each brand tested. The sums of squares were computed as SSE $= 4773.3$ and SSTr $= 591.2$. State the hypotheses of interest (including word definitions of parameters), and use the F test of ANOVA ($\alpha = .05$) to decide whether there are any differences in true average lumen outputs among the three brands for this type of bulb by obtaining as much information as possible about the P-value.

4. It is common practice in many countries to destroy (shred) refrigerators at the end of their useful lives. In this process material from insulating foam may be released into the atmosphere. The article "Release of Fluorocarbons from Insulation Foam in Home Appliances during Shredding" (*J. of the Air and Waste Mgmt. Assoc.,* 2007: 1452–1460) gave the following data on foam density (g/L) for each of two refrigerators produced by four different manufacturers:

1. 30.4, 29.2	2. 27.7, 27.1
3. 27.1, 24.8	4. 25.5, 28.8

Does it appear that true average foam density is not the same for all these manufacturers? Carry out an appropriate test of hypotheses by obtaining as much P-value information as possible, and summarize your analysis in an ANOVA table.

5. Consider the following summary data on the modulus of elasticity ($\times\ 10^6$ psi) for lumber of three different grades [in close agreement with values in the article "Bending Strength and Stiffness of Second-Growth Douglas-Fir Dimension Lumber" (*Forest Products J.,* 1991: 35–43), except that the sample sizes there were larger]:

Grade	J	$\bar{x}_{i\cdot}$	s_i
1	10	1.63	.27
2	10	1.56	.24
3	10	1.42	.26

Use this data and a significance level of .01 to test the null hypothesis of no difference in mean modulus of elasticity for the three grades.

6. The article "Origin of Precambrian Iron Formations" (*Econ. Geology,* 1964: 1025–1057) reports the following data on total Fe for four types of iron formation (1 = carbonate, 2 = silicate, 3 = magnetite, 4 = hematite).

1:	20.5	28.1	27.8	27.0	28.0
	25.2	25.3	27.1	20.5	31.3
2:	26.3	24.0	26.2	20.2	23.7
	34.0	17.1	26.8	23.7	24.9
3:	29.5	34.0	27.5	29.4	27.9
	26.2	29.9	29.5	30.0	35.6
4:	36.5	44.2	34.1	30.3	31.4
	33.1	34.1	32.9	36.3	25.5

Carry out an analysis of variance F test at significance level .01, and summarize the results in an ANOVA table.

7. An experiment was carried out to compare electrical resistivity for six different low-permeability concrete bridge deck mixtures. There were 26 measurements on concrete cylinders for each mixture; these were obtained 28 days after casting. The entries in the accompanying ANOVA table are based on information in the article "In-Place Resistivity of Bridge Deck Concrete Mixtures" (*ACI Materials J.,* 2009: 114–122). Fill in the remaining entries and test appropriate hypotheses.

Source	df	Sum of Squares	Mean Square	f
Mixture				
Error			13.929	
Total		5664.415		

8. A study of the properties of metal plate-connected trusses used for roof support ("Modeling Joints Made with Light-Gauge Metal Connector Plates," *Forest Products J.,* 1979: 39–44) yielded the following observations on axial-stiffness index (kips/in.) for plate lengths 4, 6, 8, 10, and 12 in:

4:	309.2	409.5	311.0	326.5	316.8	349.8	309.7
6:	402.1	347.2	361.0	404.5	331.0	348.9	381.7
8:	392.4	366.2	351.0	357.1	409.9	367.3	382.0
10:	346.7	452.9	461.4	433.1	410.6	384.2	362.6
12:	407.4	441.8	419.9	410.7	473.4	441.2	465.8

Does variation in plate length have any effect on true average axial stiffness? State and test the relevant hypotheses using analysis of variance with $\alpha = .01$. Display your results in an ANOVA table. [*Hint*: $\sum\sum x_{ij}^2 = 5{,}241{,}420.79$.]

9. Six samples of each of four types of cereal grain grown in a certain region were analyzed to determine thiamin content, resulting in the following data (μg/g):

Wheat	5.2	4.5	6.0	6.1	6.7	5.8
Barley	6.5	8.0	6.1	7.5	5.9	5.6
Maize	5.8	4.7	6.4	4.9	6.0	5.2
Oats	8.3	6.1	7.8	7.0	5.5	7.2

Does this data suggest that at least two of the grains differ with respect to true average thiamin content? Use a level $\alpha = .05$ test based on the P-value method.

10. In single-factor ANOVA with I treatments and J observations per treatment, let $\mu = (1/I)\sum \mu_i$.

a. Express $E(\bar{X}_{\cdot\cdot})$ in terms of μ. [*Hint:* $\bar{X}_{\cdot\cdot} = (1/I)\sum \bar{X}_{i\cdot}$]

b. Determine $E(\bar{X}_{i\cdot}^2)$. [*Hint:* For any rv Y, $E(Y^2) = V(Y) + [E(Y)]^2$.]

c. Determine $E(\bar{X}_{\cdot\cdot}^2)$.

d. Determine E(SSTr) and then show that

$$E(\text{MSTr}) = \sigma^2 + \frac{J}{I-1}\sum(\mu_i - \mu)^2$$

e. Using the result of part (d), what is E(MSTr) when H_0 is true? When H_0 is false, how does E(MSTr) compare to σ^2?

25. Lipids provide much of the dietary energy in the bodies of infants and young children. There is a growing interest in the quality of the dietary lipid supply during infancy as a major determinant of growth, visual and neural development, and long-term health. The article "Essential Fat Requirements of Preterm Infants" (*Amer. J. of Clinical Nutrition,* 2000: 245S–250S) reported the following data on total polyunsaturated fats (%) for infants who were randomized to four different feeding regimens: breast milk, corn-oil-based formula, soy-oil-based formula, or soy-and-marine-oil-based formula:

Regimen	Sample Size	Sample Mean	Sample SD
Breast milk	8	43.0	1.5
CO	13	42.4	1.3
SO	17	43.1	1.2
SMO	14	43.5	1.2

a. What assumptions must be made about the four total polyunsaturated fat distributions before carrying out a single-factor ANOVA to decide whether there are any differences in true average fat content?

b. Carry out the test suggested in part (a). What can be said about the *P*-value?

26. Samples of six different brands of diet/imitation margarine were analyzed to determine the level of physiologically active polyunsaturated fatty acids (PAPFUA, in percentages), resulting in the following data:

Imperial	14.1	13.6	14.4	14.3	
Parkay	12.8	12.5	13.4	13.0	12.3
Blue Bonnet	13.5	13.4	14.1	14.3	
Chiffon	13.2	12.7	12.6	13.9	
Mazola	16.8	17.2	16.4	17.3	18.0
Fleischmann's	18.1	17.2	18.7	18.4	

(The preceding numbers are fictitious, but the sample means agree with data reported in the January 1975 issue of *Consumer Reports.*)

a. Use ANOVA to test for differences among the true average PAPFUA percentages for the different brands.

b. Compute CIs for all $(\mu_i - \mu_j)$'s.

c. Mazola and Fleischmann's are corn-based, whereas the others are soybean-based. Compute a CI for

$$\frac{(\mu_1 + \mu_2 + \mu_3 + \mu_4)}{4} - \frac{(\mu_5 + \mu_6)}{2}$$

[*Hint:* Modify the expression for $V(\hat{\theta})$ that led to (10.5) in the previous section.]

27. Although tea is the world's most widely consumed beverage after water, little is known about its nutritional value. Folacin is the only B vitamin present in any significant amount in tea, and recent advances in assay methods have made accurate determination of folacin content feasible. Consider the accompanying data on folacin content for randomly selected specimens of the four leading brands of green tea.

1:	7.9	6.2	6.6	8.6	8.9	10.1	9.6
2:	5.7	7.5	9.8	6.1	8.4		
3:	6.8	7.5	5.0	7.4	5.3	6.1	
4:	6.4	7.1	7.9	4.5	5.0	4.0	

(Data is based on "Folacin Content of Tea," *J. of the Amer. Dietetic Assoc.*, 1983: 627–632.) Does this data suggest that true average folacin content is the same for all brands?

a. Carry out a test using $\alpha = .05$ via the *P*-value method.

b. Assess the plausibility of any assumptions required for your analysis in part (a).

c. Perform a multiple comparisons analysis to identify significant differences among brands.

28. For a single-factor ANOVA with sample sizes J_i $(i = 1, 2, \ldots I)$, show that $\text{SSTr} = \sum J_i(\bar{X}_{i.} - \bar{X}_{..})^2 = \sum J_i \bar{X}_{i.}^2 - n\bar{X}_{..}^2$ where $n = \sum J_i$.

29. When sample sizes are equal ($J_i = J$), the parameters $\alpha_1, \alpha_2, \ldots \alpha_I$ of the alternative parameterization are restricted by $\Sigma\alpha_i = 0$. For unequal sample sizes, the most natural restriction is $\sum J_i \alpha_i = 0$. Use this to show that

$$E(\text{MSTr}) = \sigma^2 + \frac{1}{I-1}\sum J_i \alpha_i^2$$

What is $E(\text{MSTr})$ when H_0 is true? [This expectation is correct if $\sum J_i \alpha_i = 0$ is replaced by the restriction $\sum \alpha_i = 0$ (or any other single linear restriction on the α_i's used to reduce the model to I independent parameters), but $\sum J_i \alpha_i = 0$ simplifies the algebra and yields natural estimates for the model parameters (in particular, $\hat{\alpha}_i = \bar{x}_{i.} - \bar{x}_{..}$).]

30. Reconsider Example 10.8 involving an investigation of the effects of different heat treatments on the yield point of steel ingots.

a. If $J = 8$ and $\sigma = 1$, what is β for a level .05 F test when $\mu_1 = \mu_2$, $\mu_3 = \mu_1 - 1$, and $\mu_4 = \mu_1 + 1$?

b. For the alternative of part (a), what value of J is necessary to obtain $\beta = .05$?

c. If there are $I = 5$ heat treatments, $J = 10$, and $\sigma = 1$, what is β for the level .05 F test when four of the μ_i's are equal and the fifth differs by 1 from the other four?

31. When sample sizes are not equal, the noncentrality parameter is $\sum J_i \alpha_i^2/\sigma^2$ and $\phi^2 = (1/I)\sum J_i \alpha_i^2/\sigma^2$. Referring to Exercise 22, what is the power of the test when $\mu_2 = \mu_3$, $\mu_1 = \mu_2 - \sigma$, and $\mu_4 = \mu_2 + \sigma$?

32. In an experiment to compare the quality of four different brands of magnetic recording tape, five 2400-ft reels of each brand (A–D) were selected and the number of flaws in each reel was determined.

A:	10	5	12	14	8
B:	14	12	17	9	8
C:	13	18	10	15	18
D:	17	16	12	22	14

It is believed that the number of flaws has approximately a Poisson distribution for each brand. Analyze the data at level .01 to see whether the expected number of flaws per reel is the same for each brand.

33. Suppose that X_{ij} is a binomial variable with parameters n and p_i (so approximately normal when $np_i \geq 10$ and $nq_i \geq 10$). Then since $\mu_i = np_i$, $V(X_{ij}) = \sigma_i^2 = np_i(1 - p_i) = \mu_i(1 - \mu_i/n)$. How should the X_{ij}'s be transformed so as to stabilize the variance? [*Hint:* $g(\mu_i) = \mu_i(1 - \mu_i/n)$.]

34. Simplify $E(\text{MSTr})$ for the random effects model when $J_1 = J_2 = \cdots = J_I = J$.

SUPPLEMENTARY EXERCISES (35–46)

35. An experiment was carried out to compare flow rates for four different types of nozzle.

a. Sample sizes were 5, 6, 7, and 6, respectively, and calculations gave $f = 3.68$. State and test the relevant hypotheses using $\alpha = .01$

b. Analysis of the data using a statistical computer package yielded P-value $= .029$. At level .01, what would you conclude, and why?

36. The article "Computer-Assisted Instruction Augmented with Planned Teacher/Student Contacts" (*J. of Exp. Educ.*, Winter, 1980–1981: 120–126) compared five different methods for teaching descriptive statistics. The five methods were traditional lecture and discussion (L/D), programmed textbook instruction (R), programmed text with lectures (R/L), computer instruction (C), and computer instruction with lectures (C/L). Forty-five students were randomly assigned, 9 to each method. After completing the course, the students took a 1-hour exam. In addition, a 10-minute retention test was administered 6 weeks later. Summary quantities are given.

	Exam		Retention Test	
Method	$\bar{x}_{i.}$	s_i	$\bar{x}_{i.}$	s_i
L/D	29.3	4.99	30.20	3.82
R	28.0	5.33	28.80	5.26
R/L	30.2	3.33	26.20	4.66
C	32.4	2.94	31.10	4.91
C/L	34.2	2.74	30.20	3.53

The grand mean for the exam was 30.82, and the grand mean for the retention test was 29.30.

a. Does the data suggest that there is a difference among the five teaching methods with respect to true mean exam score? Use $\alpha = .05$.

b. Using a .05 significance level, test the null hypothesis of no difference among the true mean retention test scores for the five different teaching methods.

37. Numerous factors contribute to the smooth running of an electric motor ("Increasing Market Share Through Improved Product and Process Design: An Experimental Approach," *Quality Engineering*, 1991: 361–369). In particular, it is desirable to keep motor noise and vibration to a minimum. To study the effect that the brand of bearing has on motor vibration, five different motor bearing brands were examined by installing each type of bearing on different random samples of six motors. The amount of motor vibration (measured in microns) was recorded when each of the 30 motors was running. The data for this study follows. State and test the relevant hypotheses at significance level .05, and then carry out a multiple comparisons analysis if appropriate.

							Mean
1:	13.1	15.0	14.0	14.4	14.0	11.6	13.68
2:	16.3	15.7	17.2	14.9	14.4	17.2	15.95
3:	13.7	13.9	12.4	13.8	14.9	13.3	13.67
4:	15.7	13.7	14.4	16.0	13.9	14.7	14.73
5:	13.5	13.4	13.2	12.7	13.4	12.3	13.08

38. An article in the British scientific journal *Nature* ("Sucrose Induction of Hepatic Hyperplasia in the Rat," August 25, 1972: 461) reports on an experiment in which each of five groups consisting of six rats was put on a diet with a different carbohydrate. At the conclusion of the experiment, the DNA content of the liver of each rat was determined (mg/g liver), with the following results:

Carbohydrate	$\bar{x}_{i.}$
Starch	2.58
Sucrose	2.63
Fructose	2.13
Glucose	2.41
Maltose	2.49

Assuming also that $\sum\sum x_{ij}^2 = 183.4$, does the data indicate that true average DNA content is affected by the type of carbohydrate in the diet? Construct an ANOVA table and use a .05 level of significance.

39. Referring to Exercise 38, construct a t CI for

$$\theta = \mu_1 - (\mu_2 + \mu_3 + \mu_4 + \mu_5)/4$$

which measures the difference between the average DNA content for the starch diet and the combined average for the four other diets. Does the resulting interval include zero?

40. Refer to Exercise 38. What is β for the test when true average DNA content is identical for three of the diets and falls below this common value by 1 standard deviation (σ) for the other two diets?

41. Four laboratories (1–4) are randomly selected from a large population, and each is asked to make three determinations