Chap. 4

46.

a. $P(67 < X < 75) = P\left(\frac{67-70}{3} < \frac{X-70}{3} < \frac{75-70}{3}\right) = P(-1 < Z < 1.67) = \Phi(1.67) - \Phi(-1) = .9525 - .1587 = .7938.$

b. By the Empirical Rule, c should equal 2 standard deviations. Since $\sigma = 3$, $c = 2(3) = 6$. We can be a little more precise, as in Exercise 42, and use $c = 1.96(3) = 5.88$.

c. Let Y = the number of acceptable specimens out of 10, so $Y \sim \text{Bin}(10, p)$, where $p = .7938$ from part **a**. Then $E(Y) = np = 10(.7938) = 7.938$ specimens.

d. Now let Y = the number of specimens out of 10 that have a hardness of less than 73.84, so $Y \sim \text{Bin}(10, p)$, where

$p = P(X < 73.84) = P\left(Z < \frac{73.84-70}{3}\right) = P(Z < 1.28) = \Phi(1.28) = .8997$. Then

$$P(Y \le 8) = \sum_{y=0}^{8} \binom{10}{y} (.8997)^y (.1003)^{10-y} = .2651.$$

You can also compute $1 - P(Y = 9, 10)$ and use the binomial formula, or round slightly to $p = .9$ and use the binomial table: $P(Y \le 8) = B(8; 10, .9) = .265$.

48.

a. By symmetry, $P(-1.72 \le Z \le -.55) = P(.55 \le Z \le 1.72) = \Phi(1.72) - \Phi(.55)$.

b. $P(-1.72 \le Z \le .55) = \Phi(.55) - \Phi(-1.72) = \Phi(.55) - [1 - \Phi(1.72)]$.

No, thanks to the symmetry of the z curve about 0.

54. Use the normal approximation to the binomial, with a continuity correction. With $p = .10$ and $n = 200$, $\mu = np = 20$, and $\sigma^2 = npq = 18$. So, $\text{Bin}(200, .10) \approx N(20, \sqrt{18})$.

a. $P(X \le 30) = \Phi\left(\frac{(30+.5)-20}{\sqrt{18}}\right) = \Phi(2.47) = .9932.$

b. $P(X < 30) = P(X \le 29) = \Phi\left(\frac{(29+.5)-20}{\sqrt{18}}\right) = \Phi(2.24) = .9875.$

c. $P(15 \le X \le 25) = P(X \le 25) - P(X \le 14) = \Phi\left(\frac{(25+.5)-20}{\sqrt{18}}\right) - \Phi\left(\frac{(14+.5)-20}{\sqrt{18}}\right)$

$= \Phi(1.30) - \Phi(-1.30) = .9032 - .0968 = .8064.$

Chap. 5

37. The joint pmf of X_1 and X_2 is presented below. Each joint probability is calculated using the independence of X_1 and X_2; e.g., $p(25, 25) = P(X_1 = 25) \cdot P(X_2 = 25) = (.2)(.2) = .04$.

		x_1			
	$p(x_1, x_2)$	25	40	65	
x_2	25	.04	.10	.06	.2
	40	.10	.25	.15	.5
	65	.06	.15	.09	.3
		.2	.5	.3	

a. For each coordinate in the table above, calculate $\bar{x}$. The six possible resulting $\bar{x}$ values and their corresponding probabilities appear in the accompanying pmf table.

$\bar{x}$	25	32.5	40	45	52.5	65
$p(\bar{x})$	.04	.20	.25	.12	.30	.09

From the table, $E(\bar{X}) = (25)(.04) + 32.5(.20) + \ldots + 65(.09) = 44.5$. From the original pmf, $\mu = 25(.2) + 40(.5) + 65(.3) = 44.5$. So, $E(\bar{X}) = \mu$.

b. For each coordinate in the joint pmf table above, calculate $s^2 = \frac{1}{2-1}\sum_{i=1}^{2}(x_i - \bar{x})^2$. The four possible resulting s^2 values and their corresponding probabilities appear in the accompanying pmf table.

s^2	0	112.5	312.5	800
$p(s^2)$	.38	.20	.30	.12

From the table, $E(S^2) = 0(.38) + \ldots + 800(.12) = 212.25$. From the original pmf, $\sigma^2 = (25 - 44.5)^2(.2) + (40 - 44.5)^2(.5) + (65 - 44.5)^2(.3) = 212.25$. So, $E(S^2) = \sigma^2$.

38.

a. Since each X is 0 or 1 or 2, the possible values of T_o are 0, 1, 2, 3, 4.
$P(T_o = 0) = P(X_1 = 0 \text{ and } X_2 = 0) = (.2)(.2) = .04$ since X_1 and X_2 are independent.
$P(T_o = 1) = P(X_1 = 1 \text{ and } X_2 = 0, \text{ or } X_1 = 0 \text{ and } X_2 = 1) = (.5)(.2) + (.2)(.5) = .20$.
Similarly, $P(T_o = 2) = .37$, $P(T_o = 3) = .30$, and $P(T_o = 4) = .09$. These values are displayed in the pmf table below.

t_o	0	1	2	3	4
$p(t_o)$	.04	.20	.37	.30	.09

b. $E(T_o) = 0(.04) + 1(.20) + 2(.37) + 3(.30) + 4(.09) = 2.2$. This is exactly twice the population mean: $E(T_o) = 2\mu$.

c. First, $E(T_o^2) = 0^2(.04) + 1^2(.20) + 2^2(.37) + 3^2(.30) + 4^2(.09) = 5.82$. Then $V(T_o) = 5.82 - (2.2)^2 = .98$. This is exactly twice the population variance: $V(T_o) = 2\sigma^2$.

d. Assuming the pattern persists (and it does), when $T_o = X_1 + X_2 + X_3 + X_4$ we have $E(T_o) = 4\mu = 4(1.1) = 4.4$ and $V(T_o) = 4\sigma^2 = 4(.49) = 1.96$.

e. The event $\{T_o = 8\}$ occurs iff we encounter 2 lights on all four trips; i.e., $X_i = 2$ for each X_i. So, assuming the X_i are independent,
$P(T_o = 8) = P(X_1 = 2 \cap X_2 = 2 \cap X_3 = 2 \cap X_4 = 2) = P(X_1 = 2) \cdots P(X_4 = 2) = (.3)^4 = 0081$.
Similarly, $T_o = 7$ iff exactly three of the X_i are 2 and the remaining X_i is 1. The probability of that event is $P(T_o = 7) = (.3)(.3)(.3)(.5) + (.3)(.3)(.5)(.3) + \ldots = 4(.3)^3(.5) = .054$. Therefore, $P(T_o \geq 7) = P(T_o = 7) + P(T_o = 8) = .054 + .0081 = .0621$.

Chap. 5

40.

a. There are only three possible values of M: 0, 5, and 10. Let's find the probabilities associated with 0 and 10, since they're the easiest.
$P(M = 0) = P(\text{all three draws are } 0) = P(X_1 = 0) \cdot P(X_2 = 0) \cdot P(X_3 = 0) = (5/10)(5/10)(5/10) = .125$.
$P(M = 10) = P(\text{at least one draw is a } 10) = 1 - P(\underline{\text{none}} \text{ of the three draws is a } 10) =$
$1 - P(X_1 \neq 10) \cdot P(X_2 \neq 10) \cdot P(X_3 \neq 10) = 1 - (8/10)(8/10)(8/10) = .488$.
Calculating all the options for $M = 5$ would be complicated; however, the three probabilities must sum to 1, so $P(M = 5) = 1 - [.125 + .488] = .387$. The probability distribution of M is displayed in the pmf table below.

m	0	5	10
$p(m)$	.125	.387	.488

An alternative solution would be to list all 27 possible combinations using a tree diagram and computing probabilities directly from the tree.

b. The statistic of interest is M, the maximum of X_1, X_2, or X_3. The population distribution for the X_i is as follows:

x	0	5	10
$p(x)$	5/10	3/10	2/10

Write a computer program to generate the digits 0-9 from a uniform distribution. Assign a value of $x = 0$ to the digits 0-4, a value of $x = 5$ to digits 5-7, and a value of $x = 10$ to digits 8 and 9. Generate samples of increasing sizes, keeping the number of replications constant, and compute $M = \max(X_1, \ldots, X_n)$ from each sample. As n, the sample size, increases, $P(M = 0)$ goes to zero and $P(M = 10)$ goes to one. Furthermore, $P(M = 5)$ goes to zero, but at a slower rate than $P(M = 0)$.

46.

a. The sampling distribution of $\bar{X}$ is centered at $E(\bar{X}) = \mu = 12$ cm, and the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \frac{\sigma_{\bar{X}}}{\sqrt{n}} = \frac{.04}{\sqrt{16}} = .01$ cm.

b. With $n = 64$, the sampling distribution of $\bar{X}$ is still centered at $E(\bar{X}) = \mu = 12$ cm, but the standard deviation of the $\bar{X}$ distribution is $\sigma_{\bar{X}} = \frac{\sigma_{\bar{X}}}{\sqrt{n}} = \frac{.04}{\sqrt{64}} = .005$ cm.

c. $\bar{X}$ is more likely to be within .01 cm of the mean (12 cm) with the second, larger, sample. This is due to the decreased variability of $\bar{X}$ that comes with a larger sample size.

50.

a. $P(9{,}900 \leq \bar{X} \leq 10{,}200) \approx P\left(\frac{9{,}900 - 10{,}000}{500/\sqrt{40}} \leq Z \leq \frac{10{,}200 - 10{,}000}{500/\sqrt{40}}\right)$
$= P(-1.26 \leq Z \leq 2.53) = \Phi(2.53) - \Phi(-1.26) = .9943 - .1038 = .8905$.

b. According to the guideline given in Section 5.4, n should be greater than 30 in order to apply the CLT, thus using the same procedure for $n = 15$ as was used for $n = 40$ would not be appropriate.

Chap 5

73.

a. Both are approximately normal by the Central Limit Theorem.

b. The difference of two rvs is just an example of a linear combination, and a linear combination of normal rvs has a normal distribution, so $\bar{X}-\bar{Y}$ has approximately a normal distribution with $\mu_{\bar{X}-\bar{Y}}=5$ and $\sigma_{\bar{X}-\bar{Y}}=\sqrt{\frac{8^2}{40}+\frac{6^2}{35}}=1.621$.

c. $P\left(-1\le \bar{X}-\bar{Y}\le 1\right)\approx P\left(\frac{-1-5}{1.6213}\le Z\le \frac{1-5}{1.6213}\right)=P(-3.70\le Z\le -2.47)\approx .0068.$

d. $P\left(\bar{X}-\bar{Y}\ge 10\right)\approx P\left(Z\ge \frac{10-5}{1.6213}\right)=P(Z\ge 3.08)=.0010.$ This probability is quite small, so such an occurrence is unlikely if $\mu_1-\mu_2=5$, and we would thus doubt this claim.