

An experiment to study the relationship between the time spent exercising ( $x$ , in minutes) and the amount of oxygen consumed during the exercise period ( $y$ , in  $cm^3$ ) resulted in the following summary statistics.

$$\begin{array}{lll} n = 20 & \sum_i x_i = 50 & \sum_i y_i = 16705 \\ \sum_i (x_i - \bar{x})^2 = 25 & \sum_i (y_i - \bar{y})^2 = 241379.8 & \sum_i (x_i - \bar{x})(y_i - \bar{y}) = 2431.5 \end{array}$$

Assume that a simple linear regression model  $y = \beta_0 + \beta_1 x + \epsilon$  is adequate.

1. Compute the least squares estimates  $\hat{\beta}_0 = b_0$ ,  $\hat{\beta}_1 = b_1$ , and the variance estimate  $\hat{\sigma}^2 = s^2$ .

*Solution:*  $b_1 = 2431.5/25 = 97.26$ ,  $b_0 = 16705/20 - 97.26(50)/20 = 592.1$ ,  $SSE = 241379.8 - (2431.5)^2/25 = 4892.11$ ,  $s^2 = 4892.11/18 = 271.78$ .

2. Construct a 95% confidence interval for the slope  $\beta_1$ .

*Solution:*  $97.26 \pm 2.101\sqrt{271.78/25}$ , or  $(90.33, 104.19)$ .

3. You are about to spend 3 minutes on a weight machine. Predict your oxygen consumption during the 3 minute period using a 95% prediction interval.

*Solution:*  $\hat{y} = 592.1 + 97.26(3) = 883.88$ ,  $\bar{x} = 50/20 = 2.5$ . A 95% PI is  $883.88 \pm 2.101\sqrt{271.78(1/20 + (3 - 2.5)^2/25 + 1)}$ , or  $(848.22, 919.54)$ .