# Seasonal ARIMA Model

Many time series data demonstrate certain periodic behavior due to seasonal effect. For example, monthly data often contain a yearly component with a periodicity of 12.

To model such data, one may superimpose ARIMA models on the two time scales, which yield ARIMA models that are of high order, albeit structured, on the finer scale.

To express models on the coarser time scale with period $s$, one may use the seasonal backward shift operator, $B^s$, and the seasonal difference operator, $\nabla_s = 1 - B^s$.

As an example, consider a seasonal ARIMA model with $s = 12$,

$$\nabla\nabla_{12} z_t = (1 - \theta B)(1 - \Theta B^{12}) a_t,$$

which is denoted as a $(0,1,1) \times (0,1,1)_{12}$ model.

# Estimation and Forecasting

Since a seasonal ARIMA model is simply a high order ARIMA model with many 0 coefficients, no new techniques are needed for estimation and forecasting.

In R, one can fit seasonal ARIMA models using `arima` in `ts`.

```
series.G<-ts(scan("airline-pass"),1949,,12)
plot(log(series.G),xlim=c(1949,1963),ylim=c(4.5,7))
fit.G<-arima(log(series.G),order=c(0,1,1),
                 seasonal=list(order=c(0,1,1),period=12))
pred.G<-predict(fit.G,3*12)
lines(pred.G$pred,col=2)
lines(pred.G$pred+pred.G$se,col=3)
lines(pred.G$pred-pred.G$se,col=5)
```

## Model Identification and Checking

Model identification is more complicated, but ACF and AIC/BIC remain the primary tools. PACF seems not as useful for identifying seasonal AR models, however.

Series G provides a “clean” example for model identification and checking, where only difference and MA are involved.

```
plot(series.G); plot(log(series.G))
acf(log(series.G),5*12); PP.test(log(series.G))
acf(diff(log(series.G)),5*12)
acf(diff(diff(log(series.G)),12),5*12)
acf(fit.G$resid); Box.test(fit.G$resid,5,,2)
```