Sometimes A and B have no outcomes in common, so that the intersection of A and B contains no outcomes.

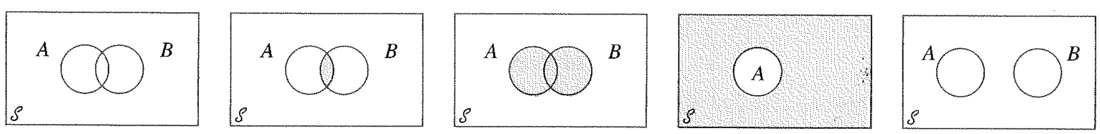
DEFINITION

Let $\varnothing$ denote the *null event* (the event consisting of no outcomes whatsoever). When $A \cap B = \varnothing$, A and B are said to be **mutually exclusive** or **disjoint** events.

Example 2.10 A small city has three automobile dealerships: a GM dealer selling Chevrolets and Buicks; a Ford dealer selling Fords and Lincolns; and a Toyota dealer. If an experiment consists of observing the brand of the next car sold, then the events $A = \{\text{Chevrolet, Buick}\}$ and $B = \{\text{Ford, Lincoln}\}$ are mutually exclusive because the next car sold cannot be both a GM product and a Ford product (at least until the two companies merge!). ■

The operations of union and intersection can be extended to more than two events. For any three events A, B, and C, the event $A \cup B \cup C$ is the set of outcomes contained in at least one of the three events, whereas $A \cap B \cap C$ is the set of outcomes contained in all three events. Given events $A_1, A_2, A_3, \ldots$, these events are said to be mutually exclusive (or pairwise disjoint) if no two events have any outcomes in common.

A pictorial representation of events and manipulations with events is obtained by using Venn diagrams. To construct a Venn diagram, draw a rectangle whose interior will represent the sample space $\mathcal{S}$. Then any event A is represented as the interior of a closed curve (often a circle) contained in $\mathcal{S}$. Figure 2.1 shows examples of Venn diagrams.



(a) Venn diagram of events A and B (b) Shaded region is $A \cap B$ (c) Shaded region is $A \cup B$ (d) Shaded region is A' (e) Mutually exclusive events

Figure 2.1 Venn diagrams

EXERCISES Section 2.1 (1–10)

1. Four universities—1, 2, 3, and 4—are participating in a holiday basketball tournament. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will play for the championship, and the two losers will also play. One possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in first-round games, and then 1 beats 3 and 2 beats 4).

a. List all outcomes in $\mathcal{S}$.

b. Let A denote the event that 1 wins the tournament. List outcomes in A.

c. Let B denote the event that 2 gets into the championship game. List outcomes in B.

d. What are the outcomes in $A \cup B$ and in $A \cap B$? What are the outcomes in A'?

2. Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.

a. List all outcomes in the event A that all three vehicles go in the same direction.

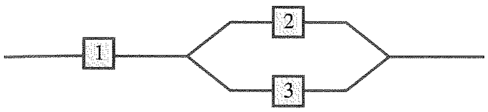
b. List all outcomes in the event B that all three vehicles take different directions.

c. List all outcomes in the event C that exactly two of the three vehicles turn right.

d. List all outcomes in the event D that exactly two vehicles go in the same direction.

e. List outcomes in D', $C \cup D$, and $C \cap D$.

3. Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2–3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2–3 subsystem.



The experiment consists of determining the condition of each component [S (success) for a functioning component and F (failure) for a nonfunctioning component].

a. Which outcomes are contained in the event A that exactly two out of the three components function?
b. Which outcomes are contained in the event B that at least two of the components function?
c. Which outcomes are contained in the event C that the system functions?
d. List outcomes in C', $A \cup C$, $A \cap C$, $B \cup C$, and $B \cap C$.

4. Each of a sample of four home mortgages is classified as fixed rate (F) or variable rate (V).
a. What are the 16 outcomes in $\mathcal{S}$?
b. Which outcomes are in the event that exactly three of the selected mortgages are fixed rate?
c. Which outcomes are in the event that all four mortgages are of the same type?
d. Which outcomes are in the event that at most one of the four is a variable-rate mortgage?
e. What is the union of the events in parts (c) and (d), and what is the intersection of these two events?
f. What are the union and intersection of the two events in parts (b) and (c)?

5. A family consisting of three persons—A, B, and C—goes to a medical clinic that always has a doctor at each of stations 1, 2, and 3. During a certain week, each member of the family visits the clinic once and is assigned at random to a station. The experiment consists of recording the station number for each member. One outcome is (1, 2, 1) for A to station 1, B to station 2, and C to station 1.
a. List the 27 outcomes in the sample space.
b. List all outcomes in the event that all three members go to the same station.
c. List all outcomes in the event that all members go to different stations.
d. List all outcomes in the event that no one goes to station 2.

6. A college library has five copies of a certain text on reserve. Two copies (1 and 2) are first printings, and the other three (3, 4, and 5) are second printings. A student examines these books in random order, stopping only when a second printing has been selected. One possible outcome is 5, and another is 213.
a. List the outcomes in $\mathcal{S}$.
b. Let A denote the event that exactly one book must be examined. What outcomes are in A?
c. Let B be the event that book 5 is the one selected. What outcomes are in B?
d. Let C be the event that book 1 is not examined. What outcomes are in C?

7. An academic department has just completed voting by secret ballot for a department head. The ballot box contains four slips with votes for candidate A and three slips with votes for candidate B. Suppose these slips are removed from the box one by one.
a. List all possible outcomes.
b. Suppose a running tally is kept as slips are removed. For what outcomes does A remain ahead of B throughout the tally?

8. An engineering construction firm is currently working on power plants at three different sites. Let A_i denote the event that the plant at site i is completed by the contract date. Use the operations of union, intersection, and complementation to describe each of the following events in terms of A_1, A_2, and A_3, draw a Venn diagram, and shade the region corresponding to each one.
a. At least one plant is completed by the contract date.
b. All plants are completed by the contract date.
c. Only the plant at site 1 is completed by the contract date.
d. Exactly one plant is completed by the contract date.
e. Either the plant at site 1 or both of the other two plants are completed by the contract date.

9. Use Venn diagrams to verify the following two relationships for any events A and B (these are called De Morgan's laws):
a. $(A \cup B)' = A' \cap B'$
b. $(A \cap B)' = A' \cup B'$

[*Hint:* In each part, draw a diagram corresponding to the left side and another corresponding to the right side.]

10. **a.** In Example 2.10, identify three events that are mutually exclusive.
b. Suppose there is no outcome common to all three of the events A, B, and C. Are these three events necessarily mutually exclusive? If your answer is yes, explain why; if your answer is no, give a counterexample using the experiment of Example 2.10.

2.2 Axioms, Interpretations, and Properties of Probability

Given an experiment and a sample space $\mathcal{S}$, *the objective of probability is to assign to each event A a number P(A), called the probability of the event A, which will give a precise measure of the chance that A will occur.* To ensure that the probability

Now consider an event A, with $N(A)$ denoting the number of outcomes contained in A. Then

$$P(A) = \sum_{E_i \text{ in } A} P(E_i) = \sum_{E_i \text{ in } A} \frac{1}{N} = \frac{N(A)}{N}$$

Thus when outcomes are equally likely, computing probabilities reduces to counting: determine both the number of outcomes $N(A)$ in A and the number of outcomes N in $\mathcal{S}$, and form their ratio.

Example 2.16 You have six unread mysteries on your bookshelf and six unread science fiction books. The first three of each type are hardcover, and the last three are paperback. Consider randomly selecting one of the six mysteries and then randomly selecting one of the six science fiction books to take on a post-finals vacation to Acapulco (after all, you need something to read on the beach). Number the mysteries 1, 2, . . . , 6, and do the same for the science fiction books. Then each outcome is a pair of numbers such as (4, 1), and there are $N = 36$ possible outcomes (For a visual of this situation, refer to the table in Example 2.3 and delete the first row and column). With random selection as described, the 36 outcomes are equally likely. Nine of these outcomes are such that both selected books are paperbacks (those in the lower right-hand corner of the referenced table): (4,4), (4,5), . . . , (6,6). So the probability of the event A that both selected books are paperbacks is

$$P(A) = \frac{N(A)}{N} = \frac{9}{36} = .25$$

■

EXERCISES Section 2.2 (11–28)

11. A mutual fund company offers its customers a variety of funds: a money-market fund, three different bond funds (short, intermediate, and long-term), two stock funds (moderate and high-risk), and a balanced fund. Among customers who own shares in just one fund, the percentages of customers in the different funds are as follows:

Money-market	20%	High-risk stock	18%
Short bond	15%	Moderate-risk stock	25%
Intermediate bond	10%	Balanced	7%
Long bond	5%		

A customer who owns shares in just one fund is randomly selected.

a. What is the probability that the selected individual owns shares in the balanced fund?

b. What is the probability that the individual owns shares in a bond fund?

c. What is the probability that the selected individual does not own shares in a stock fund?

12. Consider randomly selecting a student at a certain university, and let A denote the event that the selected individual has a Visa credit card and B be the analogous event for a MasterCard. Suppose that $P(A) = .5$, $P(B) = .4$, and $P(A \cap B) = .25$.

a. Compute the probability that the selected individual has at least one of the two types of cards (i.e., the probability of the event $A \cup B$).

b. What is the probability that the selected individual has neither type of card?

c. Describe, in terms of A and B, the event that the selected student has a Visa card but not a MasterCard, and then calculate the probability of this event.

13. A computer consulting firm presently has bids out on three projects. Let $A_i = \{\text{awarded project } i\}$, for $i = 1, 2, 3$, and suppose that $P(A_1) = .22$, $P(A_2) = .25$, $P(A_3) = .28$, $P(A_1 \cap A_2) = .11$, $P(A_1 \cap A_3) = .05$, $P(A_2 \cap A_3) = .07$, $P(A_1 \cap A_2 \cap A_3) = .01$. Express in words each of the following events, and compute the probability of each event:

a. $A_1 \cup A_2$

b. $A_1' \cap A_2'$ [*Hint:* $(A_1 \cup A_2)' = A_1' \cap A_2'$]

c. $A_1 \cup A_2 \cup A_3$

d. $A_1' \cap A_2' \cap A_3'$

e. $A_1' \cap A_2' \cap A_3$

f. $(A_1' \cap A_2') \cup A_3$

14. Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

a. What is the probability that a randomly selected adult regularly consumes both coffee and soda?

b. What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

23. The computers of six faculty members in a certain department are to be replaced. Two of the faculty members have selected laptop machines and the other four have chosen desktop machines. Suppose that only two of the setups can be done on a particular day, and the two computers to be set up are randomly selected from the six (implying 15 equally likely outcomes; if the computers are numbered 1, 2, . . . , 6, then one outcome consists of computers 1 and 2, another consists of computers 1 and 3, and so on).

a. What is the probability that both selected setups are for laptop computers?

b. What is the probability that both selected setups are desktop machines?

c. What is the probability that at least one selected setup is for a desktop computer?

d. What is the probability that at least one computer of each type is chosen for setup?

24. Show that if one event A is contained in another event B (i.e., A is a subset of B), then $P(A) \le P(B)$. [*Hint:* For such A and B, A and $B \cap A'$ are disjoint and $B = A \cup (B \cap A')$, as can be seen from a Venn diagram.] For general A and B, what does this imply about the relationship among $P(A \cap B)$, $P(A)$ and $P(A \cup B)$?

25. The three most popular options on a certain type of new car are a built-in GPS (A), a sunroof (B), and an automatic transmission (C). If 40% of all purchasers request A, 55% request B, 70% request C, 63% request A or B, 77% request A or C, 80% request B or C, and 85% request A or B or C, determine the probabilities of the following events. [*Hint:* "A or B" is the event that at least one of the two options is requested; try drawing a Venn diagram and labeling all regions.]

a. The next purchaser will request at least one of the three options.

b. The next purchaser will select none of the three options.

c. The next purchaser will request only an automatic transmission and not either of the other two options.

d. The next purchaser will select exactly one of these three options.

26. A certain system can experience three different types of defects. Let A_i ($i = 1,2,3$) denote the event that the system has a defect of type i. Suppose that

$$P(A_1) = .12 \quad P(A_2) = .07 \quad P(A_3) = .05$$
$$P(A_1 \cup A_2) = .13 \quad P(A_1 \cup A_3) = .14$$
$$P(A_2 \cup A_3) = .10 \quad P(A_1 \cap A_2 \cap A_3) = .01$$

a. What is the probability that the system does not have a type 1 defect?

b. What is the probability that the system has both type 1 and type 2 defects?

c. What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?

d. What is the probability that the system has at most two of these defects?

27. An academic department with five faculty members—Anderson, Box, Cox, Cramer, and Fisher—must select two of its members to serve on a personnel review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting the names on identical pieces of paper and then randomly selecting two.

a. What is the probability that both Anderson and Box will be selected? [*Hint:* List the equally likely outcomes.]

b. What is the probability that at least one of the two members whose name begins with C is selected?

c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years' teaching experience there?

28. In Exercise 5, suppose that any incoming individual is equally likely to be assigned to any of the three stations irrespective of where other individuals have been assigned. What is the probability that

a. All three family members are assigned to the same station?

b. At most two family members are assigned to the same station?

c. Every family member is assigned to a different station?

2.3 Counting Techniques

When the various outcomes of an experiment are equally likely (the same probability is assigned to each simple event), the task of computing probabilities reduces to counting. Letting N denote the number of outcomes in a sample space and $N(A)$ represent the number of outcomes contained in an event A,

$$P(A) = \frac{N(A)}{N} \tag{2.1}$$

If a list of the outcomes is easily obtained and N is small, then N and $N(A)$ can be determined without the benefit of any general counting principles.

There are, however, many experiments for which the effort involved in constructing such a list is prohibitive because N is quite large. By exploiting some

EXERCISES Section 2.3 (29–44)

29. As of April 2006, roughly 50 million .com web domain names were registered (e.g., yahoo.com).

- **a.** How many domain names consisting of just two letters in sequence can be formed? How many domain names of length two are there if digits as well as letters are permitted as characters? [*Note:* A character length of three or more is now mandated.]
- **b.** How many domain names are there consisting of three letters in sequence? How many of this length are there if either letters or digits are permitted? [*Note:* All are currently taken.]
- **c.** Answer the questions posed in (b) for four-character sequences.
- **d.** As of April 2006, 97,786 of the four-character sequences using either letters or digits had not yet been claimed. If a four-character name is randomly selected, what is the probability that it is already owned?

30. A friend of mine is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot, and 12 of cabernet (he only drinks red wine), all from different wineries.

- **a.** If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?
- **b.** If 6 bottles of wine are to be randomly selected from the 30 for serving, how many ways are there to do this?
- **c.** If 6 bottles are randomly selected, how many ways are there to obtain two bottles of each variety?
- **d.** If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen?
- **e.** If 6 bottles are randomly selected, what is the probability that all of them are the same variety?

31.

- **a.** Beethoven wrote 9 symphonies, and Mozart wrote 27 piano concertos. If a university radio station announcer wishes to play first a Beethoven symphony and then a Mozart concerto, in how many ways can this be done?
- **b.** The station manager decides that on each successive night (7 days per week), a Beethoven symphony will be played, followed by a Mozart piano concerto, followed by a Schubert string quartet (of which there are 15). For roughly how many years could this policy be continued before exactly the same program would have to be repeated?

32. A stereo store is offering a special price on a complete set of components (receiver, compact disc player, speakers, turntable). A purchaser is offered a choice of manufacturer for each component:

Receiver: Kenwood, Onkyo, Pioneer, Sony, Sherwood
Compact disc player: Onkyo, Pioneer, Sony, Technics
Speakers: Boston, Infinity, Polk
Turntable: Onkyo, Sony, Teac, Technics

A switchboard display in the store allows a customer to hook together any selection of components (consisting of one of each type). Use the product rules to answer the following questions:

- **a.** In how many ways can one component of each type be selected?
- **b.** In how many ways can components be selected if both the receiver and the compact disc player are to be Sony?
- **c.** In how many ways can components be selected if none is to be Sony?
- **d.** In how many ways can a selection be made if at least one Sony component is to be included?
- **e.** If someone flips switches on the selection in a completely random fashion, what is the probability that the system selected contains at least one Sony component? Exactly one Sony component?

33. Again consider a Little League team that has 15 players on its roster.

- **a.** How many ways are there to select 9 players for the starting lineup?
- **b.** How many ways are there to select 9 players for the starting lineup and a batting order for the 9 starters?
- **c.** Suppose 5 of the 15 players are left-handed. How many ways are there to select 3 left-handed outfielders and have all 6 other positions occupied by right-handed players?

34. Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.

- **a.** How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?
- **b.** In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect?
- **c.** If a sample of 5 keyboards is randomly selected, what is the probability that at least 4 of these will have a mechanical defect?

35. A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

- **a.** How many selections result in all 6 workers coming from the day shift? What is the probability that all 6 selected workers will be from the day shift?
- **b.** What is the probability that all 6 selected workers will be from the same shift?
- **c.** What is the probability that at least two different shifts will be represented among the selected workers?
- **d.** What is the probability that at least one of the shifts will be unrepresented in the sample of workers?

36. An academic department with five faculty members narrowed its choice for department head to either candidate *A* or candidate *B*. Each member then voted on a slip of paper for one of the candidates. Suppose there are actually three votes for *A* and two for *B*. If the slips are selected for tallying in random order, what is the probability that *A* remains ahead of *B* throughout the vote count (e.g., this event occurs if the selected ordering is *AABAB*, but not for *ABBAA*)?

37. An experimenter is studying the effects of temperature, pressure, and type of catalyst on yield from a certain chemical reaction. Three different temperatures, four different pressures, and five different catalysts are under consideration.

a. If any particular experimental run involves the use of a single temperature, pressure, and catalyst, how many experimental runs are possible?

b. How many experimental runs are there that involve use of the lowest temperature and two lowest pressures?

c. Suppose that five different experimental runs are to be made on the first day of experimentation. If the five are randomly selected from among all the possibilities, so that any group of five has the same probability of selection, what is the probability that a different catalyst is used on each run?

38. A box in a certain supply room contains four 40-W lightbulbs, five 60-W bulbs, and six 75-W bulbs. Suppose that three bulbs are randomly selected.

a. What is the probability that exactly two of the selected bulbs are rated 75-W?

b. What is the probability that all three of the selected bulbs have the same rating?

c. What is the probability that one bulb of each type is selected?

d. Suppose now that bulbs are to be selected one by one until a 75-W bulb is found. What is the probability that it is necessary to examine at least six bulbs?

39. Fifteen telephones have just been received at an authorized service center. Five of these telephones are cellular, five are cordless, and the other five are corded phones. Suppose that these components are randomly allocated the numbers 1, 2, . . . , 15 to establish the order in which they will be serviced.

a. What is the probability that all the cordless phones are among the first ten to be serviced?

b. What is the probability that after servicing ten of these phones, phones of only two of the three types remain to be serviced?

c. What is the probability that two phones of each type are among the first six serviced?

40. Three molecules of type *A*, three of type *B*, three of type *C*, and three of type *D* are to be linked together to form a chain molecule. One such chain molecule is *ABCDABCDABCD*, and another is *BCDDAAABDBCC*.

a. How many such chain molecules are there? [*Hint:* If the three *A*'s were distinguishable from one another—A_1, A_2, A_3—and the *B*'s, *C*'s, and *D*'s were also, how many molecules would there be? How is this number reduced when the subscripts are removed from the *A*'s?]

b. Suppose a chain molecule of the type described is randomly selected. What is the probability that all three molecules of each type end up next to one another (such as in *BBBAAADDDCCC*)?

41. An ATM personal identification number (PIN) consists of four digits, each a 0, 1, 2, . . . 8, or 9, in succession.

a. How many different possible PINs are there if there are no restrictions on the choice of digits?

b. According to a representative at the author's local branch of Chase Bank, there are in fact restrictions on the choice of digits. The following choices are prohibited: (i) all four digits identical (ii) sequences of consecutive ascending or descending digits, such as 6543 (iii) any sequence starting with 19 (birth years are too easy to guess). So if one of the PINs in (a) is randomly selected, what is the probability that it will be a legitimate PIN (that is, not be one of the prohibited sequences)?

c. Someone has stolen an ATM card and knows that the first and last digits of the PIN are 8 and 1, respectively. He has three tries before the card is retained by the ATM (but does not realize that). So he randomly selects the 2nd and 3rd digits for the first try, then randomly selects a different pair of digits for the second try, and yet another randomly selected pair of digits for the third try (the individual knows about the restrictions described in (b) so selects only from the legitimate possibilities). What is the probability that the individual gains access to the account?

d. Recalculate the probability in (c) if the first and last digits are 1 and 1, respectively.

42. A starting lineup in basketball consists of two guards, two forwards, and a center.

a. A certain college team has on its roster three centers, four guards, four forwards, and one individual (X) who can play either guard or forward. How many different starting lineups can be created? [*Hint*: Consider lineups without X, then lineups with X as guard, then lineups with X as forward.]

b. Now suppose the roster has 5 guards, 5 forwards, 3 centers, and 2 "swing players" (X and Y) who can play either guard or forward. If 5 of the 15 players are randomly selected, what is the probability that they constitute a legitimate starting lineup?

43. In five-card poker, a straight consists of five cards with adjacent denominations (e.g., 9 of clubs, 10 of hearts, jack of hearts, queen of spades, and king of clubs). Assuming that aces can be high or low, if you are dealt a five-card hand, what is the probability that it will be a straight with high card 10? What is the probability that it will be a straight? What is the probability that it will be a straight flush (all cards in the same suit)?

44. Show that $\binom{n}{k} = \binom{n}{n-k}$. Give an interpretation involving subsets.

Next to each branch corresponding to a positive test result, the multiplication rule yields the recorded probabilities. Therefore, $P(B) = .00099 + .01998 = .02097$, from which we have

$$P(A_1 | B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{.00099}{.02097} = .047$$

This result seems counterintuitive; the diagnostic test appears so accurate that we expect someone with a positive test result to be highly likely to have the disease, whereas the computed conditional probability is only .047. However, the rarity of the disease implies that most positive test results arise from errors rather than from diseased individuals. The probability of having the disease has increased by a multiplicative factor of 47 (from prior .001 to posterior .047); but to get a further increase in the posterior probability, a diagnostic test with much smaller error rates is needed. ■

EXERCISES Section 2.4 (45–69)

45. The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The accompanying *joint probability table* gives the proportions of individuals in the various ethnic group–blood group combinations.

		Blood Group			
		O	**A**	**B**	**AB**
Ethnic Group	**1**	.082	.106	.008	.004
	2	.135	.141	.018	.006
	3	.215	.200	.065	.020

Suppose that an individual is randomly selected from the population, and define events by $A = \{\text{type A selected}\}$, $B = \{\text{type B selected}\}$, and $C = \{\text{ethnic group 3 selected}\}$.

a. Calculate $P(A)$, $P(C)$, and $P(A \cap C)$.

b. Calculate both $P(A | C)$ and $P(C | A)$, and explain in context what each of these probabilities represents.

c. If the selected individual does not have type B blood, what is the probability that he or she is from ethnic group 1?

46. Suppose an individual is randomly selected from the population of all adult males living in the United States. Let A be the event that the selected individual is over 6 ft in height, and let B be the event that the selected individual is a professional basketball player. Which do you think is larger, $P(A | B)$ or $P(B | A)$? Why?

47. Return to the credit card scenario of Exercise 12 (Section 2.2), where $A = \{\text{Visa}\}$, $B = \{\text{MasterCard}\}$, $P(A) = .5$, $P(B) = .4$, and $P(A \cap B) = .25$. Calculate and interpret each of the following probabilities (a Venn diagram might help).

a. $P(B | A)$ **b.** $P(B' | A)$

c. $P(A | B)$ **d.** $P(A' | B)$

e. Given that the selected individual has at least one card, what is the probability that he or she has a Visa card?

48. Reconsider the system defect situation described in Exercise 26 (Section 2.2).

a. Given that the system has a type 1 defect, what is the probability that it has a type 2 defect?

b. Given that the system has a type 1 defect, what is the probability that it has all three types of defects?

c. Given that the system has at least one type of defect, what is the probability that it has exactly one type of defect?

d. Given that the system has both of the first two types of defects, what is the probability that it does not have the third type of defect?

49. The accompanying table gives information on the type of coffee selected by someone purchasing a single cup at a particular airport kiosk.

	Small	**Medium**	**Large**
Regular	14%	20%	26%
Decaf	20%	10%	10%

Consider randomly selecting such a coffee purchaser.

a. What is the probability that the individual purchased a small cup? A cup of decaf coffee?

b. If we learn that the selected individual purchased a small cup, what now is the probability that he/she chose decaf coffee, and how would you interpret this probability?

c. If we learn that the selected individual purchased decaf, what now is the probability that a small size was selected, and how does this compare to the corresponding unconditional probability of (a)?

50. A department store sells sport shirts in three sizes (small, medium, and large), three patterns (plaid, print, and stripe), and two sleeve lengths (long and short). The accompanying tables give the proportions of shirts sold in the various category combinations.

are the probabilities associated with 0, 1, and 2 defective components being in the batch under each of the following conditions?

a. Neither tested component is defective.

b. One of the two tested components is defective. [*Hint:* Draw a tree diagram with three first-generation branches for the three different types of batches.]

62. A company that manufactures video cameras produces a basic model and a deluxe model. Over the past year, 40% of the cameras sold have been of the basic model. Of those buying the basic model, 30% purchase an extended warranty, whereas 50% of all deluxe purchasers do so. If you learn that a randomly selected purchaser has an extended warranty, how likely is it that he or she has a basic model?

63. For customers purchasing a refrigerator at a certain appliance store, let A be the event that the refrigerator was manufactured in the U.S., B be the event that the refrigerator had an icemaker, and C be the event that the customer purchased an extended warranty. Relevant probabilities are

$$P(A) = .75 \quad P(B|A) = .9 \quad P(B|A') = .8$$
$$P(C|A \cap B) = .8 \quad P(C|A \cap B') = .6$$
$$P(C|A' \cap B) = .7 \quad P(C|A' \cap B') = .3$$

a. Construct a tree diagram consisting of first-, second-, and third-generation branches, and place an event label and appropriate probability next to each branch.

b. Compute $P(A \cap B \cap C)$.

c. Compute $P(B \cap C)$.

d. Compute $P(C)$.

e. Compute $P(A|B \cap C)$, the probability of a U.S. purchase given that an icemaker and extended warranty are also purchased.

64. The Reviews editor for a certain scientific journal decides whether the review for any particular book should be short (1–2 pages), medium (3–4 pages), or long (5–6 pages). Data on recent reviews indicates that 60% of them are short, 30% are medium, and the other 10% are long. Reviews are submitted in either Word or LaTeX. For short reviews, 80% are in Word, whereas 50% of medium reviews are in Word and 30% of long reviews are in Word. Suppose a recent review is randomly selected.

a. What is the probability that the selected review was submitted in Word format?

b. If the selected review was submitted in Word format, what are the posterior probabilities of it being short, medium, or long?

65. A large operator of timeshare complexes requires anyone interested in making a purchase to first visit the site of interest. Historical data indicates that 20% of all potential purchasers select a day visit, 50% choose a one-night visit, and 30% opt for a two-night visit. In addition, 10% of day visitors ultimately make a purchase, 30% of one-night visitors buy a unit, and 20% of those visiting for two nights decide to buy. Suppose a visitor is randomly selected and is found to have made a purchase. How likely is it that this person made a day visit? A one-night visit? A two-night visit?

66. Consider the following information about travelers on vacation (based partly on a recent Travelocity poll): 40% check work email, 30% use a cell phone to stay connected to work, 25% bring a laptop with them, 23% both check work email and use a cell phone to stay connected, and 51% neither check work email nor use a cell phone to stay connected nor bring a laptop. In addition, 88 out of every 100 who bring a laptop also check work email, and 70 out of every 100 who use a cell phone to stay connected also bring a laptop.

a. What is the probability that a randomly selected traveler who checks work email also uses a cell phone to stay connected?

b. What is the probability that someone who brings a laptop on vacation also uses a cell phone to stay connected?

c. If the randomly selected traveler checked work email and brought a laptop, what is the probability that he/she uses a cell phone to stay connected?

67. There has been a great deal of controversy over the last several years regarding what types of surveillance are appropriate to prevent terrorism. Suppose a particular surveillance system has a 99% chance of correctly identifying a future terrorist and a 99.9% chance of correctly identifying someone who is not a future terrorist. If there are 1000 future terrorists in a population of 300 million, and one of these 300 million is randomly selected, scrutinized by the system, and identified as a future terrorist, what is the probability that he/she actually is a future terrorist? Does the value of this probability make you uneasy about using the surveillance system? Explain.

68. A friend who lives in Los Angeles makes frequent consulting trips to Washington, D.C.; 50% of the time she travels on airline #1, 30% of the time on airline #2, and the remaining 20% of the time on airline #3. For airline #1, flights are late into D.C. 30% of the time and late into L.A. 10% of the time. For airline #2, these percentages are 25% and 20%, whereas for airline #3 the percentages are 40% and 25%. If we learn that on a particular trip she arrived late at exactly one of the two destinations, what are the posterior probabilities of having flown on airlines #1, #2, and #3?Assume that the chance of a late arrival in L.A. is unaffected by what happens on the flight to D.C. [*Hint:* From the tip of each first-generation branch on a tree diagram, draw three second-generation branches labeled, respectively, 0 late, 1 late, and 2 late.]

69. In Exercise 59, consider the following additional information on credit card usage:

70% of all regular fill-up customers use a credit card.
50% of all regular non-fill-up customers use a credit card.
60% of all plus fill-up customers use a credit card.
50% of all plus non-fill-up customers use a credit card.

75. One of the assumptions underlying the theory of control charting (see Chapter 16) is that successive plotted points are independent of one another. Each plotted point can signal either that a manufacturing process is operating correctly or that there is some sort of malfunction. Even when a process is running correctly, there is a small probability that a particular point will signal a problem with the process. Suppose that this probability is .05. What is the probability that at least one of 10 successive points indicates a problem when in fact the process is operating correctly? Answer this question for 25 successive points.

76. In October, 1994, a flaw in a certain Pentium chip installed in computers was discovered that could result in a wrong answer when performing a division. The manufacturer initially claimed that the chance of any particular division being incorrect was only 1 in 9 billion, so that it would take thousands of years before a typical user encountered a mistake. However, statisticians are not typical users; some modern statistical techniques are so computationally intensive that a billion divisions over a short time period is not outside the realm of possibility. Assuming that the 1 in 9 billion figure is correct and that results of different divisions are independent of one another, what is the probability that at least one error occurs in one billion divisions with this chip?

77. An aircraft seam requires 25 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability.

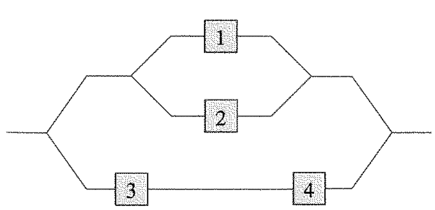
a. If 20% of all seams need reworking, what is the probability that a rivet is defective?

b. How small should the probability of a defective rivet be to ensure that only 10% of all seams need reworking?

78. A boiler has five identical relief valves. The probability that any particular valve will open on demand is .95. Assuming independent operation of the valves, calculate P(at least one valve opens) and P(at least one valve fails to open).

79. Two pumps connected in parallel fail independently of one another on any given day. The probability that only the older pump will fail is .10, and the probability that only the newer pump will fail is .05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?

80. Consider the system of components connected as in the accompanying picture. Components 1 and 2 are connected in parallel, so that subsystem works iff either 1 or 2 works; since 3 and 4 are connected in series, that subsystem works iff both 3 and 4 work. If components work independently of one another and $P(\text{component works}) = .9$, calculate $P(\text{system works})$.



81. Refer back to the series-parallel system configuration introduced in Example 2.35, and suppose that there are only two cells rather than three in each parallel subsystem [in Figure 2.14(a), eliminate cells 3 and 6, and renumber cells 4 and 5 as 3 and 4]. Using $P(A_i) = .9$, the probability that system lifetime exceeds t_0 is easily seen to be .9639. To what value would .9 have to be changed in order to increase the system lifetime reliability from .9639 to .99? [*Hint:* Let $P(A_i) = p$, express system reliability in terms of p, and then let $x = p^2$.]

82. Consider independently rolling two fair dice, one red and the other green. Let A be the event that the red die shows 3 dots, B be the event that the green die shows 4 dots, and C be the event that the total number of dots showing on the two dice is 7. Are these events pairwise independent (i.e., are A and B independent events, are A and C independent, and are B and C independent)? Are the three events mutually independent?

83. Components arriving at a distributor are checked for defects by two different inspectors (each component is checked by both inspectors). The first inspector detects 90% of all defectives that are present, and the second inspector does likewise. At least one inspector does not detect a defect on 20% of all defective components. What is the probability that the following occur?

a. A defective component will be detected only by the first inspector? By exactly one of the two inspectors?

b. All three defective components in a batch escape detection by both inspectors (assuming inspections of different components are independent of one another)?

84. Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities:

a. P(all of the next three vehicles inspected pass)

b. P(at least one of the next three inspected fails)

c. P(exactly one of the next three inspected passes)

d. P(at most one of the next three vehicles inspected passes)

e. Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass (a conditional probability)?

85. A quality control inspector is inspecting newly produced items for faults. The inspector searches an item for faults in a series of independent fixations, each of a fixed duration. Given that a flaw is actually present, let p denote the probability that the flaw is detected during any one fixation (this model is discussed in "Human Performance in Sampling Inspection," *Human Factors*, 1979: 99–105).

a. Assuming that an item has a flaw, what is the probability that it is detected by the end of the second fixation (once a flaw has been detected, the sequence of fixations terminates)?

b. Give an expression for the probability that a flaw will be detected by the end of the nth fixation.