EXERCISES Section 2.3 (29–44)

29. As of April 2006, roughly 50 million .com web domain names were registered (e.g., yahoo.com).

a. How many domain names consisting of just two letters in sequence can be formed? How many domain names of length two are there if digits as well as letters are permitted as characters? [*Note:* A character length of three or more is now mandated.]

b. How many domain names are there consisting of three letters in sequence? How many of this length are there if either letters or digits are permitted? [*Note:* All are currently taken.]

c. Answer the questions posed in (b) for four-character sequences.

d. As of April 2006, 97,786 of the four-character sequences using either letters or digits had not yet been claimed. If a four-character name is randomly selected, what is the probability that it is already owned?

30. A friend of mine is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot, and 12 of cabernet (he only drinks red wine), all from different wineries.

a. If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?

b. If 6 bottles of wine are to be randomly selected from the 30 for serving, how many ways are there to do this?

c. If 6 bottles are randomly selected, how many ways are there to obtain two bottles of each variety?

d. If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen?

e. If 6 bottles are randomly selected, what is the probability that all of them are the same variety?

31. The composer Beethoven wrote 9 symphonies, 5 piano concertos (music for piano and orchestra), and 32 piano sonatas (music for solo piano).

a. How many ways are there to play first a Beethoven symphony and then a Beethoven piano concerto?

b. The manager of a radio station decides that on each successive evening (7 days per week), a Beethoven symphony will be played followed by a Beethoven piano concerto followed by a Beethoven piano sonata. For how many years could this policy be continued before exactly the same program would have to be repeated?

32. An electronics store is offering a special price on a complete set of components (receiver, compact disc player, speakers, turntable). A purchaser is offered a choice of manufacturer for each component:

Receiver: Kenwood, Onkyo, Pioneer, Sony, Sherwood
Compact disc player: Onkyo, Pioneer, Sony, Technics
Speakers: Boston, Infinity, Polk
Turntable: Onkyo, Sony, Teac, Technics

A switchboard display in the store allows a customer to hook together any selection of components (consisting of one of each type). Use the product rules to answer the following questions:

a. In how many ways can one component of each type be selected?

b. In how many ways can components be selected if both the receiver and the compact disc player are to be Sony?

c. In how many ways can components be selected if none is to be Sony?

d. In how many ways can a selection be made if at least one Sony component is to be included?

e. If someone flips switches on the selection in a completely random fashion, what is the probability that the system selected contains at least one Sony component? Exactly one Sony component?

33. Again consider a Little League team that has 15 players on its roster.

a. How many ways are there to select 9 players for the starting lineup?

b. How many ways are there to select 9 players for the starting lineup and a batting order for the 9 starters?

c. Suppose 5 of the 15 players are left-handed. How many ways are there to select 3 left-handed outfielders and have all 6 other positions occupied by right-handed players?

34. Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.

a. How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?

b. In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect?

c. If a sample of 5 keyboards is randomly selected, what is the probability that at least 4 of these will have a mechanical defect?

35. A production facility employs 10 workers on the day shift, 8 workers on the swing shift, and 6 workers on the graveyard shift. A quality control consultant is to select 5 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 5 workers has the same chance of being selected as does any other group (drawing 5 slips without replacement from among 24).

a. How many selections result in all 5 workers coming from the day shift? What is the probability that all 5 selected workers will be from the day shift?

b. What is the probability that all 5 selected workers will be from the same shift?

c. What is the probability that at least two different shifts will be represented among the selected workers?

d. What is the probability that at least one of the shifts will be unrepresented in the sample of workers?

36. An academic department with five faculty members narrowed its choice for department head to either candidate *A* or candidate *B*. Each member then voted on a slip of paper for one of the candidates. Suppose there are actually three votes for *A* and two for *B*. If the slips are selected for tallying in random order, what is the probability that *A* remains ahead of *B* throughout the vote count (e.g., this event occurs if the selected ordering is *AABAB*, but not for *ABBAA*)?

37. An experimenter is studying the effects of temperature, pressure, and type of catalyst on yield from a certain chemical reaction. Three different temperatures, four different pressures, and five different catalysts are under consideration.

a. If any particular experimental run involves the use of a single temperature, pressure, and catalyst, how many experimental runs are possible?

b. How many experimental runs are there that involve use of the lowest temperature and two lowest pressures?

c. Suppose that five different experimental runs are to be made on the first day of experimentation. If the five are randomly selected from among all the possibilities, so that any group of five has the same probability of selection, what is the probability that a different catalyst is used on each run?

38. A sonnet is a 14-line poem in which certain rhyming patterns are followed. The writer Raymond Queneau published a book containing just 10 sonnets, each on a different page. However, these were structured such that other sonnets could be created as follows: the first line of a sonnet could come from the first line on any of the 10 pages, the second line could come from the second line on any of the 10 pages, and so on (successive lines were perforated for this purpose).

a. How many sonnets can be created from the 10 in the book?

b. If one of the sonnets counted in part (a) is selected at random, what is the probability that none of its lines came from either the first or the last sonnet in the book?

39. A box in a supply room contains 15 compact fluorescent lightbulbs, of which 5 are rated 13-watt, 6 are rated 18-watt, and 4 are rated 23-watt. Suppose that three of these bulbs are randomly selected.

a. What is the probability that exactly two of the selected bulbs are rated 23-watt?

b. What is the probability that all three of the bulbs have the same rating?

c. What is the probability that one bulb of each type is selected?

d. If bulbs are selected one by one until a 23-watt bulb is obtained, what is the probability that it is necessary to examine at least 6 bulbs?

40. Three molecules of type *A*, three of type *B*, three of type *C*, and three of type *D* are to be linked together to form a chain molecule. One such chain molecule is *ABCDABCDABCD*, and another is *BCDDAAABDBCC*.

a. How many such chain molecules are there? [*Hint:* If the three *A*'s were distinguishable from one another—A_1, A_2, A_3—and the *B*'s, *C*'s, and *D*'s were also, how many molecules would there be? How is this number reduced when the subscripts are removed from the *A*'s?]

b. Suppose a chain molecule of the type described is randomly selected. What is the probability that all three molecules of each type end up next to one another (such as in *BBBAAADDDCCC*)?

41. An ATM personal identification number (PIN) consists of four digits, each a 0, 1, 2,… 8, or 9, in succession.

a. How many different possible PINs are there if there are no restrictions on the choice of digits?

b. According to a representative at the author's local branch of Chase Bank, there are in fact restrictions on the choice of digits. The following choices are prohibited: (i) all four digits identical (ii) sequences of consecutive ascending or descending digits, such as 6543 (iii) any sequence starting with 19 (birth years are too easy to guess). So if one of the PINs in (a) is randomly selected, what is the probability that it will be a legitimate PIN (that is, not be one of the prohibited sequences)?

c. Someone has stolen an ATM card and knows that the first and last digits of the PIN are 8 and 1, respectively. He has three tries before the card is retained by the ATM (but does not realize that). So he randomly selects the 2nd and 3rd digits for the first try, then randomly selects a different pair of digits for the second try, and yet another randomly selected pair of digits for the third try (the individual knows about the restrictions described in (b) so selects only from the legitimate possibilities). What is the probability that the individual gains access to the account?

d. Recalculate the probability in (c) if the first and last digits are 1 and 1, respectively.

42. A starting lineup in basketball consists of two guards, two forwards, and a center.

a. A certain college team has on its roster three centers, four guards, four forwards, and one individual (X)

66. Consider the following information about travelers on vacation (based partly on a recent Travelocity poll): 40% check work email, 30% use a cell phone to stay connected to work, 25% bring a laptop with them, 23% both check work email and use a cell phone to stay connected, and 51% neither check work email nor use a cell phone to stay connected nor bring a laptop. In addition, 88 out of every 100 who bring a laptop also check work email, and 70 out of every 100 who use a cell phone to stay connected also bring a laptop.

a. What is the probability that a randomly selected traveler who checks work email also uses a cell phone to stay connected?

b. What is the probability that someone who brings a laptop on vacation also uses a cell phone to stay connected?

c. If the randomly selected traveler checked work email and brought a laptop, what is the probability that he/she uses a cell phone to stay connected?

67. There has been a great deal of controversy over the last several years regarding what types of surveillance are appropriate to prevent terrorism. Suppose a particular surveillance system has a 99% chance of correctly identifying a future terrorist and a 99.9% chance of correctly identifying someone who is not a future terrorist. If there are 1000 future terrorists in a population of 300 million, and one of these 300 million is randomly selected, scrutinized by the system, and identified as a future terrorist, what is the probability that he/she actually is a future terrorist? Does the value of this probability make you uneasy about using the surveillance system? Explain.

68. A friend who lives in Los Angeles makes frequent consulting trips to Washington, D.C.; 50% of the time she travels on airline #1, 30% of the time on airline #2, and the remaining 20% of the time on airline #3. For airline #1, flights are late into D.C. 30% of the time and late into L.A. 10% of the time. For airline #2, these percentages are 25% and 20%, whereas for airline #3 the percentages are 40% and 25%. If we learn that on a particular trip she arrived late at exactly one of the two destinations, what are the posterior probabilities of having flown on airlines #1, #2, and #3? Assume that the chance of a late arrival in L.A. is unaffected by what happens on the flight to D.C. [*Hint:* From the tip of each first-generation branch on a tree diagram, draw three second-generation branches labeled, respectively, 0 late, 1 late, and 2 late.]

69. In Exercise 59, consider the following additional information on credit card usage:

70% of all regular fill-up customers use a credit card.

50% of all regular non-fill-up customers use a credit card.

60% of all plus fill-up customers use a credit card.

50% of all plus non-fill-up customers use a credit card.

50% of all premium fill-up customers use a credit card.

40% of all premium non-fill-up customers use a credit card.

Compute the probability of each of the following events for the next customer to arrive (a tree diagram might help).

a. {plus and fill-up and credit card}

b. {premium and non-fill-up and credit card}

c. {premium and credit card}

d. {fill-up and credit card}

e. {credit card}

f. If the next customer uses a credit card, what is the probability that premium was requested?

2.5 Independence

The definition of conditional probability enables us to revise the probability $P(A)$ originally assigned to A when we are subsequently informed that another event B has occurred; the new probability of A is $P(A|B)$. In our examples, it frequently happened that $P(A|B)$ differed from the unconditional probability $P(A)$. Then the information "B has occurred" resulted in a change in the likelihood of A occurring. Often the chance that A will occur or has occurred is not affected by knowledge that B has occurred, so that $P(A|B) = P(A)$. It is then natural to regard A and B as independent events, meaning that the occurrence or nonoccurrence of one event has no bearing on the chance that the other will occur.

DEFINITION

Two events A and B are **independent** if $P(A|B) = P(A)$ and are **dependent** otherwise.

75. One of the assumptions underlying the theory of control charting (see Chapter 16) is that successive plotted points are independent of one another. Each plotted point can signal either that a manufacturing process is operating correctly or that there is some sort of malfunction. Even when a process is running correctly, there is a small probability that a particular point will signal a problem with the process. Suppose that this probability is .05. What is the probability that at least one of 10 successive points indicates a problem when in fact the process is operating correctly? Answer this question for 25 successive points.

76. In October, 1994, a flaw in a certain Pentium chip installed in computers was discovered that could result in a wrong answer when performing a division. The manufacturer initially claimed that the chance of any particular division being incorrect was only 1 in 9 billion, so that it would take thousands of years before a typical user encountered a mistake. However, statisticians are not typical users; some modern statistical techniques are so computationally intensive that a billion divisions over a short time period is not outside the realm of possibility. Assuming that the 1 in 9 billion figure is correct and that results of different divisions are independent of one another, what is the probability that at least one error occurs in one billion divisions with this chip?

77. An aircraft seam requires 25 rivets. The seam will have to be reworked if any of these rivets is defective. Suppose rivets are defective independently of one another, each with the same probability.

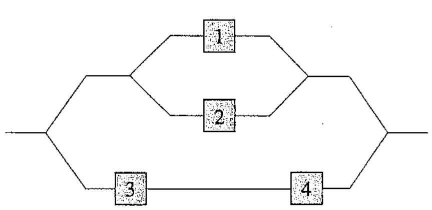
a. If 20% of all seams need reworking, what is the probability that a rivet is defective?

b. How small should the probability of a defective rivet be to ensure that only 10% of all seams need reworking?

78. A boiler has five identical relief valves. The probability that any particular valve will open on demand is .95. Assuming independent operation of the valves, calculate P(at least one valve opens) and P(at least one valve fails to open).

79. Two pumps connected in parallel fail independently of one another on any given day. The probability that only the older pump will fail is .10, and the probability that only the newer pump will fail is .05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?

80. Consider the system of components connected as in the accompanying picture. Components 1 and 2 are connected in parallel, so that subsystem works iff either 1 or 2 works; since 3 and 4 are connected in series, that subsystem works iff both 3 and 4 work. If components work independently of one another and P(component works) = .9, calculate P(system works).



81. Refer back to the series-parallel system configuration introduced in Example 2.36, and suppose that there are only two cells rather than three in each parallel subsystem [in Figure 2.14(a), eliminate cells 3 and 6, and renumber cells 4 and 5 as 3 and 4]. Using $P(A_i) = .9$, the probability that system lifetime exceeds t_0 is easily seen to be .9639. To what value would .9 have to be changed in order to increase the system lifetime reliability from .9639 to .99? [*Hint:* Let $P(A_i) = p$, express system reliability in terms of p, and then let $x = p^2$.]

82. Consider independently rolling two fair dice, one red and the other green. Let A be the event that the red die shows 3 dots, B be the event that the green die shows 4 dots, and C be the event that the total number of dots showing on the two dice is 7. Are these events pairwise independent (i.e., are A and B independent events, are A and C independent, and are B and C independent)? Are the three events mutually independent?

83. Components arriving at a distributor are checked for defects by two different inspectors (each component is checked by both inspectors). The first inspector detects 90% of all defectives that are present, and the second inspector does likewise. At least one inspector does not detect a defect on 20% of all defective components. What is the probability that the following occur?

a. A defective component will be detected only by the first inspector? By exactly one of the two inspectors?

b. All three defective components in a batch escape detection by both inspectors (assuming inspections of different components are independent of one another)?

84. Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities:

a. P(all of the next three vehicles inspected pass)

b. P(at least one of the next three inspected fails)

c. P(exactly one of the next three inspected passes)

d. P(at most one of the next three vehicles inspected passes)

e. Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass (a conditional probability)?

85. A quality control inspector is inspecting newly produced items for faults. The inspector searches an item for faults in a series of independent fixations, each of a fixed duration. Given that a flaw is actually present, let p denote the probability that the flaw is detected during any one fixation (this model is discussed in "Human Performance in Sampling Inspection," *Human Factors,* 1979: 99–105).

a. Assuming that an item has a flaw, what is the probability that it is detected by the end of the second fixation (once a flaw has been detected, the sequence of fixations terminates)?

b. Give an expression for the probability that a flaw will be detected by the end of the nth fixation.