51.

a. If a red ball is drawn from the first box, the composition of the second box becomes eight red and three green. Use the multiplication rule:

$P(\text{R from 1}^{\text{st}} \cap \text{R from 2}^{\text{nd}}) = P(\text{R from 1}^{\text{st}}) \times P(\text{R from 2}^{\text{nd}} | \text{R from 1}^{\text{st}}) = \frac{6}{10} \times \frac{8}{11} = .436$.

b. $P(\text{same numbers as originally}) = P(\text{both selected balls are the same color}) = P(\text{both R}) + P(\text{both G}) =$ $\frac{6}{10} \times \frac{8}{11} + \frac{4}{10} \times \frac{4}{11} = .581$.

54.

a. $P(A_2 | A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{.11}{.22} = .50$. If the firm is awarded project 1, there is a 50% chance they will also be awarded project 2.

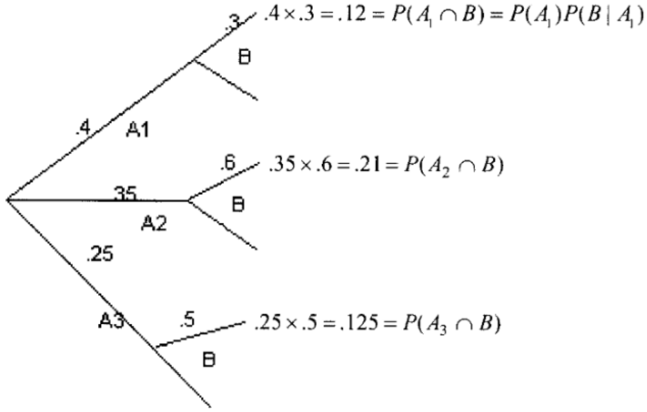
b. $P(A_2 \cap A_3 | A_1) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{.01}{.22} = .0455$. If the firm is awarded project 1, there is a 4.55% chance they will also be awarded projects 2 and 3.

c. $P(A_2 \cup A_3 | A_1) = \frac{P[A_1 \cap (A_2 \cup A_3)]}{P(A_1)} = \frac{P[(A_1 \cap A_2) \cup (A_1 \cap A_3)]}{P(A_1)}$

$= \frac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{.15}{.22} = .682$. If the firm is awarded project 1, there is a 68.2% chance they will also be awarded at least one of the other two projects.

d. $P(A_1 \cap A_2 \cap A_3 | A_1 \cup A_2 \cup A_3) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.01}{.53} = .0189$. If the firm is awarded at least one of the projects, there is a 1.89% chance they will be awarded all three projects.

59. The required probabilities appear in the tree diagram below.

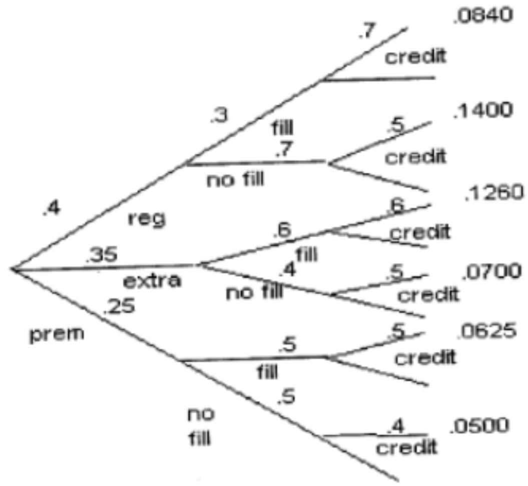


a. $P(A_2 \cap B) = .21$.

b. By the law of total probability, $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .455$.

c. Using Bayes' theorem, $P(A_1 \mid B) = \dfrac{P(A_1 \cap B)}{P(B)} = \dfrac{.12}{.455} = .264$; $P(A_2 \mid B) = \dfrac{.21}{.455} = .462$; $P(A_3 \mid B) = 1 - .264 - .462 = .274$. Notice the three probabilities sum to 1.

69. The tree diagram below summarizes the information in the exercise (plus the previous information in Exercise 59). Probabilities for the branches corresponding to paying with credit are indicated at the far right. ("extra" = "plus")



a. $P(\text{plus} \cap \text{fill} \cap \text{credit}) = (.35)(.6)(.6) = .1260$.

b. $P(\text{premium} \cap \text{no fill} \cap \text{credit}) = (.25)(.5)(.4) = .05$.

c. From the tree diagram, $P(\text{premium} \cap \text{credit}) = .0625 + .0500 = .1125$.

d. From the tree diagram, $P(\text{fill} \cap \text{credit}) = .0840 + .1260 + .0625 = .2725$.

e. $P(\text{credit}) = .0840 + .1400 + .1260 + .0700 + .0625 + .0500 = .5325$.

f. $P(\text{premium} \mid \text{credit}) = \frac{P(\text{premium} \cap \text{credit})}{P(\text{credit})} = \frac{.1125}{.5325} = .2113$.

74. Using subscripts to differentiate between the selected individuals,
$P(O_1 \cap O_2) = P(O_1)P(O_2) = (.45)(.45) = .2025$.
$P(\text{two individuals match}) = P(A_1 \cap A_2) + P(B_1 \cap B_2) + P(AB_1 \cap AB_2) + P(O_1 \cap O_2) = .40^2 + .11^2 + .04^2 + .45^2 = .3762$.

78.

$$P(\text{at least one opens}) = 1 - P(\text{none opens}) = 1 - 0.04^5 = 0.999999898$$

$$P(\text{at least one fails to open}) = 1 - P(\text{all open}) = 1 - 0.96^5 = 0.1846273.$$

80.

Let A_i denote the event that component i works ($i = 1, 2, 3, 4$). The event "the system works" is $(A_1 \cup A_2) \cup (A_3 \cap A_4)$. $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.9 + 0.9 - 0.9 \times 0.9 = 0.99$, and $P(A_3 \cap A_4) = P(A_3) \times P(A_4) = 0.8 \times 0.8 = 0.64$.

Now use the additional rule and independence for the system: $P((A_1 \cup A_2) \cup (A_3 \cap A_4)) = P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \times P(A_3 \cap A_4) = 0.99 + 0.64 - 0.99 \times 0.64 = 0.9964$.

83. We'll need to know *P*(both detect the defect) = 1 – *P*(at least one doesn't) = 1 – .2 = .8.

a. $P(1^{st}$ detects $\cap$ 2^{nd} doesn't$)$ = $P(1^{st}$ detects$)$ – $P(1^{st}$ does $\cap$ 2^{nd} does$)$ = .9 – .8 = .1.
Similarly, $P(1^{st}$ doesn't $\cap$ 2^{nd} does$)$ = .1, so *P*(exactly one does)= .1 + .1= .2.

b. *P*(neither detects a defect) = 1 – [*P*(both do) + *P*(exactly 1 does)] = 1 – [.8+.2] = 0. That is, under this model there is a 0% probability neither inspector detects a defect. As a result, *P*(all 3 escape) = (0)(0)(0) = 0.

84. a. P(all three components function properly throughout the warranty period)

$= P(A_1)P(A_2)P(A_3) = (.95)(.98)(.80) = .7448$

b. P(at least one component need service during the warranty period) = $1 - P(A_1)P(A_2)P(A_3) = .2552$

c. P(all three components need service during the warranty period)

$= P(A_1')P(A_2')P(A_3') = (1-.95)(1-.98)(1-.80) = .0002$

d. P(only the receiver need service during the warranty period)

$= P(A_1')P(A_2)P(A_3) = (1-.95)(.98)(.80) = .0392$

e. P(exactly one of the three components need service during the warranty period)

$= P(A_1')P(A_2)P(A_3) + P(A_1)P(A_2')P(A_3) + P(A_1)P(A_2)P(A_3')$

$= (1-.95)(.98)(.80) + (.95)(1-.98)(.80) + (.95)(.98)(1-.80) = .2406$

Bonus Question:

Let D = a random patient has the disease

$+_i$ = a random patient gets a positive result in the i^{th} test $(i = 1, 2)$

We know: $P(D) = 0.001$

$P(+_i \mid D) = 0.99$ and $P(+_i \mid D^c) = 1 - 0.98 = 0.02$ $(i = 1, 2)$

and $P(+_1 \cap +_2 \mid D) = P(+_1 \mid D)\, P(+_2 \mid D)$ and

$P(+_1 \cap +_2 \mid D^c) = P(+_1 \mid D^c)\, P(+_2 \mid D^c)$. (By conditional independence)

(a)

$$P(+_2 \mid +_1) = \frac{P(+_1 \cap +_2)}{P(+_1)} = \frac{P(+_1 \cap +_2 \mid D) P(D) + P(+_1 \cap +_2 \mid D^c) P(D^c)}{P(+_1 \mid D) P(D) + P(+_1 \mid D^c) P(D^c)}$$

(By the law of total probability)

$$= \frac{P(+_1 \mid D) P(+_2 \mid D) P(D) + P(+_1 \mid D^c) P(+_2 \mid D^c) P(D^c)}{P(+_1 \mid D) P(D) + P(+_2 \mid D^c) P(D^c)}$$

$$= \frac{(0.99)^2 \cdot (0.001) + (0.02)^2 \cdot (0.999)}{(0.99)(0.001) + (0.02)(0.999)}$$

$$= \frac{0.0013797}{0.02097} = 0.065794$$

(b)

$$P(D \mid +_1 \cap +_2) = \frac{P(+_1 \cap +_2 \cap D)}{P(+_1 \cap +_2)} = \frac{P(+_1 \cap +_2 \mid D)\, P(D)}{P(+_1 \cap +_2)}$$

$$= \frac{P(+_1 \mid D)\, P(+_2 \mid D)\, P(D)}{P(+_1 \cap +_2)}$$

$$= \frac{(0.99)^2 \cdot (0.001)}{0.0013797} = 0.710372$$ (By part a)