
\]
41.a). Let $X$ denote the number of flaws along a 100-m reel of magnetic tape.

\[ X \sim N(\mu = 25, \sigma = 5) \]

\[
P(20 \leq X \leq 40) = P\left(\frac{20 - 0.5 - 25}{5} \leq \frac{X - \mu}{\sigma} \leq \frac{40 + 0.5 - 25}{5}\right)
\]

\[
= P(-1.1 \leq Z \leq 3.1)
\]

\[
= \text{area to the left of 3.1} - \text{area to the left of -1.1}
\]

\[
= 0.9990 - 0.1357
\]

\[
= 0.8633
\]

b). \[ P(X \leq 30) = P\left(\frac{X - \mu}{\sigma} \leq \frac{30 + 0.5 - 25}{5}\right) \]

\[
= P(Z \leq 1.1)
\]

\[
= \text{area to the left of 1.1}
\]

\[
= 0.8643
\]

\[
P(X < 30) = P\left(\frac{X - \mu}{\sigma} < \frac{30 - 0.5 - 25}{5}\right)
\]

\[
= P(Z < 0.9)
\]

\[
= \text{area to the left of 0.9}
\]

\[
= 0.8159
\]