Chap 8

9.

a. R_1 is most appropriate, because x either too large or too small contradicts $p = .5$ and supports $p \neq .5$.

b. A type I error consists of judging one of the two companies favored over the other when in fact there is a 50-50 split in the population. A type II error involves judging the split to be 50-50 when it is not.

c. X has a binomial distribution with $n = 25$ and $p = 0.5$.
$\alpha = P(\text{type I error}) = P(X \le 7 \text{ or } X \ge 18 \text{ when } X \sim \text{Bin}(25, .5)) = B(7; 25, .5) + [1 - B(17; 25, .5)] = .044$.

d. $\beta(.4) = P(8 \le X \le 17 \text{ when } p = .4) = B(17; 25, .4) - B(7, 25, .4) = .845$; $\beta(.6) = .845$ also. Similarly, $\beta(.3) = B(17; 25, .3) - B(7; 25, .3) = .488 = \beta(.7)$.

e. $x = 6$ is in the rejection region R_1, so H_0 is rejected in favor of H_a.

11.

a. $H_0: \mu = 10$ v. $H_a: \mu \neq 10$.

b. $\alpha = P(\text{rejecting } H_0 \text{ when } H_0 \text{ is true}) = P(\bar{X} \ge 10.1032 \text{ or } \bar{X} \le 9.8968 \text{ when } \mu = 10)$.
Since $\bar{X}$ is normally distributed with standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{.2}{5} = .04$, $\alpha = P(Z \ge 2.58 \text{ or } Z \le -2.58) =$
$.005 + .005 = .01$.

c. When $\mu = 10.1$, $E(\bar{X}) = 10.1$, so $\beta(10.1) = P(9.8968 < \bar{X} < 10.1032 \text{ when } \mu = 10.1) =$
$P(-5.08 < Z < .08) = .5319$. Similarly, $\beta(9.8) = P(2.42 < Z < 7.58) = .0078$.

d. $c = \pm 2.58$

e. Now $\frac{\sigma}{\sqrt{n}} = \frac{.2}{3.162} = .0632$. Thus 10.1032 is replaced by c, where $\frac{c - 10}{.0632} = 1.96$ i.e. $c = 10.124$.
Similarly, 9.8968 is replaced by 9.876.

f. $\bar{x} = 10.020$. Since $\bar{x}$ is neither ≥ 10.124 nor ≤ 9.876, , it is not in the rejection region. H_0 is not rejected; it is still plausible that $\mu = 10$.

g. $\bar{x} \ge 10.1032$ or ≤ 9.8968 iff $z \ge 2.58$ or $z \le -2.58$.

18.

a. $\frac{72.3 - 75}{1.8} = -1.5$ so 72.3 is 1.5 SDs (of $\bar{x}$) below 75.

b. H_0 is rejected if $z \le -2.33$; since $z = -1.5$ is not ≤ -2.33, don't reject H_0.

c. α = area under standard normal curve below $-2.88 = \Phi(-2.88) = .0020$.

d. $\Phi\left(-2.88 + \frac{75 - 70}{9/5}\right) = \Phi(-.1) = .4602$ so $\beta(70) = .5398$.

e. $n = \left[\frac{9(2.88 + 2.33)}{75 - 70}\right]^2 = 87.95$, so use $n = 88$.

f. Zero. By definition, a type I error can only occur when H_0 is true, but $\mu = 76$ means that H_0 is actually false.

Chap 8

25.

a. H_0: $\mu = 5.5$ v. H_a: $\mu \neq 5.5$; for a level .01 test, (not specified in the problem description), reject H_0 if either $z \geq 2.58$ or $z \leq -2.58$. Since $z = \frac{5.25-5.5}{.075} = -3.33 \leq -2.58$, reject H_0.

b. $1-\beta(5.6) = 1-\Phi\left(2.58+\frac{(-.1)}{.075}\right)+\Phi\left(-2.58+\frac{(-.1)}{.075}\right) = 1-\Phi(1.25)+\Phi(-3.91) = .105$.

c. $n = \left[\frac{.3(2.58+2.33)}{-.1}\right]^2 = 216.97$, so use $n = 217$.

Chap 9

1.

a. $E(\bar{X}-\bar{Y}) = E(\bar{X}) - E(\bar{Y}) = 4.1-4.5 = -.4$, irrespective of sample sizes.

b. $V(\bar{X}-\bar{Y}) = V(\bar{X})+V(\bar{Y}) = \frac{\sigma_1^2}{m}+\frac{\sigma_2^2}{n} = \frac{(1.8)^2}{100}+\frac{(2.0)^2}{100} = .0724$, and the SD of $\bar{X}-\bar{Y}$ is $\bar{X}-\bar{Y} = \sqrt{.0724} = .2691$.

c. A normal curve with mean and s.d. as given in **a** and **b** (because $m = n = 100$, the CLT implies that both $\bar{X}$ and $\bar{Y}$ have approximately normal distributions, so $\bar{X}-\bar{Y}$ does also). The shape is not necessarily that of a normal curve when $m = n = 10$, because the CLT cannot be invoked. So if the two lifetime population distributions are not normal, the distribution of $\bar{X}-\bar{Y}$ will typically be quite complicated.

3. The test statistic value is $z = \frac{(\bar{x}-\bar{y})-5000}{\sqrt{\frac{s_1^2}{m}+\frac{s_2^2}{n}}}$, and H_0 will be rejected at level .01 if $z \geq 2.33$. We compute $z = \frac{(42{,}500-36{,}800)-5000}{\sqrt{\frac{2200^2}{45}+\frac{1500^2}{45}}} = \frac{700}{396.93} = 1.76$, which is not ≥ 2.33, so we don'*t* reject H_0 and conclude that the true average life for radials does not exceed that for economy brand by significantly more than 500.

19. For the given hypotheses, the test statistic is $t = \frac{115.7-129.3+10}{\sqrt{\frac{5.03^2}{6}+\frac{5.38^2}{6}}} = \frac{-3.6}{3.007} = -1.20$, and the df is

$$\nu = \frac{(4.2168+4.8241)^2}{\frac{(4.2168)^2}{5}+\frac{(4.8241)^2}{5}} = 9.96$$

, so use df = 9. We will reject H_0 if $t \leq -t_{.01,9} = -2.764$; since $-1.20 > -2.764$, we don't reject H_0.

Chap 9

25. We calculate the degrees of freedom $\nu = \dfrac{\left(\frac{5.5^2}{28}+\frac{7.8^2}{31}\right)^2}{\dfrac{\left(\frac{5.5^2}{28}\right)^2}{27}+\dfrac{\left(\frac{7.8^2}{31}\right)^2}{30}} = 53.95$, or about 54 (normally we would round down to 53, but this number is very close to 54 – of course for this large number of df, using either 53 or 54 won't make much difference in the critical t value) so the desired confidence interval is $(91.5-88.3)\pm 1.68\sqrt{\frac{5.5^2}{28}+\frac{7.8^2}{31}} = 3.2 \pm 2.931 = (.269, 6.131)$. Because 0 does not lie inside this interval, we can be reasonably certain that the true difference $\mu_1 - \mu_2$ is not 0 and, therefore, that the two population means are not equal. For a 95% interval, the t value increases to about 2.01 or so, which results in the interval 3.2 ± 3.506. Since this interval does contain 0, we can no longer conclude that the means are different if we use a 95% confidence interval.

34.

a. Following the usual format for most confidence intervals: statistic ± (critical value)(standard error), a pooled variance confidence interval for the difference between two means is $(\bar{x}-\bar{y}) \pm t_{\alpha/2,m+n-2} \cdot s_p\sqrt{\frac{1}{m}+\frac{1}{n}}$.

b. The sample means and standard deviations of the two samples are $\bar{x} = 13.90$, $s_1 = 1.225$, $\bar{y} = 12.20$, $s_2 = 1.010$. The pooled variance estimate is $s_p^2 =$
$\left(\frac{m-1}{m+n-2}\right)s_1^2 + \left(\frac{n-1}{m+n-2}\right)s_2^2 = \left(\frac{4-1}{4+4-2}\right)(1.225)^2 + \left(\frac{4-1}{4+4-2}\right)(1.010)^2 = 1.260$, so $s_p = 1.1227$.
With df $= m + n - 1 = 6$ for this interval, $t_{.025,6} = 2.447$ and the desired interval is $(13.90-12.20) \pm (2.447)(1.1227)\sqrt{\frac{1}{4}+\frac{1}{4}} = 1.7 \pm 1.945 = (-.24, 3.64)$. This interval contains 0, so it does not support the conclusion that the two population means are different.

c. Using the two-sample t interval discussed earlier, we use the CI as follows: First, we need to calculate the degrees of freedom. $\nu = \dfrac{\left(\frac{1.225^2}{4}+\frac{1.01^2}{4}\right)^2}{\dfrac{\left(\frac{1.225^2}{4}\right)^2}{3}+\dfrac{\left(\frac{1.01^2}{4}\right)^2}{3}} = \dfrac{.3971}{.0686} = 5.78 \searrow 5$ and $t_{.025,5} = 2.571$. Then the interval is $(13.9-12.2) \pm 2.571\sqrt{\frac{1.225^2}{4}+\frac{1.01^2}{4}} = 1.70 \pm 2.571(.7938) = (-.34, 3.74)$. This interval is slightly wider, but it still supports the same conclusion.

40. From the data, $n = 10$, $\bar{d} = 105.7$, $s_D = 103.845$.

a. Let μ_D = true mean difference in TBBMC, postweaning minus lactation. We wish to test the hypotheses H_0: $\mu_D \le 25$ v. H_a: $\mu_D > 25$. The test statistic is $t = \dfrac{105.7-25}{103.845/\sqrt{10}} = 2.46$; at 9df, the corresponding P-value is around .018. Hence, at the 5% significance level, we reject H_0 and conclude that true average TBBMC during postweaning does exceed the average during lactation by more than 25 grams.

b. A 95% upper confidence bound for $\mu_D = \bar{d} + t_{.05,9}s_D/\sqrt{n} = 105.7 + 1.833(103.845)/\sqrt{10} = 165.89$ grams.

c. No. If we pretend the two samples are independent, the new standard error is is roughly 235, far greater than $103.845/\sqrt{10}$. In turn, the resulting t statistic is just $t = 0.45$, with estimated df = 17 and P-value = .329 (all using a computer).