1. Let $\{a_t\}_{-\infty}^{\infty}$ be a white noise process with $E[a_t] = 0$ and $\text{var}[a_t] = \sigma_a^2$. Define $z_t = \mu + a_t - .5a_{t-1}$. Find the mean, autocovariance, and autocorrelation of z_t, and verify that $\{z_t\}_{-\infty}^{\infty}$ is stationary.

2. Let $\{y_t\}$ be a stationary process with mean μ_y and autocovariance $\gamma_y(s) = \text{cov}[y_t, y_{t-s}]$. Define $z_t = y_t - y_{t-1}$. Obtain the mean and autocovariance of $\{z_t\}_{-\infty}^{\infty}$ in terms of those of y_t and verify that it is stationary.

3. Let $\{y_t\}_{-\infty}^{\infty}$ and $\{z_t\}_{-\infty}^{\infty}$ be two stationary processes with means μ_y and μ_z and autocovariances $\gamma_y(s)$ and $\gamma_z(s)$, independent of each other. Find the mean and autocovariance of $w_t = ay_t + bz_t$, where a and b are constants, and show that $\{w_t\}_{-\infty}^{\infty}$ is stationary.

4. Let a_i, b_i be independent r.v.'s with $E[a_i] = E[b_i] = 0$ and $\text{var}[a_i] = \text{var}[b_i] = \sigma_i^2$. Compute the mean and autocovariance of $z_t = \sum_{i=1}^{m}(a_i \cos 2\pi\omega_i t + b_i \sin 2\pi\omega_i t)$ and show that it is stationary. [Hint: you may want to use the trigonometric identity $\cos x \cos y + \sin x \sin y = \cos(x - y)$.]

5. Let $\{a_t\}_1^{\infty}$ be a white noise process with mean 0 and variance σ_a^2. Define $z_0 = 0$, $z_t = \phi z_{t-1} + a_t$, $t = 1, 2, \ldots$, where $|\phi| < 1$.
   (a) Express z_t explicitly in terms of a_t.
   (b) Calculate the autocovariance $\text{cov}[z_t, z_{t+s}]$ for $s > 0$, and show that for large t, z_t is approximately stationary.

6. Observing $z_1, \ldots, z_N$ from a stationary process with autocovariance γ_k and autocorrelation $\rho_k = \gamma_k/\gamma_0$. It is known that $\text{var}[\bar{z}] = (\gamma_0/N)[1 + 2\sum_{k=1}^{N-1}(1 - k/N)\rho_k]$.
   (a) If $\rho_k \to 0$ as $k \to \infty$, show that $\text{var}[\bar{z}] \to 0$ as $N \to \infty$.
   (b) Compare $\text{var}[\bar{z}]$ with the following autocorrelations: (i) $\rho_k = 0$, $k \neq 0$; (ii) $\rho_1 = .8$, $\rho_2 = .55$, $\rho_k = 0$, $k > 2$.

7. Problem 2.1 in the text (p. 569 in 3rd ed; p.701 in 4th ed.), plus
   (d) After inspecting the graphs in (a)-(c), do you think the series is stationary?
   (e) Calculate and plot the sample ACF for lags up to 6.
   (f) Assume $\rho_k = 0$, $k > 2$. Obtain approximate standard errors for r_1, r_2, and r_k, $k > 2$.
   (g) Assume $\rho_k = 0$, $k > 2$. Obtain approximate correlation between r_4 and r_5.

Due January 22, 2024