

# STAT 514 Homework 4

Due June 14

1. (18 points) Ten needles were randomly selected from a branch of a loblolly pine tree. The stomata (microscopic breathing holes) are arranged in rows. On each needle, four rows are randomly selected and the number of stomata per centimeter for each of the rows was determined. The data below is in the file named “stomata.dat” on the course website.

										Needle									
1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
149	136	143	121	148	129	127	134	117	129	143	139	142	133	121	134	130	137	128	132
138	129	124	126	124	127	123	119	117	131	131	143	134	130	128	113	125	130	118	137

- (a) (2 points) Why is the random effects model appropriate here?
- (b) (3 points) Estimate all relevant variance components (Do not use the REML estimators from *proc mixed* ).
- (c) (2 points) What percentage of the overall variation in stomata number per centimeter is due to the needle?
- (d) (2 points) Construct a 95% confidence interval for this ratio.
- (e) (4 points) Construct 95% confidence intervals for the within needle and between needle variance components.
- (f) (2 points) Construct a 95% confidence interval for the average number of stomata per centimeter (i.e. grand mean  $\mu$ ).
- (g) (3 points) Is there significant variability in average number of stomata per centimeter among different needles (use  $\alpha=0.05$ )?
2. (12 points) A sociologist is interested in studying the IQs of teachers from low income areas of a major city. Six schools were randomly chosen from low income areas and from each of these schools, five teachers were randomly chosen. The following table summarizes the mean IQ for each of these schools (Note: These numbers are all made up and are not intended to reflect teachers’ true IQ scores).

School	1	2	3	4	5	6
Mean	97	99	95	107	98	103

- (a) (3 points) If  $MS_E = 45$ , is there significant variability in average IQ among schools in low income areas (use  $\alpha=0.01$ )?
- (b) (3 points) Estimate all variance components.
- (c) (3 points) How much power does this study have if the true variances were such that  $2\sigma^2 = \sigma_\tau^2$  and  $n$  was increased to 12?
- (d) (3 points) Suppose the national average IQ for teachers is 105. Test the null hypothesis that the average IQ of these teachers is not lower than the national average ( $\alpha = 0.05$ ).
3. (13 points) An industrial engineer is investigating the effects of four assembly methods (A, B, C, D) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses the Latin square design shown below.

Order of Assembly	Operator			
	1	2	3	4
1	$C = 10$	$D = 14$	$A = 7$	$B = 8$
2	$B = 7$	$C = 16$	$D = 11$	$A = 7$
3	$A = 5$	$B = 10$	$C = 11$	$D = 9$
4	$D = 10$	$A = 10$	$B = 12$	$C = 14$

- (a) (3 points) Test if there is a difference between the four assembly methods. State the hypotheses and use  $\alpha = 0.05$ .
- (b) (2 points) Obtain the estimates of the treatment effects.
- (c) (4 points) Use Tukey's method to perform pairwise comparison of treatments by hand.
- (d) (4 points) Use residuals to check normality and independence assumptions.
4. (7 points) Suppose in Problem 3 the engineer suspects that the workplaces used by the four operators may represent an additional source of variation. A fourth factor, workplace ( $\alpha, \beta, \gamma$  and  $\delta$ ) needs to be considered and another experiment is conducted. The layout of the experiment and the data are given below.

Order of Assembly	Operator			
	1	2	3	4
1	$C\beta = 11$	$B\gamma = 10$	$D\delta = 14$	$A\alpha = 8$
2	$B\alpha = 8$	$C\delta = 12$	$A\gamma = 10$	$D\beta = 12$
3	$A\delta = 10$	$D\alpha = 11$	$B\beta = 7$	$C\gamma = 15$
4	$D\gamma = 9$	$A\beta = 8$	$C\alpha = 18$	$B\delta = 7$

- (a) (2 points) What design is employed in this experiment? Why?
- (b) (2 points) Test if the four assembly methods are different ( $\alpha = 0.05$ ).
- (c) (3 points) Is your conclusion consistent with that from Problem 3? If your answer is no, what are the possible causes for the inconsistency?
5. (Bonus: 7 points) For a  $p \times p$  Latin square design with fixed treatment and block effects:
- (a) (4 points) Find a mathematical expression for the minimum power for detecting a difference of size  $D$  between any two treatments when the error variance is  $\sigma^2$ .
- (b) (3 points) Find an expression for a  $100(1 - \alpha)\%$  confidence interval for  $\tau_i - \tau_j$ .
6. (Bonus: 8 points) For a RCBD with  $a$  treatments with fixed treatment effects and  $b$  blocks with random block effects, find an expression for an approximate  $100(1 - \alpha)\%$  confidence interval for  $\tau_i$ . Hint: You will need to use the Welch-Satterthwaite equation for the effective degrees of freedom.