1. BJR Exercise 5.1.

   Solution:

   (a) For (1), $\hat{z}_t(1) = 0.5z_t$, $\hat{z}_t(2) = 0.5\hat{z}_t(1) = 0.25z_t$.
   For (2), $\hat{z}_t(1) = z_t - 0.5a_t$, $\hat{z}_t(2) = \hat{z}_t(1)$.
   For (3), $\hat{z}_t(1) = 1.6z_t - 0.6z_{t-1}$, $\hat{z}_t(2) = 1.6\hat{z}_t(1) - 0.6z_t$.

   (b) For (1), $\hat{z}_t(1) = 0.5z_t$, $\hat{z}_t(2) = 0.25z_t$.
   For (2), $\hat{z}_t(1) = z_t - 0.5a_t$, $\hat{z}_t(2) = z_t - 0.5a_t$.
   For (3), $\hat{z}_t(1) = 1.6z_t - 0.6z_{t-1}$, $\hat{z}_t(2) = 1.96z_t - 0.96z_{t-1}$.

   (c) For (1) and (3), same as in (b).
   For (2), $\pi_j^{(2)} = \pi_j = (0.5)^j$, so $\hat{z}_t(2) = \hat{z}_t(1) = \sum_{j=1}^{\infty}(0.5)^j z_{t+1-j}$.

2. BJR Exercise 5.2.

   Solution:

   (a) My calculation yields $a_{100} = 3.731520$ and $a_{99} = -0.376177$, so $\hat{z}_{100}(1) = 167.7900$, $\hat{z}_{100}(l) = 168.8348$, $l > 1$.

   (b) $\hat{\sigma}(l)$ are given by (1.050238, 1.055476,1.072272, 1.088809, 1.105098, 1.121151, 1.136977, 1.152585, 1.167986, 1.183185, 1.198192, 1.213014).

3. BJR Exercise 5.4.

   Solution:

   (a) $a_{101} = z_{101} - \hat{z}_{100}(1) = 174 - 167.79 = 6.21$, $\psi_1 = -0.1$, and $\psi_l = 0.18$, $l > 1$, so $\hat{z}_{101}(1) = \hat{z}_{100}(2) + \psi_1 a_{101} = 168.8348 - 0.1(6.21) = 168.2138$, $\hat{z}_{101}(l) = \hat{z}_{100}(l + 1) + \psi_l a_{101} = 168.8348 + 0.18(6.21) = 169.9526$, $l > 1$.

   (b) $\hat{z}_{101}(1) = z_{101} - 1.1a_{101} + 0.28a_{100} = 174 - 1.1(6.21) + 0.28(3.731520) = 168.2138$, $\hat{z}_{101}(2) = 168.2138 + 0.28(6.21) = 169.9526$, $\hat{z}_{101}(l) = \hat{z}_{101}(l - 1)$, $l > 2$.

4. BJR Exercise 5.5.

   Solution:

   (a) $e_t(1) = a_{t+1}$, $e_t(2) = a_{t+2} - 0.1a_{t+1}$, $e_t(l) = a_{t+l} - 0.1a_{t+l-1} + 0.18(a_{t+l-2} + \cdots + a_{t+1})$, $l > 2$.

   (b) $e_t(3) = a_{t+3} - 0.1a_{t+2} + 0.18a_{t+1}$, an MA(2) process.

$$
\begin{aligned}
\rho_1 &= (-0.1 - 0.018)/(1 + 0.1^2 + 0.18^2) = -0.1132003, \\
\rho_2 &= 0.18/(1 + 0.1^2 + 0.18^2) = 0.1726784, \\
\rho_k &= 0, \quad k > 2.
\end{aligned}
$$

(c)

$$\begin{aligned}
&\operatorname{corr}(e_t(2), e_t(1)) = -0.1/\sqrt{1+0.1^2} = -0.0995037,\\
&\operatorname{corr}(e_t(2), e_t(2)) = 1,\\
&\operatorname{corr}(e_t(2), e_t(3)) = (-0.1-0.018)/\sqrt{(1+0.1^2)(1+0.1^2+0.18^2)} = -0.1150017,\\
&\operatorname{corr}(e_t(2), e_t(4)) = 0.9(0.18)/\sqrt{(1+0.1^2)(1+0.1^2+2(0.18)^2)} = 0.1554857,\\
&\operatorname{corr}(e_t(2), e_t(5)) = 0.9(0.18)/\sqrt{(1+0.1^2)(1+0.1^2+3(0.18)^2)} = 0.1531938,\\
&\operatorname{corr}(e_t(2), e_t(6)) = 0.9(0.18)/\sqrt{(1+0.1^2)(1+0.1^2+4(0.18)^2)} = 0.1510004.
\end{aligned}$$

5. BJR Exercise 5.7.

Solution:

(a)

$$\begin{aligned}
\hat{z}_{48}(1) &= z_{48} - a_{48} + 0.5a_{47} + 0.5 = 130 - 0.2 + 0.5(-0.3) + 0.5 = 130.15,\\
\hat{z}_{48}(2) &= \hat{z}_{48}(1) + 0.5a_{48} + 0.5 = 130.15 + 0.5(0.2) + 0.5 = 130.75,\\
\hat{z}_{48}(l) &= \hat{z}_{48}(l-1) + 0.5 = \hat{z}_{48}(2) + 0.5(l-2) = 130.75 + 0.5(l-2), \quad l > 2.
\end{aligned}$$

(b) $e_t(l)$ only involves the stochastic part. Since $\psi_1 = 0$, $\psi_l = 0.5$, $l > 1$, one has $\sigma(1) = \sigma(2) = 0.2$, $\sigma(l) = 0.2\sqrt{1+(l-2)(0.25)}$, $l > 2$.

(c) $z_t = \sum_{j=0}^{t-49} \psi_j a_{t-j} + C_{48}(t-48) + 0.5(t-48)$, $t > 48$, $\hat{z}_{48}(l) = C_{48}(l) + 0.5l$, where $C_{48}(1) = z_{48} - a_{48} + 0.5a_{47}$, $C_{48}(2) = z_{48} - 0.5a_{48} + 0.5a_{47}$, and $C_{48}(l) = C_{48}(l-1)$, $l > 2$.