1. For an AR(1) process $z_t = \phi z_{t-1} + a_t$, $|\phi| < 1$, its spectral density can be shown to be $f(\omega) = (\sigma_2^2/\gamma_0)|1 - \phi e^{-i2\pi\omega}|^{-2}$. Verify that $f(\omega) = (1 - \phi^2)/(1 - 2\phi \cos 2\pi\omega + \phi^2)$.

2. For an MA(1) process $z_t = a_t - \theta a_{t-1}$, its spectral density can be shown to be $f(\omega) = (\sigma_2^2/\gamma_0)|1 - \theta e^{-i2\pi\omega}|^2$. Verify that $f(\omega) = 1 - 2\theta \cos 2\pi\omega/(1 + \theta^2)$.

3. Let $\{y_t\}$ be a stationary process with the power spectrum $p_y(\omega) = \sum_{k=-\infty}^{\infty} \gamma_y(k)e^{-i2\pi k\omega}$. Define $z_t = y_t - y_{t-1}$. Obtain the power spectrum of z_t in terms of $p_y(\omega)$.

4. Let $\{y_t\}_{-\infty}^{\infty}$ and $\{z_t\}_{-\infty}^{\infty}$ be two stationary processes, independent of each other, with power spectrums $p_y(\omega)$ and $p_z(\omega)$. Find the power spectrum of the stationary process $w_t = ay_t + bz_t$ in terms of $p_y(\omega)$ and $p_z(\omega)$, where a and b are constants.

5. Let a_i, b_i be independent r.v.'s with $E[a_i] = E[b_i] = 0$ and $\text{var}[a_i] = \text{var}[b_i] = \sigma_i^2$. Find the spectral distribution of the stationary process $z_t = \sum_{i=1}^{m}(a_i \cos 2\pi\omega_i t + b_i \sin 2\pi\omega_i t)$. Note that the spectral density does not exist since the spectral distribution is discrete.