1. Consider the MA(2) process $z_t = a_t - a_{t-1} + \frac{2}{9}a_{t-2}$, $\text{var}[a_t] = \sigma_a^2 = 3$.
   (a) Find the variance and autocorrelation function of $\{z_t\}$.
   (b) Determine if the process is invertible, and if so express the process in the infinite autoregressive form, $z_t = \sum_{j=1}^{\infty} \pi_j z_{t-j} + a_t$, providing numerical values for $\pi_1, \ldots, \pi_5$.
   (c) Show that $\pi_j - \theta_1\pi_{j-1} - \theta_2\pi_{j-2} = 0$ for $j > 0$, where $\pi_{-1} = 0$. Find a general expression for π_j.

2. Consider the AR(2) process $z_t = 1.6z_{t-1} - .89z_{t-2} + a_t$, $\sigma_a^2 = 2$.
   (a) Verify that $\{z_t\}$ is stationary, and find the variance γ_0.
   (b) Solve the first 2 Yule-Walker equations for ρ_1 and ρ_2, and then compute ρ_k for $k = 3, \ldots, 8$ using the recursive equation $\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2}$.
   (c) Give a general formula for ρ_k as the solution of the linear difference equation, $\rho_k - 1.6\rho_{k-1} + .89\rho_{k-2} = 0$, $k > 0$. Do express your result in terms of real numbers and real functions.
   (d) Consider the infinite moving average form of the process, $z_t = a_t + \sum_{j=1}^{\infty} \psi_j a_{t-j}$. Show that $\psi_j - 1.6\psi_{j-1} + .89\psi_{j-2} = 0$, $j > 0$, where $\psi_{-1} = 0$, and find a general expression for ψ_j.

3. Assume that the stationary process $\{z_t\}$ is generated by a low order AR model with $p \leq 4$, and that $\{z_t\}$ has the following variance and autocorrelation function,

$$\gamma_0 = 12, \quad \rho_1 = .8, \quad \rho_2 = .46, \quad \rho_3 = .152, \quad \rho_4 = -.0476.$$

   (a) For each of the possible orders 1, 2, 3, solve the first sets of Yule-Walker equations to obtain the corresponding ϕ_j's and σ_a^2.
   (b) What is the actual AR order p of this process?

4. Suppose a sequence of real numbers $\{x_t\}_0^{\infty}$ satisfy the linear difference equation,

$$x_t + ax_{t-1} + bx_{t-2} = 0, \quad t > 1.$$

   (a) Find the coefficients a, b, such that x_t is sinusoidal with period 12,

$$x_t = a_1 \cos(2\pi t/12) + a_2 \sin(2\pi t/12).$$

   (b) Find the coefficients a, b, such that x_t is damped sinusoidal with damping factor .8 and period 12,

$$x_t = (.8)^t\{a_1 \cos(2\pi t/12) + a_2 \sin(2\pi t/12)\}.$$

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