Chap 8

39.

1 p = true proportion of all donors with type A blood

2 H_0: $p = .40$

3 H_a: $p \neq .40$

4 $$z = \frac{\hat{p} - p_o}{\sqrt{p_o(1-p_o)/n}} = \frac{\hat{p} - .40}{\sqrt{.40(.60)/n}}$$

5 Reject H_0 if $z \geq 2.58$ or $z \leq -2.58$

6 $$z = \frac{82/150 - .40}{\sqrt{.40(.60)/150}} = \frac{.147}{.04} = 3.667$$

7 Reject H_0. The data does suggest that the percentage of all donors with type A blood differs from 40%. (at the .01 significance level). Since the z critical value for a significance level of .05 is less than that of .01, the conclusion would not change.

Chap 9

50.

a. H_0 will be rejected if $|z| \geq 1.96$. With $\hat{p}_1 = \frac{63}{300} = .2100$, and $\hat{p}_2 = \frac{75}{180} = .4167$, $\hat{p} = \frac{63+75}{300+180} = .2875$, $z = \frac{.2100 - .4167}{\sqrt{(.2875)(.7125)(\frac{1}{300} + \frac{1}{180})}} = \frac{-.2067}{.0427} = -4.84$. Since $-4.84 \leq -1.96$, H_0 is rejected.

Chap 14

7. We test $H_o: p_1 = p_2 = p_3 = p_4 = .25$ vs. H_a: at least one proportion $\neq .25$, and df = 3. We will reject H_0 if the p-value < .01.

Cell	1	2	3	4
Observed	328	334	372	327
Expected	340.25	340.25	340.25	34.025
χ^2 term	.4410	.1148	2.9627	.5160

$\chi^2 = 4.0345$, and with 3 df., p-value > .10, so we fail to reject H_0. The data fails to indicate a seasonal relationship with incidence of violent crime.

17. $\hat{\lambda} = \frac{380}{120} = 3.167$, so $\hat{p} = e^{-3.167}\frac{(3.167)^x}{x!}$.

x	0	1	2	3	4	5	6	≥ 7
$\hat{p}$	.0421	.1334	.2113	.2230	.1766	.1119	.0590	.0427
$n\hat{p}$	5.05	16.00	25.36	26.76	21.19	13.43	7.08	5.12
obs	24	16	16	18	15	9	6	16

The resulting value of $\chi^2 = 103.98$, and when compared to $\chi^2_{.01,7} = 18.474$, it is obvious that the Poisson model fits very poorly.

Chap 14

19. With $A = 2n_1 + n_4 + n_5$, $B = 2n_2 + n_4 + n_6$, and $C = 2n_3 + n_5 + n_6$, the likelihood is proportional to $\theta_1^A \theta_2^B (1-\theta_1-\theta_2)^C$, where $A + B + C = 2n$. Taking the natural log and equating both $\frac{\partial}{\partial\theta_1}$ and $\frac{\partial}{\partial\theta_2}$ to zero gives $\frac{A}{\theta_1} = \frac{C}{1-\theta_1-\theta_2}$ and $\frac{B}{\theta_2} = \frac{C}{1-\theta_1-\theta_2}$, whence $\theta_2 = \frac{B\theta_1}{A}$. Substituting this into the first equation gives $\theta_1 = \frac{A}{A+B+C}$, and then $\theta_2 = \frac{B}{A+B+C}$. Thus $\hat{\theta}_1 = \frac{2n_1+n_4+n_5}{2n}$, $\hat{\theta}_2 = \frac{2n_2+n_4+n_6}{2n}$, and $(1-\hat{\theta}_1-\hat{\theta}_2) = \frac{2n_3+n_5+n_6}{2n}$. Substituting the observed n_i's yields $\hat{\theta}_1 = \frac{2(49)+20+53}{400} = .4275$, $\hat{\theta}_2 = \frac{110}{400} = .2750$, and $(1-\hat{\theta}_1-\hat{\theta}_2) = .2975$, from which $\hat{p}_1 = (.4275)^2 = .183$, $\hat{p}_2 = .076$, $\hat{p}_3 = .089$, $\hat{p}_4 = 2(.4275)(.275) = .235$, $\hat{p}_5 = .254$, $\hat{p}_6 = .164$.

Category	**1**	**2**	**3**	**4**	**5**	**6**
np	36.6	15.2	17.8	47.0	50.8	32.8
observed	49	26	14	20	53	38

This gives $\chi^2 = 29.1$. With $\chi^2_{.01,6-1-2} = \chi^2_{.01,3} = 11.344$, and $\chi^2_{.01,6-1} = \chi^2_{.01,5} = 15.085$, according to (14.15) H_0 must be rejected since $29.1 \geq 15.085$.

26. Let p_{i1} = the probability that a fruit given treatment i matures and p_{i2} = the probability that a fruit given treatment i aborts. Then H_0: $p_{i1} = p_{i2}$ for i = 1, 2, 3, 4, 5 will be rejected if $\chi^2 \geq \chi^2_{.01,4} = 13.277$.

Observed		Estimated Expected		
Matured	Aborted	Matured	Aborted	n_i
141	206	110.7	236.3	347
28	69	30.9	66.1	97
25	73	31.3	66.7	98
24	78	32.5	69.5	102
20	82	32.5	69.5	102
		238	508	746

Thus $\chi^2 = \frac{(141-110.7)^2}{110.7} + \ldots + \frac{(82-69.5)^2}{69.5} = 24.82 \geq 13.277$, so H_0 is rejected at level .01.

Chap 14

32. $$\chi^2 = \frac{(479-494.4)^2}{494.4} + \frac{(173-151.5)^2}{151.5} + \frac{(119-125.2)^2}{125.2} + \frac{(214-177.0)^2}{177.0} + \frac{(47-54.2)^2}{54.2}$$
$$= \frac{(15-44.8)^2}{44.8} + \frac{(172-193.6)^2}{193.6} + \frac{(45-59.3)^2}{59.3} + \frac{(85-49.0)^2}{49.0} = 64.65 \geq \chi^2_{.01,4} = 13.277$$ so the independence hypothesis is rejected in favor of the conclusion that political views and level of marijuana usage are related.

36.

a.

Observed

13	19	28	60
7	11	22	40
20	30	50	100

Estimated Expected

12	18	30
8	12	20

$\chi^2 = \frac{(13-12)^2}{12} + \ldots + \frac{(22-20)^2}{20} = .6806$. Because $.6806 < \chi^2_{.10,2} = 4.605$, H_0 is not rejected.

b. Each observation count here is 10 times what it was in **a**, and the same is true of the estimated expected counts, so now $\chi^2 = 6.806 \geq 4.605$, and H_0 is rejected. With the much larger sample size, the departure from what is expected under H_0, the independence hypothesis, is statistically significant – it cannot be explained just by random variation.

c. The observed counts are .13n, .19n, .28n, .07n, .11n, .22n, whereas the estimated expected $\frac{(.60n)(.20n)}{n} =$ .12n, .18n, .30n, .08n, .12n, .20n, yielding $\chi^2 = .006806n$. H_0 will be rejected at level .10 iff $.006806n \geq 4.605$, i.e., iff $n \geq 676.6$, so the minimum n = 677.