10.

a. Possible values of T are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

b. Possible values of X are: –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6.

c. Possible values of U are: 0, 1, 2, 3, 4, 5, 6.

d. Possible values of Z are: 0, 1, 2.

15.

a. (1,2) (1,3) (1,4) (1,5) (2,3) (2,4) (2,5) (3,4) (3,5) (4,5)

b. X can only take on the values 0, 1, 2. $p(0) = P(X = 0) = P(\{(3,4)\ (3,5)\ (4,5)\}) = 3/10 = .3$; $p(2) = P(X = 2) = P(\{(1,2)\}) = 1/10 = .1$; $p(1) = P(X = 1) = 1 - [p(0) + p(2)] = .60$; and otherwise $p(x) = 0$.

c. $F(0) = P(X \le 0) = P(X = 0) = .30$;
$F(1) = P(X \le 1) = P(X = 0 \text{ or } 1) = .30 + .60 = .90$;
$F(2) = P(X \le 2) = 1$.
Therefore, the complete cdf of X is

$$F(x) = \begin{cases} 0 & x < 0 \\ .30 & 0 \le x < 1 \\ .90 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

17.

a. $p(2) = P(Y = 2) = P(\text{first 2 batteries are acceptable}) = P(AA) = (.9)(.9) = .81$.

b. $p(3) = P(Y = 3) = P(UAA \text{ or } AUA) = (.1)(.9)^2 + (.1)(.9)^2 = 2[(.1)(.9)^2] = .162$.

c. The fifth battery must be an A, and exactly one of the first four must also be an A.
Thus, $p(5) = P(AUUUA \text{ or } UAUUA \text{ or } UUAUA \text{ or } UUUAA) = 4[(.1)^3(.9)^2] = .00324$.

d. $p(y) = P(\text{the } y^{\text{th}} \text{ is an } A \text{ and so is exactly one of the first } y - 1) = (y - 1)(.1)^{y-2}(.9)^2$, for $y = 2, 3, 4, 5, \ldots$.

24.

a. Possible X values are those values at which $F(x)$ jumps, and the probability of any particular value is the size of the jump at that value. Thus we have:

x	1	3	4	6	12
$p(x)$	.30	.10	.05	.15	.40

b. $P(3 \le X \le 6) = F(6) - F(3-) = .60 - .30 = .30$; $P(4 \le X) = 1 - P(X < 4) = 1 - F(4-) = 1 - .40 = .60$.

38. (1/3.5) = \$.286, while $E[h(X)] = E\left(\frac{1}{X}\right) = \sum_{x=1}^{6}\left(\frac{1}{x}\right)\cdot p(x) = \sum_{x=1}^{6}\left(\frac{1}{x}\right)\cdot\frac{1}{6} = \frac{1}{6}\sum_{x=1}^{6}\frac{1}{x} = \$.408$.

So you expect to win more if you gamble.

Note: In general, if $h(x)$ is concave up then $E[h(X)] > h(E(X))$, while the opposite is true if $h(x)$ is concave down.

54. Let X equal the number of customers who choose an oversize racket, so $X \sim \text{Bin}(10, .60)$.

a. $P(X \ge 6) = 1 - P(X \le 5) = 1 - B(5;10,.60) = 1 - .367 = .633$.

b. $\mu = np = 10(.6) = 6$ and $\sigma = \sqrt{10(.6)(.4)} = 1.55$, so $\mu \pm \sigma = (4.45, 7.55)$.
$P(4.45 < X < 7.55) = P(5 \le X \le 7) = P(X \le 7) - P(X \le 4) = .833 - .166 = .667$.

c. This occurs iff between 3 and 7 customers want the oversize racket (otherwise, one type will run out early). $P(3 \le X \le 7) = P(X \le 7) - P(X \le 2) = .833 - .012 = .821$.

66.

a. Let Y = the number with reservations who show up, a binomial rv with $n = 6$ and $p = .8$. Since there are only four spaces available, at least one individual cannot be accommodated if Y is more than 4. The desired probability is $P(Y = 5 \text{ or } 6) = b(5; 6, .8) + b(6;6,.8) = .3932 + .2621 = .6553$.

b. Let $h(Y)$ = the number of available spaces. Then

when y is:	0	1	2	3	4	5	6
$h(y)$ is:	4	3	2	1	0	0	0

The expected number of available spaces when the limousine departs equals

$$E[h(Y)] = \sum_{y=0}^{6} h(y)\cdot b(y;6,.8) = 4(.000) + 3(.002) + 2(.015) + 1(.082) + 0 + 0 + 0 = 0.118.$$

c. Possible X values are 0, 1, 2, 3, and 4. $X = 0$ if there are 3 reservations and 0 show or 4 reservations and 0 show or 5 reservations and 0 show or 6 reservations and none show, so
$P(X = 0) = b(0; 3, .8)(.1) + b(0; 4, .8)(.2) + b(0; 5, .8)(.3) + b(0; 6, .8)(.4)$
$= .0080(.1) + .0016(.2) + .0003(.3) + .0001(.4) = .0013$.
Similarly, $P(X = 1) = b(1; 3, .8)(.1) + \ldots + b(1; 6, .8)(.4) = .0172$; $P(X = 2) = .0906$;
$P(X = 3) = .2273$; and $P(X = 4) = .6636$.

These values are displayed below.

x	0	1	2	3	4
$p(x)$	.0013	.0172	.0906	.2273	.6636

87.

a. For a two hour period the parameter of the distribution is $\mu = \alpha t = (4)(2) = 8$,

so $P(X = 10) = \dfrac{e^{-8}8^{10}}{10!} = .099$.

b. For a 30-minute period, $\alpha t = (4)(.5) = 2$, so $P(X = 0) = \dfrac{e^{-2}2^{0}}{0!} = .135$.

c. The expected value is simply $E(X) = \alpha t = 2$.

88. Let X = the number of diodes on a board that fail. Then $X \sim \text{Bin}(n = 200, p = .01)$.

a. $E(X) = np = (200)(.01) = 2$; $V(X) = npq = (200)(.01)(.99) = 1.98$, so $\sigma = 1.407$.

b. X has approximately a Poisson distribution with $\mu = np = 2$, so $P(X \geq 4) = 1 - P(X \leq 3) = 1 - F(3; 2) = 1 - .857 = .143$.

c. For any one board, $P(\text{board works properly}) = P(\text{all diodes work}) = P(X = 0) = \dfrac{e^{-2}2^{0}}{0!} = .135$.
Let Y = the number among the five boards that work, a binomial rv with $n = 5$ and $p = .135$.
Then $P(Y \geq 4) = P(Y = 4) + P(Y = 5) = \binom{5}{4}(.135)^4(.865) + \binom{5}{5}(.135)^5(.865)^0 = .00148$.

29.

Let $h_2(x)$, $h_3(x)$ and $h_4(x)$ denote the profit for 2, 3 and 4 cakes respectively. For example, if 2 cakes are baked, the profit is \$20 − 2×(\$4) = \$12 if the demand $x = 1$; 2×(\$20) − 2×(\$4) = \$32 if $x = 2$; and also \$32 if $x = 3, 4$ since additional demand isn't met. The values of $h_3(x)$ and $h_4(x)$ can be calculated similarly.

x	0	1	2	3	4
$h_2(x)$	−8	12	32	32	32
$h_3(x)$	−12	8	28	48	48
$h_4(x)$	−16	4	24	44	64
$p(x)$	0.15	0.25	0.30	0.15	0.15

Using the above summary,

$$E[h_2(x)] = \sum_{x=0}^{4} h_2(x)p(x) = (-8)\times 0.15 + 12\times 0.25 + 32\times 0.3 + 32\times 0.15 + 32\times 0.15 = 21$$

$$E[h_3(x)] = \sum_{x=0}^{4} h_3(x)p(x) = (-12)\times 0.15 + 8\times 0.25 + 28\times 0.3 + 48\times 0.15 + 48\times 0.15 = 23$$

$$E[h_4(x)] = \sum_{x=0}^{4} h_4(x)p(x) = (-16)\times 0.15 + 4\times 0.25 + 24\times 0.3 + 44\times 0.15 + 64\times 0.15 = 22$$

Therefore, baking 3 cakes daily would have a slightly higher expected profit.