

4.

- a. The $2^4 = 16$ possible outcomes have been numbered here for later reference.

Outcome	Home Mortgage Number			
	1	2	3	4
1	<i>F</i>	<i>F</i>	<i>F</i>	<i>F</i>
2	<i>F</i>	<i>F</i>	<i>F</i>	<i>V</i>
3	<i>F</i>	<i>F</i>	<i>V</i>	<i>F</i>
4	<i>F</i>	<i>F</i>	<i>V</i>	<i>V</i>
5	<i>F</i>	<i>V</i>	<i>F</i>	<i>F</i>
6	<i>F</i>	<i>V</i>	<i>F</i>	<i>V</i>
7	<i>F</i>	<i>V</i>	<i>V</i>	<i>F</i>
8	<i>F</i>	<i>V</i>	<i>V</i>	<i>V</i>
9	<i>V</i>	<i>F</i>	<i>F</i>	<i>F</i>
10	<i>V</i>	<i>F</i>	<i>F</i>	<i>V</i>
11	<i>V</i>	<i>F</i>	<i>V</i>	<i>F</i>
12	<i>V</i>	<i>F</i>	<i>V</i>	<i>V</i>
13	<i>V</i>	<i>V</i>	<i>F</i>	<i>F</i>
14	<i>V</i>	<i>V</i>	<i>F</i>	<i>V</i>
15	<i>V</i>	<i>V</i>	<i>V</i>	<i>F</i>
16	<i>V</i>	<i>V</i>	<i>V</i>	<i>V</i>

- b. Outcome numbers 2, 3, 5, 9 above.
- c. Outcome numbers 1, 16 above.
- d. Outcome numbers 1, 2, 3, 5, 9 above.
- e. In words, the union of (c) and (d) is the event that either all of the mortgages are variable, or that at most one of them is variable-rate: outcomes 1, 2, 3, 5, 9, 16. The intersection of (c) and (d) is the event that all of the mortgages are fixed-rate: outcome 1.
- f. The union of (b) and (c) is the event that either exactly three are fixed, or that all four are the same: outcomes 1, 2, 3, 5, 9, 16. The intersection of (b) and (c) is the event that exactly three are fixed and all four are the same type. This cannot happen (the events have no outcomes in common), so the intersection of (b) and (c) is \emptyset .

6.

- a. $S = \{123, 124, 125, 213, 214, 215, 13, 14, 15, 23, 24, 25, 3, 4, 5\}$.
- b. $A = \{3, 4, 5\}$.
- c. $B = \{125, 215, 15, 25, 5\}$.
- d. $C = \{23, 24, 25, 3, 4, 5\}$.

18. The only reason that at least two bills must be selected to obtain the first \$10 bill is if the first selection was not a \$10 bill. There are $4 + 6 = 10$ non-\$10 bills out of $5 + 4 + 6 = 15$ bills in the wallet, so the probability of this event is simply $\frac{10}{15} = \frac{2}{3}$.

21. The 12 simple events are:

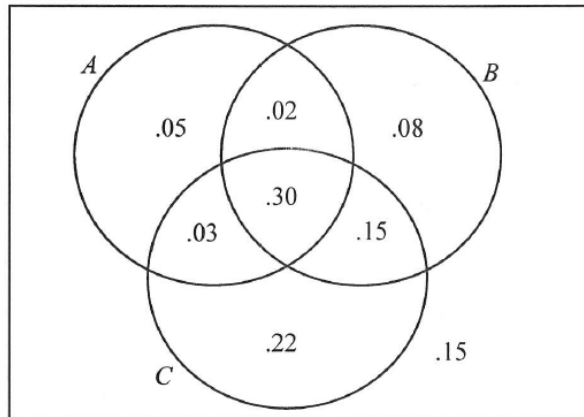
$\{LN\}, \{LL\}, \{LM\}, \{LH\}, \{MN\}, \{ML\}, \{MM\}, \{MH\}, \{HN\}, \{HL\}, \{HM\}, \{HH\}$.

In what follows, the first letter refers to the auto deductible and the second letter refers to the homeowner's deductible.

- a. $P(MH) = .10$.
- b. $P(\text{low auto deductible}) = P(\{LN, LL, LM, LH\}) = .04 + .06 + .05 + .03 = .18$. Following a similar pattern, $P(\text{low homeowner's deductible}) = .06 + .10 + .03 = .19$.
- c. $P(\text{same deductible for both}) = P(\{LL, MM, HH\}) = .06 + .20 + .15 = .41$.
- d. $P(\text{deductibles are different}) = 1 - P(\text{same deductible for both}) = 1 - .41 = .59$.
- e. $P(\text{at least one low deductible}) = P(\{LN, LL, LM, LH, ML, HL\}) = .04 + .06 + .05 + .03 + .10 + .03 = .31$.
- f. $P(\text{neither deductible is low}) = 1 - P(\text{at least one low deductible}) = 1 - .31 = .69$.

25. By rearranging the addition rule, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .40 + .55 - .63 = .32$. By the same method, $P(A \cap C) = .40 + .70 - .77 = .33$ and $P(B \cap C) = .55 + .70 - .80 = .45$. Finally, rearranging the addition rule for 3 events gives $P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C) = .85 - .40 - .55 - .70 + .32 + .33 + .45 = .30$.

These probabilities are reflected in the Venn diagram below.



- a. $P(A \cup B \cup C) = .85$, as given.
- b. $P(\text{none selected}) = 1 - P(\text{at least one selected}) = 1 - P(A \cup B \cup C) = 1 - .85 = .15$.
- c. From the Venn diagram, $P(\text{only automatic transmission selected}) = .22$.
- d. From the Venn diagram, $P(\text{exactly one of the three}) = .05 + .08 + .22 = .35$.

27. There are 10 equally likely outcomes: {A, B} {A, Co} {A, Cr} {A, F} {B, Co} {B, Cr} {B, F} {Co, Cr} {Co, F} and {Cr, F}.
- $P(\{A, B\}) = \frac{1}{10} = .1$.
 - $P(\text{at least one } C) = P(\{A, Co\} \text{ or } \{A, Cr\} \text{ or } \{B, Co\} \text{ or } \{B, Cr\} \text{ or } \{Co, Cr\} \text{ or } \{Co, F\} \text{ or } \{Cr, F\}) = \frac{7}{10} = .7$.
 - Replacing each person with his/her years of experience, $P(\text{at least 15 years}) = P(\{3, 14\} \text{ or } \{6, 10\} \text{ or } \{6, 14\} \text{ or } \{7, 10\} \text{ or } \{7, 14\} \text{ or } \{10, 14\}) = \frac{6}{10} = .6$.

30.

- Because order is important, we'll use $P_{3,8} = (8)(7)(6) = 336$.
- Order doesn't matter here, so we use $\binom{30}{6} = 593,775$.
- The number of ways to choose 2 zinfandels from the 8 available is $\binom{8}{2}$. Similarly, the number of ways to choose the merlots and cabernets are $\binom{10}{2}$ and $\binom{12}{2}$, respectively. Hence, the total number of options (using the Fundamental Counting Principle) equals $\binom{8}{2}\binom{10}{2}\binom{12}{2} = (28)(45)(66) = 83,160$.
- The numerator comes from part c and the denominator from part b: $\frac{83,160}{593,775} = .140$.
- We use the same denominator as in part d. The number of ways to choose all zinfandel is $\binom{8}{6}$, with similar answers for all merlot and all cabernet. Since these are disjoint events, $P(\text{all same}) = P(\text{all zin}) + P(\text{all merlot}) + P(\text{all cab}) = \frac{\binom{8}{6} + \binom{10}{6} + \binom{12}{6}}{\binom{30}{6}} = \frac{1162}{593,775} = .002$.

32.

- Since there are 5 receivers, 4 CD players, 3 speakers, and 4 turntables, the total number of possible selections is $(5)(4)(3)(4) = 240$.
- We now only have 1 choice for the receiver and CD player: $(1)(1)(3)(4) = 12$.
- Eliminating Sony leaves 4, 3, 3, and 3 choices for the four pieces of equipment, respectively: $(4)(3)(3)(3) = 108$.
- From a, there are 240 possible configurations. From c, 108 of them involve zero Sony products. So, the number of configurations with at least one Sony product is $240 - 108 = 132$.
- Assuming all 240 arrangements are equally likely, $P(\text{at least one Sony}) = \frac{132}{240} = .55$.

Next, $P(\text{exactly one component Sony}) = P(\text{only the receiver is Sony}) + P(\text{only the CD player is Sony}) + P(\text{only the turntable is Sony})$. Counting from the available options gives

$$P(\text{exactly one component Sony}) = \frac{(1)(3)(3)(3) + (4)(1)(3)(3) + (4)(3)(3)(1)}{240} = \frac{99}{240} = .413$$