37

$$E(X) = 1\cdot\frac{1}{n} + 2\cdot\frac{1}{n} + \cdots + n\cdot\frac{1}{n} = \frac{1+2+\cdots+n}{n}$$

$$= \frac{n(n+1)/2}{n} = \frac{n+1}{2} \#$$

$$Var(X) = E(X^2) - (E(X))^2 = \left[1^2\cdot\frac{1}{n} + 2^2\cdot\frac{1}{n} + \cdots + n^2\frac{1}{n}\right] - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{n(n+1)(2n+1)/6}{n} - \left(\frac{n+1}{2}\right)^2 = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12} \#$$

39.

$$E(X) = 1(.2) + 2(.4) + 3(.3) + 4(.1) = .2 + .8 + .9 + .4$$

$$= 2.3 \#$$

$$E(X^2) = 1^2(.2) + 2^2(.4) + 3^2(.3) + 4^2(.1) = .2 + 1.6 + 2.7 + 1.6$$

$$= 6.1$$

$$Var(X) = E(X^2) - (E(X))^2 = 6.1 - (2.3)^2 = .81$$

$$E[\#\text{ of pounds left}] = E[100-5X] = 100 - 5E(X) = 100 - 5(2.3)$$

$$= 88.5 \#$$

$$Var(100-5X) = 25\,Var(X) = 25(.81) = 20.25 \#$$

59. In this example, $X \sim Bin(25, p)$ with p unknown.

a. P(rejecting claim when $p = .8$) $= P(X \le 15$ when $p = .8)$

$= B(15; 25, .8) = .017$

b. P(<u>not</u> rejecting claim when $p = .7$) $= P(X > 15$ when $p = .7)$

$= 1 - P(X \le 15$ when $p = .7) = 1 - B(15; 25, .7) = 1 - .189 = .811$

For $p = .6$, this probability is $= 1 - B(15; 25, .6) = 1 - .575 = .425$

c. The probability of rejecting the claim when $p = .8$ becomes $B(14; 25, .8) = .006$ smaller than in a. above. However, the probabilities of b. above increase to .902 and .586, respectively. So, by changing 15 to 14, we're making it less likely that we will reject the claim when it is true (p really is $\ge .8$) but more likely that we'll "fail" to reject the claim when it's false (p really is $< .8$)

60. Using the hint, $h(X) = 1 \cdot X + 2.25(25 - X) = 62.5 - 1.5X$, which is a linear function. Since the mean of X is $E(X) = np = (25)(.6) = 15$, $E(h(X)) = 62.5 - 1.5\,E(X) = 62.5 - 1.5 \cdot 15 = \$40$

91. a. For a quarter-acre (.25 acre) plot, the mean parameter is $\mu = (80)(.25) = 20$,
so $P(X \le 16) = F(16; 20) = .221$

b. The expected number of trees is $\alpha \cdot$(area) $= 80$ trees / acre $\cdot$ (85,000 acres)

$= 6{,}800{,}000$ trees

c. The area of the circle is $\pi r^2 = \pi(.1)^2 = 0.01\pi = 0.031416$ square miles, which is equivalent to $.031416 \cdot (640) = 20.106$ acres. Thus X has a Poisson distribution with parameter $\mu = \alpha(20.106) = 80 \cdot (20.106) = 1608.5$.

That's the pmf of X is the function $p(x; 1608.5)$.