STAT 514 Homework 3

Due June 05

1. (10 points) Recall Problem 4 from Homework 2.

   (a) (5 points) If the scientist wants to guarantee that the power of the test in part a) is larger than 0.99 when there is at least one pair of treatments that differs by 2σ, how many observations does he need in every group?

   (b) (5 points) Compare the treatments (the 4 different pesticides) with the control at $\alpha = 0.05$ using the Bonferroni method and Dunnett's method. Compare the results from these two methods.

2. (12 points) An experiment is run to determine whether four specific firing temperatures have different effects on the density of a certain brick. The experiment generates the following data ("temperature.dat" from the course website):

```
temperature density
1 22.8 1 22.5 1 21.5 1 21.6 1 22.1
2 21.2 2 19.5 2 20.3 2 20.6 2 19.8
3 20.8 3 21   3 22.2 3 21.6 3 20.4
4 23.7 4 23.3 4 22.4 4 22.6 4 22.9
```

   where the temperature levels are 100, 125, 150 and 175 coded as 1, 2, 3 and 4 respectively.

   (a) (2 points) Test if the firing temperatures have different effects. Use $\alpha = 0.05$.

   (b) (5 points) Since temperature is a quantitative factor, the experimenter is also interested in modelling the functional relationship between brick density and temperature. Use orthogonal contrasts to fit an orthogonal polynomial model. Test if the linear, quadratic and cubic effects are significant ($\alpha = 0.05$).

   (c) (Bonus: 5 points) Use the polynomial model obtained in b), which only includes the significant terms, to find the temperature that produces the lowest density.

3. (12 points) Suppose you performed an ANOVA with $a = 4$ treatments and $n = 5$ observations per treatment. If the $MS_E = 16$ and $\alpha = 0.05$, what would the minimum difference have to be between any two sample means for you to conclude that the population means were significantly different if you performed

   (a) (2 points) the LSD comparison procedure?

   (b) (2 points) the Bonferroni comparison procedure?

   (c) (2 points) Tukey's multiple comparison procedure?

(d) (2 points) Scheffe's procedure?

(e) (4 points) Explain the relationship between power and the minimum difference. Also state which of the above four is the most powerful and least powerful comparison procedure.

4. (23 points) A clay tile company is interested in studying the effect of cooling temperature on strength. The company has five ovens which produce the tiles. Four tiles were baked in each oven and then randomly assigned to one of the four cooling temperatures. The data are shown below.

Cooling Temp	Oven 1	2	3	4	5	mean
5°	3	10	7	4	3	5.4
10°	3	8	12	2	4	5.8
15°	9	13	15	3	10	10
20°	7	12	9	8	13	9.8
Mean	5.50	10.75	10.75	4.25	7.50	7.75

(a) (6 points) Which type of design was employed? Describe how the fundamental principles of experimental design were followed in this design.

(b) (4 points) If $MS_E = 6.275$, compute the F-statistic to determine if there is a difference among the four cooling temperatures (use $\alpha = 0.05$).

(c) (4 points) Estimate the relative efficiency and interpret your result.

(d) (3 points) If there is a difference among the four temperatures, perform pairwise comparisons using Tukey's procedure. Calculate by hands first, then use SAS to verify your calculations.

(e) (3 points) Suppose the company believes there is a jump in the strength at 12.5°, but otherwise cooling temperature has no effect, that is, 5° and 10° are not different, neither are 15° and 20°, but these two groups of temperatures have different effects. Find a set of orthogonal contrasts that would allow you to test this.

(f) (3 points) Test these contrasts using SAS (or by hand). State your conclusions.

5. (3 points) An experiment was designed to study the performance of four different detergents for cleaning clothes. The following "cleanness" readings (higher=cleaner) were obtained with specially designed equipment for three different types of common stains:

	stain 1	stain 2	stain 3
detergent 1	45	43	41
detergent 2	47	46	52
detergent 3	48	50	55
detergent 4	42	37	49

The conclusion from ANOVA is that the detergents are different. However one researcher suspects that it may not be proper to assume an additive model. Use Tukey's test for non-additivity to settle this issue.

6. (Bonus: 5 points) Let SS_{C_1} and SS_{C_2} be single-degree-of-freedom sums of squares for the orthogonal contrasts C_1 and C_2 in a balanced single-factor experiment with 3 treatments. Prove that $SS_{Treatments} = SS_{C_1} + SS_{C_2}$. You can choose any 2 specific orthogonal contrasts for the proof. Hint: You may have to use the fact that $\sum\sum_{i<j} y_{i\cdot} y_{j\cdot} = \frac{1}{2}(y_{\cdot\cdot}^2 - \sum_i y_{i\cdot}^2)$.