STAT 514 Homework 5

Due June 20

1. (5 points) A pork producer is interested in five different chemical treatments to reduce the percentage of PSE meat. Each animal carcass can only be split in halves and each half can be treated with one chemical only. Generate a design necessary for this experiment assuming that there will be a total of ten carcasses. What is the name of the design? Describe the design in detail along with all variables and specify all design parameters.

2. (22 points) An engineer is studying the mileage performance characteristics of 5 types of gasoline additives. In the road test, he wishes to use cars as blocks; however, because of a time constraint, he must use an incomplete block design. He runs a balanced design with the following five blocks.

	car				
additive	1	2	3	4	5
1		17	14	13	12
2	14	14		13	10
3	12		13	12	9
4	13	11	11	12	
5	11	12	10		8

   (a) (3 points) Verify that this is a balanced incomplete block design.

   (b) (3 points) Test if there is a difference between the five additives (use $\alpha = 0.05$).

   (c) (5 points) Obtain the estimates of treatment means (i.e., the adjusted means).

   (d) (2 points) Calculate the standard error of the difference between any two treatment mean estimates.

   (e) (5 points) Calculate the critical difference for Tukey's pairwise comparisons of all treatment means by hand and draw the conclusions (use $\alpha = 0.05$). Are they consistent with the results from SAS (with the following code)?

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lsmeans trt / pdiff adjust = tukey;
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   (f) (4 points) Suppose the engineer wants to know whether the combination of additives 1, 2 and 3 has the same characteristics as the combination of additives 4 and 5 on average. Use a proper contrast to address this issue and write your conclusion (use $\alpha = 0.05$).

3. (28 points) An experiment is conducted to study the influence of operating temperature and three types of face-plate glass on the light output of an oscilloscope tube. The following data are collected:

	Temperature		
Glass Type	100	125	150
1	58.0,56.8,57.2	107,106.7,106.5	129.2,128.0,128.8
2	55,53,57.9	107,103.7,105	117.8,116.2,109.6
3	54.6,57.3,59.9	106.3,107.3,108.6	101.7,105.4,103.9

   (a) (3 points) Write down the full statistical model along with the constraints.

   (b) (4 points) Test if the involved factorial effects are significant. State the hypotheses and use $\alpha = 0.05$.

   (c) (4 points) Obtain the estimates of the main effects and interaction effects.

   (d) (4 points) Use diagnostic plots to check the model assumptions.

   (e) (2 points) Generate the interaction plot for glass type and temperature and interpret the plot.

   (f) (2 points) Use the Bonferroni procedure to perform pairwise comparisons for glass type level means (i.e., row means) with $\alpha = 0.05$ and write down your conclusions.

   (g) (3 points) Use Tukey's method for the pairwise comparisons between treatment (or cell) means and write down your conclusions.

   (h) (4 points) It is clear that glass type is a categorical factor and temperature is a continuous factor. Use quadratic regression to derive the functional relationships between the response (light output) and temperature for the three types of glass separately.

   (i) (2 points) Predict the average light output using the third glass type and temperature 140.

4. (6 points) An experiment was performed to investigate the capability of a measurement system. Ten parts were randomly selected, and two randomly selected operators measured each part three times. The tests were made in random order resulting in the following data.

	Operator 1			Operator 2		
Parts	1	2	3	1	2	3
1	52	50	50	52	50	50
2	51	51	51	51	50	50
3	52	50	49	53	48	50
4	50	51	50	51	48	49
5	47	46	49	46	47	48
6	50	49	50	50	48	51
7	52	52	51	51	51	51
8	53	50	50	54	52	51
9	49	51	50	48	50	51
10	48	49	48	48	49	48

(a) (4 points) Test the significance of all possible variance components using $\alpha = 0.05$.

(b) (2 points) Find the estimates of the variance components using the analysis of variance method.

5. (11 points) Now you will analyze the data from Problem 4 assuming that the operators are fixed, using the restricted form of mixed effects models.

   (a) (4 points) Test the significance of the variance components using $\alpha = 0.05$.

   (b) (2 points) Find the estimates of the variance components using the analysis of variance method.

   (c) (5 points) Find exact (or at least, approximate) 95% confidence intervals of the variance components.

6. (8 points) Consider a balanced three-factor ANOVA study with factors A, B, and C. Suppose both B and C have fixed effects, but A has a random effect. Following the "Rules for Expected Mean Squares", work out the EMS table, and propose an F-test for each set of the main effects. Assume that we use the restricted mixed effects model.

7. (Bonus: 8 points) Consider the three-factor model:

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + \epsilon_{ijk}$$

   for $i = 1, 2, \cdots, a; j = 1, 2, \cdots, b; k = 1, 2, \cdots, c$. Note that there is only one replicate. Assuming all the factors are fixed, what would you use as the "error", or more precisely, "experimental error" to test hypotheses? Write down the degrees of freedom and the expected mean squares for different sources of variation in the ANOVA table.

8. (Bonus: 12 points) The yield of a chemical process is to be measured using two acid concentrations (0.1 and 0.2 moles/L) and two standing times (8 hours and 10 hours). You have to compare the effects of different acid concentrations and standing times. You have time and money to collect 24 different observations, each with a specific combination of acid concentration and standing time. However, due to the size of the batches of the material, you can collect only 6 observations from each batch.

(a) (6 points) Propose two **balanced** experimental designs that use all 24 observations and present ANOVA tables with sources of variation and degrees of freedom.

(b) (6 points) Which of the two designs proposed in part (a) is better in terms of the precision of 95% confidence intervals for any treatment difference?

Hint: There are $2 \times 2 = 4$ treatments in this experiment.