1. Consider the model $(1 - .5B)(1 - B)z_t = 2 + (1 + .5B)a_t$ with $\sigma_a^2 = .25$.

(12 pts.) (a) Given $z_{85} = 5$, $z_{84} = 2$, $a_{85} = -1$, $a_{84} = .5$, obtain $\hat{z}_{85}(l)$, $l = 1, 2, 3$.

(8 pts.) (b) Find the ψ weights ψ_j, $j = 1, 2, 3$. State the recursive formula one may use to calculate ψ_j for $j > 3$.

(8 pts.) (c) Calculate the variances of the forecasting errors $e_{85}(l) = z_{85+l} - \hat{z}_{85}(l)$, $l = 1, 2, 3$, and construct 95% prediction intervals for z_{86}, z_{87}, z_{88}.

(8 pts.) (d) State the difference equation satisfied by the "eventual" forecasting function, and give an expression of the general form of the forecasting function $\hat{z}_t(l)$ with all deterministic constants specified.

2. Consider $z_t = x_t + b_t$, where $(1 - B^4)x_t = a_t$, and a_t, b_t are independent white noise with variances σ_a^2, σ_b^2.

(12 pts.) (a) Show that z_t is a seasonal ARIMA model. Find its orders and derive the relations between σ_a^2, σ_b^2 and the parameters of the seasonal ARIMA model.

(10 pts.) (b) Based on the form of the seasonal ARIMA model, give an expression for the forecasting function $\hat{z}_t(l)$; specify all deterministic constants.
[Hint: the roots of $1 - B^4$ are ± 1, $\pm i$.]

3. Let $x_t = \alpha \sin(\pi t/3) + \beta \cos(\pi t/3) + n_t$, where n_t satisfies $n_t = \phi n_{t-1} + a_t - \theta a_{t-1}$ with $|\phi| < 1$ and a_t a white noise, $E[a_t] = \sigma_a^2$.

(12 pts.) (a) Passing x_t through a linear filter with transfer function $v(B) = 1 - B + B^2$, one has $z_t = x_t - x_{t-1} + x_{t-2}$. Show that z_t is a stationary ARMA process and specify its orders.
[Hint: $1 - B + B^2 = (1 - e^{i\pi/3}B)(1 - e^{-i\pi/3}B)$, and $\alpha \sin(\pi t/3) + \beta \cos(\pi t/3) = A(e^{i(\pi t/3+\delta)} + e^{-i(\pi t/3+\delta)})$ for some A, δ depending on α, β.]

(8 pts.) (b) Give the power spectra of n_t and z_t.

4. A time series of $n = 100$ observations gave the following sample ACF,

Lag k	1	2	3	4	5
r_k	.75	.44	.16	$-.05$	$-.16$

with sample mean $\bar{z} = 12.0$ and sample variance $c_0 = 9.0$.

(10 pts.) (a) Consider a AR(2) model $(1 - \phi_1 B - \phi_2 B^2)(z_t - \mu) = a_t$. Obtain the Yule-Walker estimates $\hat{\phi}_1$ and $\hat{\phi}_2$. Also obtain estimates for μ and σ_a^2.

(12 pts.) (b) Consider a ARMA(1,1) model $(1 - \phi B)(z_t - \mu) = a_t - \theta a_{t-1}$. Find the moment estimates of ϕ, and specify an equation that the moment estimate of θ satisfies.