chap. 2

47. A Venn diagram appears at the end of this exercise.

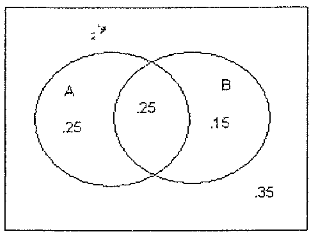
a. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.25}{.50} = .50$.

b. $P(B'|A) = \dfrac{P(A \cap B')}{P(A)} = \dfrac{.25}{.50} = .50$.

c. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.25}{.40} = .6125$.

d. $P(A'|B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{.15}{.40} = .3875$.

e. $P(A|A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A)}{P(A \cup B)} = \dfrac{.50}{.65} = .7692$. It should be clear from the Venn diagram that $A \cap (A \cup B) = A$.



48.

a. $P(A_2 \mid A_1) = \dfrac{P(A_2 \cap A_1)}{P(A_1)} = \dfrac{.06}{.12} = .50$. The numerator comes from Exercise 26.

b. $P(A_1 \cap A_2 \cap A_3 \mid A_1) = \dfrac{P([A_1 \cap A_2 \cap A_3] \cap A_1)}{P(A_1)} = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \dfrac{.01}{.12} = .0833$. The numerator simplifies because $A_1 \cap A_2 \cap A_3$ is a subset of A_1, so their intersection is just the smaller event.

c. For this example, you definitely need a Venn diagram. The seven pieces of the partition inside the three circles have probabilities .04, .05, .00, .02, .01, .01, and .01. Those add to .14 (so the chance of no defects is .86).
Let E = "exactly one defect." From the Venn diagram, $P(E)$ = .04 + .00 + .01 = .05. From the addition above, P(at least one defect) = $P(A_1 \cup A_2 \cup A_3)$ = .14. Finally, the answer to the question is

$$P(E \mid A_1 \cup A_2 \cup A_3) = \frac{P(E \cap [A_1 \cup A_2 \cup A_3])}{P(A_1 \cup A_2 \cup A_3)} = \frac{P(E)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.05}{.14} = .3571$$

The numerator simplifies because E is a subset of $A_1 \cup A_2 \cup A_3$.

d. $P(A_3' \mid A_1 \cap A_2) = \dfrac{P(A_3' \cap [A_1 \cap A_2])}{P(A_1 \cap A_2)} = \dfrac{.05}{.06} = .8333$. The numerator is Exercise 26(c), while the denominator is Exercise 26(b).

Chap. 2

68. Let's see how we can implement the hint. If she's flying airline #1, the chance of 2 late flights is (30%)(10%) = 3%; the two flights being "unaffected" by each other means we can multiply their probabilities. Similarly, the chance of 0 late flights on airline #1 is (70%)(90%) = 63%. Since percents add to 100%, the chance of exactly 1 late flight on airline #1 is 100% – (3% + 63%) = 34%. A similar approach works for the other two airlines: the probability of exactly 1 late flight on airline #2 is 35%, and the chance of exactly 1 late flight on airline #3 is 45%.

The initial ("prior") probabilities for the three airlines are $P(A_1) = 50\%$, $P(A_2) = 30\%$, and $P(A_3) = 20\%$. Given that she had exactly 1 late flight (call that event B), the conditional ("posterior") probabilities of the three airlines can be calculated using Bayes' Rule:

$$P(A_1 \mid B) = \frac{P(A_1)P(B \mid A_1)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + P(A_3)P(B \mid A_3)} = \frac{(.5)(.34)}{(.5)(.34)+(.3)(.35)+(.2)(.45)} = \frac{.170}{.365} =$$

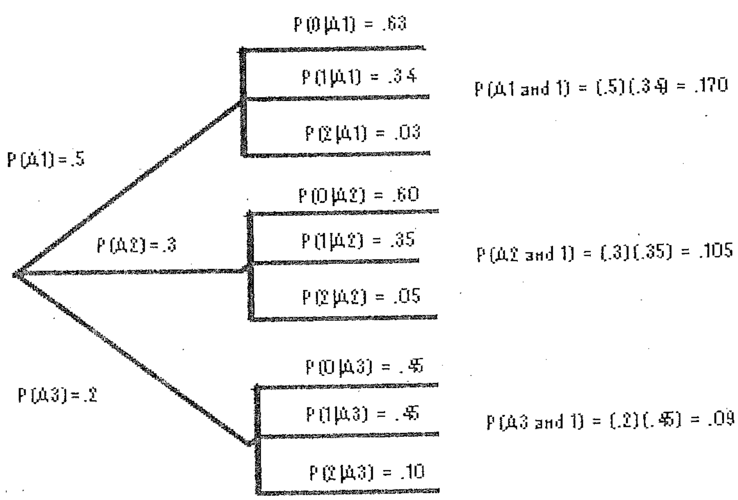
.4657;

$$P(A_2 \mid B) = \frac{P(A_2)P(B \mid A_2)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + P(A_3)P(B \mid A_3)} = \frac{(.3)(.35)}{.365} = .2877; \text{ and}$$

$$P(A_3 \mid B) = \frac{P(A_3)P(B \mid A_3)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + P(A_3)P(B \mid A_3)} = \frac{(.2)(.45)}{.365} = .2466.$$

Notice that, except for rounding error, these three posterior probabilities add to 1.

The tree diagram below shows these probabilities.



80. Let A_i denote the event that component #i works (i = 1, 2, 3, 4). Based on the design of the system, the event "the system works" is $(A_1 \cup A_2) \cup (A_3 \cap A_4)$. We'll eventually need $P(A_1 \cup A_2)$, so work that out first: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = (.9) + (.9) - (.9)(.9) = .99$. The third term uses independence of events. Also, $P(A_3 \cap A_4) = (.9)(.9) = .81$, again using independence.

Now use the addition rule and independence for the system:

$$\begin{aligned} P((A_1 \cup A_2) \cup (A_3 \cap A_4)) &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4)) \\ &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \times P(A_3 \cap A_4) \\ &= (.99) + (.81) - (.99)(.81) = .9981 \end{aligned}$$

(You could also use deMorgan's law in a couple of places.)

Chap. 3

10.

a. Possible values of T are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

b. Possible values of X are: –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6.

c. Possible values of U are: 0, 1, 2, 3, 4, 5, 6.

d. Possible values of Z are: 0, 1, 2.

12.

a. Since there are 50 seats, the flight will accommodate all ticketed passengers who show up as long as there are no more than 50. $P(Y \leq 50) = .05 + .10 + .12 + .14 + .25 + .17 = .83$.

b. This is the complement of part a: $P(Y > 50) = 1 - P(Y \leq 50) = 1 - .83 = .17$.

c. If you're the first standby passenger, you need no more than 49 people to show up (so that there's space left for you). $P(Y \leq 49) = .05 + .10 + .12 + .14 + .25 = .66$. On the other hand, if you're third on the standby list, you need no more than 47 people to show up (so that, even with the two standby passengers ahead of you, there's still room). $P(Y \leq 47) = .05 + .10 + .12 = .27$.

31. From the table in Exercise 12, $E(Y) = 45(.05) + 46(.10) + \ldots + 55(.01) = 48.84$; similarly, $E(Y^2) = 45^2(.05) + 46^2(.10) + \ldots + 55^2(.01) = 2389.84$; thus $V(Y) = E(Y^2) - [E(Y)]^2 = 2389.84 - (48.84)^2 = 4.4944$ and $\sigma_Y = \sqrt{4.4944} = 2.12$.

One standard deviation from the mean value of Y gives $48.84 \pm 2.12 = 46.72$ to 50.96. So, the probability Y is within one standard deviation of its mean value equals $P(46.72 < Y < 50.96) = P(Y = 47, 48, 49, 50) = .12 + .14 + .25 + .17 = .68$.

35. Let $h_3(X)$ and $h_4(X)$ equal the net revenue (sales revenue minus order cost) for 3 and 4 copies purchased, respectively. If 3 magazines are ordered ($6 spent), net revenue is $4 – $6 = –$2 if $X = 1$, 2($4) – $6 = $2 if $X = 2$, 3($4) – $6 = $6 if $X = 3$, and also $6 if $X = 4$, 5, or 6 (since that additional demand simply isn't met. The values of $h_4(X)$ can be deduced similarly. Both distributions are summarized below.

x	1	2	3	4	5	6
$h_3(x)$	–2	2	6	6	6	6
$h_4(x)$	–4	0	4	8	8	8
$p(x)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$

Using the table, $E[h_3(X)] = \sum_{x=1}^{6} h_3(x) \cdot p(x) = (-2)(\frac{1}{15}) + \ldots + (6)(\frac{2}{15}) = \$4.93$.

Similarly, $E[h_4(X)] = \sum_{x=1}^{6} h_4(x) \cdot p(x) = (-4)(\frac{1}{15}) + \ldots + (8)(\frac{2}{15}) = \$5.33$.

Therefore, ordering 4 copies gives slightly higher revenue, on the average.

Chap. 3

49. Let X be the number of "seconds," so $X \sim \text{Bin}(6, .10)$.

a. $P(X=1) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{6}{1}(.1)^1(.9)^5 = .3543$.

b. $P(X \geq 2) = 1 - [P(X=0) + P(X=1)] = 1 - \left[\binom{6}{0}(.1)^0(.9)^6 + \binom{6}{1}(.1)^1(.9)^5\right] = 1 - [.5314 + .3543] =$ .1143.

c. Either 4 or 5 goblets must be selected.

Select 4 goblets with zero defects: $P(X=0) = \binom{4}{0}(.1)^0(.9)^4 = .6561$.

Select 4 goblets, one of which has a defect, and the 5th is good: $\left[\binom{4}{1}(.1)^1(.9)^3\right] \times .9 = .26244$

So, the desired probability is .6561 + .26244 = .91854.

56. Let X = the number of students in the sample needing accommodation, so $X \sim \text{Bin}(25, .02)$.

a. $P(X=1) = 25(.02)(.98)^{24} = .308$.

b. $P(X \geq 1) = 1 - P(X=0) = 1 - (.98)^{25} = 1 - .603 = .397$.

c. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - [.603 + .308] = .089$.

d. $\mu = 25(.02) = .5$ and $\sigma = \sqrt{25(.02)(.98)} = \sqrt{.49} = .7$, so $\mu \pm \sigma = (-.9, 1.9)$. $P(-.9 \leq X \leq 1.9) =$ $P(X = 0 \text{ or } 1) = .911$.

e. $\dfrac{.5(4.5) + 24.5(3)}{25} = 3.03$ hours. Notice the sample size of 25 is actually irrelevant.