5.

a. $$\int_0^2 f(x)\,dx = 1$$

$$\Rightarrow \int_0^2 kx^2\,dx = \frac{k}{3}x^3\Big|_0^2 = \frac{8}{3}k = 1$$

$$\Rightarrow \boxed{k = \frac{3}{8}}$$

b. $$\int_0^1 f(x)\,dx = \int_0^1 \frac{3}{8}x^2\,dx = \frac{1}{8}x^3\Big|_0^1 = \boxed{\frac{1}{8}}$$

c. $P(1 < X < \frac{3}{2})$

$$= \int_1^{\frac{3}{2}} f(x)\,dx = \int_1^{\frac{3}{2}} \frac{3}{8}x^2\,dx = \frac{1}{8}x^3\Big|_1^{\frac{3}{2}} = \frac{1}{8}\left(\frac{27}{8} - 1\right)$$

$$= \frac{1}{8}\cdot\frac{19}{8} = \boxed{\frac{19}{64}} = .2969$$

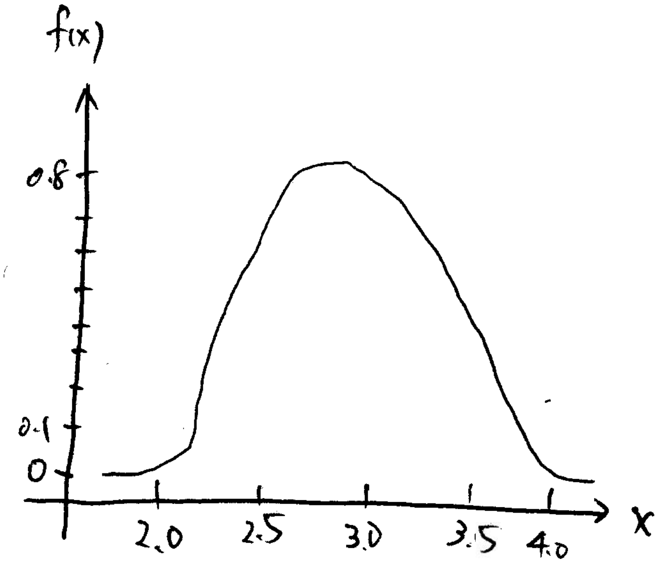
d. $P(\frac{3}{2} < X < 2)$

$$= \int_{\frac{3}{2}}^2 f(x)\,dx \quad \int_{3/2}^2 \frac{3}{8}x^2\,dx = \int_{3/2}^2 \frac{1}{8}x^3 = \frac{1}{8}\left(8 - \frac{27}{8}\right)$$

$$= \frac{1}{8}\cdot\frac{37}{8} = \boxed{\frac{37}{64}} = .5781$$

6.

a.




b.

$$\int_2^4 f(x)\,dx = 1$$

$$\int_2^4 k\left[1-(x-3)^2\right]dx = k\int_2^4 1-x^2+6x-9\;dx = k\int_2^4(-x^2+6x-8)\,dx$$

$$= k\; -\frac{1}{3}x^3+3x^2-8x\Big|_2^4 = k\cdot\frac{4}{3} = 1 \Rightarrow \boxed{k=\frac{3}{4}}$$

c.

$$P(X>3) = \int_3^4 \frac{3}{4}\left[1-(x-3)^2\right]dx = \int_3^4 \frac{3}{4}(-x^2+6x-8)\,dx$$

$$= \frac{3}{4}\; -\frac{1}{3}x^3+3x^2-8x\Big|_3^4$$

$$= \frac{3}{4}\left[\left(-\frac{1}{3}\cdot4^3+3\cdot4^2-8\cdot4\right)-\left(-\frac{1}{3}\cdot3^3+3\cdot3^2-8\cdot3\right)\right]$$

$$= \boxed{0.5}$$

6.

d.

$$P(2.75 < X < 3.25) = \int_{2.75}^{3.25} \frac{3}{4}[1-(x-3)^2]\,dx$$

$$= \int_{2.75}^{3.25} \frac{3}{4}(-x^2+6x-8)\,dx = \frac{3}{4} \left. -\frac{1}{3}x^3+3x^2-8x \right|_{2.75}^{3.25}$$

$$= \frac{3}{4}\left\{\left[-\frac{1}{3}(3.25)^3+3\cdot(3.25)^2-8(3.25)\right]-\left[-\frac{1}{3}(2.75)^3+3(2.75)^2-8(2.75)\right]\right\}$$

$$= .3672$$

e. $P(X>3.5)+P(X<2.5) = 1-P(2.5<X<3.5)$

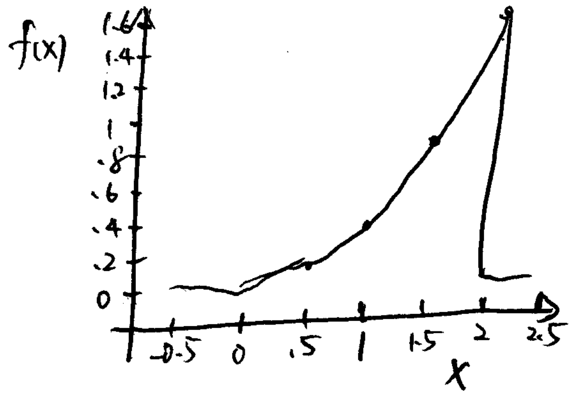
$$= 1-\int_{2.5}^{3.5}\frac{3}{4}[1-(x-3)^2]\,dx = 1-\left(\frac{3}{4}\left[-\frac{1}{3}x^3+3x^2-8x\right|_{2.75}^{3.25}\right)$$

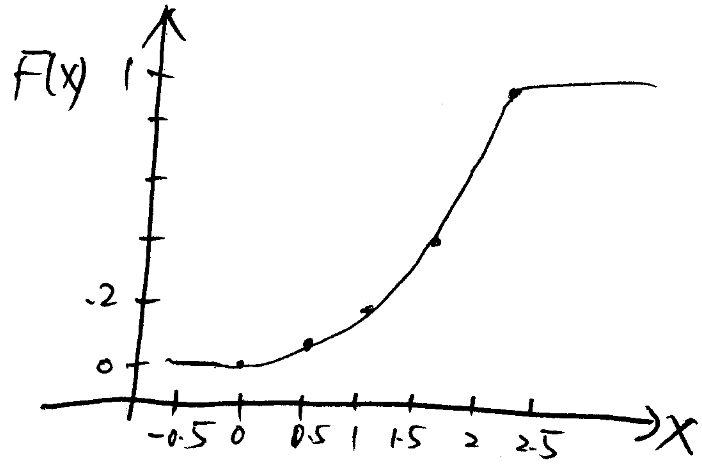
$$= 1-.6875 = \boxed{.3125}$$

16.

$$f(x) = \begin{cases} \frac{3}{8}x^2 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

a.




$$F(x) = \int_0^x f(t)\,dt = \int_0^x \frac{3}{8}t^2\,dt = \frac{1}{8}t^3\Big|_0^x = \frac{1}{8}x^3$$

$$\Rightarrow \quad F(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{8}x^3 & 0 < x < 2 \\ 1 & x \geq 2 \end{cases}$$

b.

$$F(0.5) = \frac{1}{8}(.5)^3 = \boxed{.0156}$$

c.

$$P(.25 < X \le .5) = P(.25 \le X \le .5) = F(.5) - F(.25)$$

$$= \frac{1}{8}(.5)^3 - \frac{1}{8}(.25)^3 = \boxed{0.0137}$$

16.

d. $m = 75^{th}$ percentile

$F(m) = .75$

$\Rightarrow \frac{m^3}{8} = .75 \Rightarrow m^3 = 6 \Rightarrow m = \sqrt[3]{6} = \boxed{1.8171}$

**e.** $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx = \int_0^2 x \cdot \frac{3}{8}x^2 dx = \frac{3}{8}\int_0^2 x^3 dx = \frac{3}{32}x^4 \Big]_0^2 = \frac{3}{2} = 1.5$. Similarly,

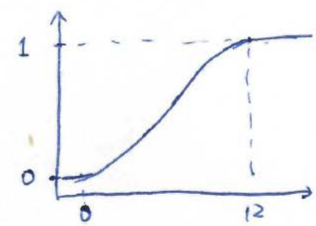
$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)dx = \int_0^2 x^2 \cdot \frac{3}{8}x^2 dx = \ldots = 2.4$, from which $V(X) = 2.4 - (1.5)^2 = .15$ and $\sigma_X = .3873$.

**f.** $\mu \pm \sigma = (1.1127, 1.8873)$. Thus, $P(\mu - \sigma \le X \le \mu + \sigma) = F(1.8873) - F(1.1127) = .8403 - .1722 = .6681$ and the probability $X$ is <u>more</u> than 1 standard deviation from its mean value equals $1 - .6681 = .3318$.

99. a. For $0 \le y \le 12$, $F(y) = \frac{1}{24}\int_0^y (u - \frac{u^2}{12})\,du = \frac{1}{24}(\frac{u^2}{2} - \frac{u^3}{36})\Big|_0^y$

$$= \frac{y^2}{48} - \frac{y^3}{864}. \text{ Thus,}$$

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{y^2}{48} - \frac{y^3}{864} & 0 \le y \le 12 \\ 1 & y > 12 \end{cases}$$




b. $P(Y \le 4) = F(4) = .259$. $P(Y > 6) = 1 - F(6) = .5$.

$P(4 \le X \le 6) = F(6) - F(4) = .5 - .259 = .241$

c. $E(Y) = \int_0^{12} y \frac{1}{24} y(1 - \frac{y}{12})\,dy = \frac{1}{24}\int_0^{12}(y^2 - \frac{y^3}{12})\,dy$

$$= \frac{1}{24}\left[\frac{y^3}{3} - \frac{y^4}{48}\right]\Big|_0^{12} = 6 \text{ inches}.$$

$E(Y^2) = \frac{1}{24}\int_0^{12}(y^3 - \frac{y^4}{12})\,dy = 43.2$

so $V(Y) = 43.2 - 36 = 7.2$.

d $P(Y < 4 \text{ or } Y > 8) = 1 - P(4 \le Y \le 8)$

$$= 1 - [F(8) - F(4)]$$

$$= .518.$$

e. The shorter segment has length equal to $\min(Y, 12 - Y)$, and

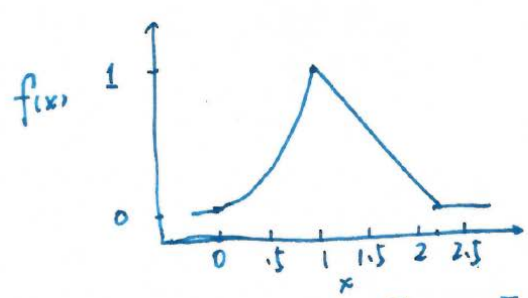
$E[\min(Y, 12 - Y)] = \int_0^{12} \min(y, 12 - y) f(y)\,dy$

$$= \int_0^6 \min(y, 12-y)\cdot f(y)\,dy + \int_6^{12} \min(y, 12-y)\cdot f(y)\,dy$$

$$= \int_0^6 y \cdot f(y)\,dy + \int_6^{12}(12-y)\cdot f(y)\,dy$$

$$= \frac{90}{24} = 3.75 \text{ inches}$$

101. a. By differentiation, $f(x) = \begin{cases} x^2 & 0 \le x < 1 \\ \frac{7}{4} - \frac{3}{4}x & 1 \le x < \frac{7}{3} \\ 0 & \text{otherwise} \end{cases}$




b. $P(.5 \le X \le 2) = F(2) - F(.5) = 1 - \frac{1}{2}(\frac{7}{3} - 2)(\frac{7}{4} - \frac{3}{4}\cdot 2) - \frac{(.5)^3}{3}$

$$= \frac{11}{12} = .917.$$

c. Using the pdf from a. $E(X) = \int_0^1 x \cdot x^2\,dx + \int_1^{7/3} x\cdot(\frac{7}{4} - \frac{3}{4}x)\,dx$

$$= \frac{131}{108} = 1.213$$