1. An animal study is being planned as a completely randomized design involving 6 treatment groups. Past studies indicate the standard deviation of a response "measurement" is no more than 7.0 units. Consider a balanced design and use 5% significance level.

   (a) What sample size per group is needed to detect a difference of 8.0 units between treatments 1 and 2 with 80% power?

   (b) What sample size is needed for the power of the overall F-test to be at least 80%, given that the researchers want to detect any pairwise difference in means of 8.0 units?

   Solutions:

   (a) Following §1.2 in notes

   ```
   2*((qnorm(.975)+qnorm(.8))/(8/7))^2
   1-pt(qt(.975,24),24,(8/7)*sqrt(13/2))+
     pt(-qt(.975,24),24,(8/7)*sqrt(13/2))
   1-pt(qt(.975,26),26,(8/7)*sqrt(14/2))+
     pt(-qt(.975,26),26,(8/7)*sqrt(14/2))
   ```

   So one needs $n = 14$ per group.

   (b) Following §2.2 in notes, for $\delta = 8$, $\phi \geq n(8/7)^2/2$.

   ```
   1-pf(qf(.95,5,13*6),5,13*6,14*(8/7)^2/2)
   wk=NULL
   for(n in 15:25)
   wk=c(wk,1-pf(qf(.95,5,(n-1)*6),5,(n-1)*6,n*(8/7)^2/2))
   wk
   ```

   So one needs $n = 21$ per group.

2. Suppose the animals used in Problem 1 are from three different suppliers, and the "supplier effect" is responsible for 60% of the "measurement" variance. Using the suppliers as blocks or using only one supplier, repeat Problem 1.

   Solutions: Repeat Problem 1 with $\sigma \leq 7\sqrt{0.4}$.

   (a) Following §1.2 in notes

   ```
   2*((qnorm(.975)+qnorm(.8))/(8/7))^2*0.4
   1-pt(qt(.975,8),8,(8/7)*sqrt(5/2/.4))+
     pt(-qt(.975,8),8,(8/7)*sqrt(5/2/.4))
   1-pt(qt(.975,10),10,(8/7)*sqrt(6/2/.4))+
     pt(-qt(.975,10),10,(8/7)*sqrt(6/2/.4))
   ```

   So one needs $n = 6$ per group.

   (b) Following §2.2 in notes, for $\delta = 8$, $\phi \geq n(8/7)^2/2/0.4$.

```
1-pf(qf(.95,5,5*6),5,5*6,6*(8/7)^2/2/.4)
wk=NULL
for(n in 7:14)
wk=c(wk,1-pf(qf(.95,5,(n-1)*6),5,(n-1)*6,n*(8/7)^2/2/.4))
wk
1-pf(qf(.95,5,46),5,46,9*(8/7)^2/2/.4)
```

So one needs $n = 9$ per group.

3. A researcher is planning a three-arm clinical trial with a time-to-event endpoint measured in days. In an earlier study involving the current standard-of-care treatment, the natural logarithm of time-to-event was approximately normal with mean 3.4 and standard deviation 0.5. The researcher thinks two new treatments might increase the average time-to-event by at least 14 days and wants to make sure he can detect this with 80% power using tests of 5% significance level. How many patients per group do you recommend him to consider?

   Solutions: $e^{3.4} = 30$, $\log 44 = 3.784$, so effect size is $\delta = (3.784 - 3.4)/0.5 = 0.768$. Despite the three arms, the purpose seems to be testing each new treatment against the standard, so we plan for one-sided two-sample test.

```
2*((qnorm(.95)+qnorm(.8))/.768)^2
1-pt(qt(.95,40),40,.768*sqrt(21/2))
1-pt(qt(.95,42),42,.768*sqrt(22/2))
```

   So one needs $n = 22$ per group.

4. An animal science researcher is planning a three-period crossover design to compare diets on milk production. Previous studies have suggested milk production to be normally distributed with a mean of 2100 liters and a standard deviation of 48 liters and that the intraclass correlation of milkings within a cow is 0.50. How many cows are needed if the researcher wants to detect differences in means of 20 liters (80% power)?

   Solutions: This is a complete block design with $a = 3$ as in §1.2 in the design notes, but $\sigma^2 = \tilde{\sigma}^2(1-\rho) = 48^2/2$ is via $\tilde{\sigma}^2 = 48^2$ and $\rho = 0.5$ as in §1.3 in the power notes. $\text{SSA}/\sigma^2 = \sum_{i,j}(\bar{Y}_{i\cdot} - \bar{Y}_{\cdot\cdot})^2/\sigma^2 = b\sum_i(\alpha_i + \bar{\epsilon}_{i\cdot} - \bar{\epsilon}_{\cdot\cdot})^2/\sigma^2$ is χ^2 with noncentrality $\phi = b\sum_i \alpha_i^2/\sigma^2 = b\sum_i(\mu_i - \mu)^2/\sigma^2$. This is similar to Problem 1, but the denominator df is $(a-1)(b-1)$ instead of $a(b-1)$.

```
2*((qnorm(.975)+qnorm(.8))/(20/(48/sqrt(2))))^2
1-pf(qf(.95,2,45*2),2,45*2,46*(20/(48/sqrt(2)))^2/2)
wk=NULL
for(n in 47:60)
wk=c(wk,1-pf(qf(.95,2,(n-1)*2),2,(n-1)*2,n*(20/(48/sqrt(2)))^2/2))
wk
```

   So one needs $n = 58$ cows.