STAT 514 Homework 1

Due May 24

Optional reading assignment: "IntroSAS.doc" from the course website or equivalent online material if not familiar with SAS programming.

1. In a study of egg cell maturation, the eggs from each of four female frogs were divided into two batches and one batch was exposed to progesterone. After two minutes, the cAMP content was measured and the data is tabulated below. It is believed that cAMP is a substance that can mediate cellular response to hormones.

Frog	control	progesterone	diff
1	6	4	2
2	4	5	-1
3	5	2	3
4	4	2	2

   (a) Use a Randomization Test to test whether there is a mean difference in cAMP between the control group and progesterone group. (Hint: It is a paired data. Under the null hypothesis, within each pair, control and progesterone are exchangeable.)

   (b) Perform the usual parametric procedure for the same hypotheses as part a and compare the P-values and conclusions from these two approaches. Also list the assumptions you had to make for this parametric approach.

2. Suppose we have 10 random variables $Z_1, \cdots, Z_{10}$ generated in the following manner. First generate a random variable X from a Normal distribution with mean 5 and variance 1. Then generate 10 i.i.d. random variables $Y_1, \cdots, Y_{10}$ from a Normal distribution with mean 0 and variance 0.5 and add the variable X to each one. In other words,

$$Z_i = X + Y_i, \quad i = 1, \ldots, 10,$$

   where X and Y_i's are mutually independent with $X \sim N(5, 1)$, $Y_i \sim N(0, 0.5)$.

   Calculate $E(Z_i)$, $Var(Z_i)$, $Cov(Z_i, Z_j)$ $(i \neq j)$, $E(\bar{Z})$ and $Var(\bar{Z})$ (where $\bar{Z} = \sum_{i=1}^{10} Z_i/10$).

3. Let us continue to study the keyboard experiment from the first lecture (last slide of Lecture 1). Let y be the amount of time used to type up a manuscript. Note that y depends on the keyboard, the manuscript, whether the manuscript has already been typed once, and experimental error. Let μ_A and μ_B denote the effects of keyboard A and B respectively, τ_i denote the effect of manuscript i for $i = 1, \cdots, 6$ and ϵ denote the experimental error. Let α_l denote the learning effect of typing the manuscript once. We are interested in estimating the difference between μ_A and μ_B. Suppose Design 2 from the lecture notes is used in the experiment, which is

$$1.A - B; 2.B - A; 3.A - B; 4.B - A; 5.A - B; 6.A - B.$$

   An additive statistical model for the amount of time used in the run $1 : A$ is

$$y_{1A} = \gamma + \mu_A + \tau_1 + \epsilon_{1A},$$

   and the model for the amount of time used in the run $1 : B$ is

$$y_{1B} = \gamma + \mu_B + \tau_1 + \alpha_l + \epsilon_{1B},$$

   where γ is some constant.

(a) Is α_l positive or negative? Why is it not included in the first model?

(b) Write down the statistical models for the other runs.

(c) Regardless of which design is used, what is the *simplest* estimate for $\mu_A - \mu_B$, using $y_{1A}, y_{1B}, \ldots, y_{6A}, y_{6B}$?

(d) Explain why the third design is the best one. (Hint: Consider the bias of the estimator of $\mu_A - \mu_B$ under the three different designs.)

4. (Bonus) You have to design an experiment to compare the typing efficiencies of three different types of keyboards denoted by A, B and C. Two typists, denoted by T_1 and T_2, are employed and six standard manuscripts $M_1, \cdots, M_6$ are used.

   (a) Propose a good experimental design.

   (b) Which principles have you used in designing the experiment? Comment on their advantages in this particular experiment.

5. In hypothesis testing, if n is fixed and significance level is decreased ($\alpha \to 0$), what happens to the Type-I and Type-II error probabilities? If α is fixed and sample size n is increased ($n \to \infty$), what happens to the Type-I and Type-II error probabilities? In order to make both Type-I and Type-II error decrease, what should we do? You may consider a one-sample Z-test.

6. A vendor submits lots of fabric to a textile manufacturer. The manufacturer wants to know if the true (i.e. population) lot average breaking strength exceeds 200 psi and accepts the lot only in that case. Past experiences indicate that a reasonable value for the true variance of breaking strength is 100 (psi)^2. The hypotheses to be tested are:

$$H_0 : \mu = 200 \text{ vs. } H_1 : \mu > 200.$$

   The manufacturer decides to randomly select a number of specimens from a specific lot, measure their breaking strengths and test the hypotheses with $\alpha = 5\%$. She also wants to guarantee that average breaking strength of 210 or higher should be detected with probability at least 95%. What is the minimum number of specimens she should select? (Hint: Directly compute the power of the test $Pr(\text{Test Statistic} > \text{Critical Value}|\mu = 210)$. You do not need to generate O.C. curves.)

7. Prove the following for single factor (i.e. one-way) ANOVA:

$$\sum_{i=1}^{a}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{a} n_i(\bar{y}_{i.} - \bar{y}_{..})^2 + \sum_{i=1}^{a}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{i.})^2$$

8. An article in the *Journal of Strain Analysis for Engineering Design* (vol.18, no.2, 1983) compares several procedures for predicting the shear strength of steel plate girders. Data for nine girders in the form of the ratio of predicted to observed load for two of these procedures, the Karlsruhe and Lehigh methods, are given below:

Girder	Karlsruhe Method	Lehigh Method
S1/1	1.180	1.061
S2/1	1.151	0.992
S3/1	1.322	1.063
S4/1	1.339	1.062
S5/1	1.200	1.065
S1/2	1.402	1.178
S2/2	1.365	1.037
S3/2	1.537	1.086
S4/2	1.559	1.052

(a) Investigate the normality of data from these two methods (i.e. two columns) and the difference of the two methods (i.e. the difference of the two columns). Which normality is more important for the analysis of this data?

(b) Is there any evidence to support a claim that there is a difference in mean performance between the two methods? Use $\alpha = 5\%$. What is the P-value?

(c) Construct a 95% confidence interval for the difference in mean predicted to observed load ratio.

(d) (Bonus) Which design principles have been used in the experiment? Comment on their advantages and disadvantages.