Chap 3

71.

a. Possible values of X are 5, 6, 7, 8, 9, 10. (In order to have less than 5 of the granite, there would have to be more than 10 of the basaltic). X is hypergeometric, with $n = 15$, $N = 20$, and $M = 10$. So, the pmf of X is

$$p(x) = h(x;\ 15, 10, 20) = \frac{\binom{10}{x}\binom{10}{15-x}}{\binom{20}{15}}.$$

The pmf is also provided in table form below.

x	5	6	7	8	9	10
$p(x)$	.0163	.1354	.3483	.3483	.1354	.0163

b. P(all 10 of one kind or the other) = $P(X = 5) + P(X = 10) = .0163 + .0163 = .0326$.

c. $\mu = n \cdot \frac{M}{N} = 15 \cdot \frac{10}{20} = 7.5$; $V(X) = \left(\frac{20-15}{20-1}\right) 15 \left(\frac{10}{20}\right)\left(1 - \frac{10}{20}\right) = .9868$; $\sigma = .9934$.

$\mu \pm \sigma = 7.5 \pm .9934 = (6.5066, 8.4934)$, so we want $P(6.5066 < X < 8.4934)$. That equals $P(X = 7) + P(X = 8) = .3483 + .3483 = .6966$.

87.

a. For a two hour period the parameter of the distribution is $\mu = \alpha t = (4)(2) = 8$, so $P(X = 10) = \frac{e^{-8}8^{10}}{10!} = .099$.

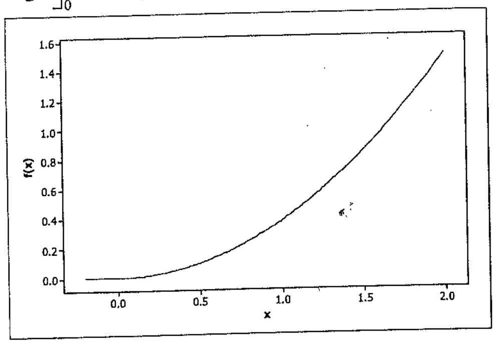
b. For a 30-minute period, $\alpha t = (4)(.5) = 2$, so $P(X = 0) = \frac{e^{-2}2^{0}}{0!} = .135$.

c. The expected value is simply $E(X) = \alpha t = 2$.

Chap 4

5.

a. $1 = \int_{-\infty}^{\infty} f(x)dx = \int_0^2 kx^2 dx = \frac{kx^3}{3}\Big]_0^2 = \frac{8k}{3} \Rightarrow k = \frac{3}{8}$.



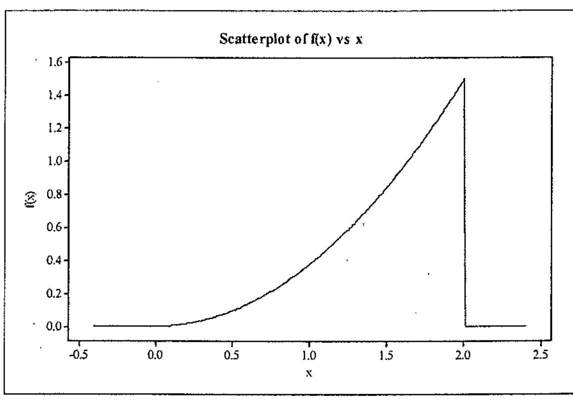
b. $P(0 \le X \le 1) = \int_0^1 \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_0^1 = \frac{1}{8} = .125$.

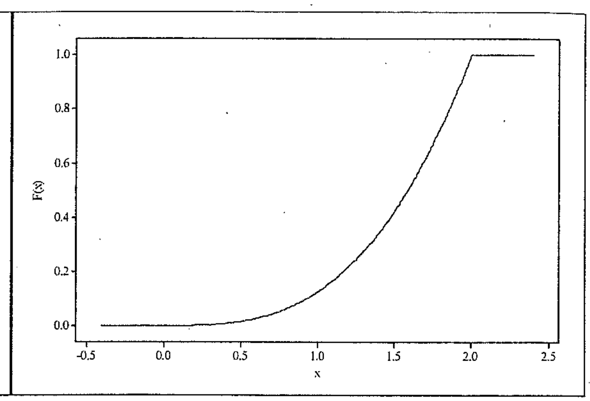
c. $P(1 \le X \le 1.5) = \int_1^{1.5} \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_1^{1.5} = \frac{1}{8}\left(\frac{3}{2}\right)^3 - \frac{1}{8}(1)^3 = \frac{19}{64} = .296875$.

d. $P(X \ge 1.5) = 1 - \int_{1.5}^2 \frac{3}{8}x^2 dx = \frac{1}{8}x^3\Big]_{1.5}^2 = \frac{1}{8}(2)^3 - \frac{1}{8}(1.5)^3 = .578125$.

16.

a. Since X is restricted to the interval [0, 2], $F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 2$.
For $0 \le x \le 2$, $F(x) = \int_0^x \frac{3}{8}y^2 dy = \frac{1}{8}y^3\Big]_0^x = \frac{x^3}{8}$. Both graphs appear below.





b. $P(X \le .5) = F(.5) = \frac{(.5)^3}{8} = \frac{1}{64} = .015625$.

c. $P(.25 < X \le .5) = F(.5) - F(.25) = .015625 - .001953125 = .0137$.
Since X is continuous, $P(.25 \le X \le .5) = P(.25 < X \le .5) = .0137$.

d. The 75th percentile is the value of x for which $F(x) = .75$: $\frac{x^3}{8} = .75 \Rightarrow x = 1.817$.

e. $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_0^2 x \cdot \frac{3}{8}x^2 dx = \frac{3}{8}\int_0^2 x^3 dx = \frac{3}{32}x^4\Big]_0^2 = \frac{3}{2} = 1.5$. Similarly,
$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)dx = \int_0^2 x^2 \cdot \frac{3}{8}x^2 dx = \ldots = 2.4$, from which $V(X) = 2.4 - (1.5)^2 = .15$ and $\sigma_X = .3873$.

f. $\mu \pm \sigma = (1.1127, 1.8873)$. Thus, $P(\mu - \sigma \le X \le \mu + \sigma) = F(1.8873) - F(1.1127) = .8403 - .1722 = .6681$, and the probability X is <u>more</u> than 1 standard deviation from its mean value equals $1 - .6681 = .3318$.

Chap 4

33.

a. $P(X \leq 50) = P\left(Z \leq \frac{50-46.8}{1.75}\right) = P(Z \leq 1.83) = \Phi(1.83) = .9664.$

b. $P(X \geq 48) = P\left(Z \geq \frac{48-46.8}{1.75}\right) = P(Z \geq 0.69) = 1 - \Phi(0.69) = 1 - .7549 = .2451.$

c. The mean and standard deviation aren't important here. The probability a normal random variable is within 1.5 standard deviations of its mean equals $P(-1.5 \leq Z \leq 1.5) =$
$\Phi(1.5) - \Phi(-1.5) = .9332 - .0668 = .8664.$

35.

a. $P(X \geq 10) = P(Z \geq .43) = 1 - \Phi(.43) = 1 - .6664 = .3336.$
Since X is continuous, $P(X > 10) = P(X \geq 10) = .3336.$

b. $P(X > 20) = P(Z > 4) \approx 0.$

c. $P(5 \leq X \leq 10) = P(-1.36 \leq Z \leq .43) = \Phi(.43) - \Phi(-1.36) = .6664 - .0869 = .5795.$

d. $P(8.8 - c \leq X \leq 8.8 + c) = .98$, so $8.8 - c$ and $8.8 + c$ are at the 1st and the 99th percentile of the given distribution, respectively. The 99th percentile of the standard normal distribution satisfies $\Phi(z) = .99$, which corresponds to $z = 2.33$.
So, $8.8 + c = \mu + 2.33\sigma = 8.8 + 2.33(2.8) \Rightarrow c = 2.33(2.8) = 6.524.$

e. From **a**, $P(X > 10) = .3336$, so $P(X \leq 10) = 1 - .3336 = .6664$. For four independent selections,
P(at least one diameter exceeds 10) = $1 - P$(none of the four exceeds 10) =
$1 - P$(first doesn't $\cap$ … fourth doesn't) = $1 - (.6664)(.6664)(.6664)(.6664)$ by independence =
$1 - (.6664)^4 = .8028.$