The exact probabilities are .2622 and .8348, respectively, so the approximations are quite good. In the last calculation, the probability $P(5 \le X \le 15)$ is being approximated by the area under the normal curve between 4.5 and 15.5—the continuity correction is used for both the upper and lower limits. ■

When the objective of our investigation is to make an inference about a population proportion p, interest will focus on the sample proportion of successes X/n rather than on X itself. Because this proportion is just X multiplied by the constant $1/n$, it will also have approximately a normal distribution (with mean $\mu = p$ and standard deviation $\sigma = \sqrt{pq/n}$) provided that both $np \ge 10$ and $nq \ge 10$. This normal approximation is the basis for several inferential procedures to be discussed in later chapters.

EXERCISES Section 4.3 (28–58)

28. Let Z be a standard normal random variable and calculate the following probabilities, drawing pictures wherever appropriate.

a. $P(0 \le Z \le 2.17)$ **b.** $P(0 \le Z \le 1)$
c. $P(-2.50 \le Z \le 0)$ **d.** $P(-2.50 \le Z \le 2.50)$
e. $P(Z \le 1.37)$ **f.** $P(-1.75 \le Z)$
g. $P(-1.50 \le Z \le 2.00)$ **h.** $P(1.37 \le Z \le 2.50)$
i. $P(1.50 \le Z)$ **j.** $P(|Z| \le 2.50)$

29. In each case, determine the value of the constant c that makes the probability statement correct.

a. $\Phi(c) = .9838$ **b.** $P(0 \le Z \le c) = .291$
c. $P(c \le Z) = .121$ **d.** $P(-c \le Z \le c) = .668$
e. $P(c \le |Z|) = .016$

30. Find the following percentiles for the standard normal distribution. Interpolate where appropriate.

a. 91st **b.** 9th **c.** 75th
d. 25th **e.** 6th

31. Determine z_α for the following:

a. $\alpha = .0055$ **b.** $\alpha = .09$
c. $\alpha = .663$

32. Suppose the force acting on a column that helps to support a building is a normally distributed random variable X with mean value 15.0 kips and standard deviation 1.25 kips. Compute the following probabilities by standardizing and then using Table A.3.

a. $P(X \le 15)$ **b.** $P(X \le 17.5)$
c. $P(X \ge 10)$ **d.** $P(14 \le X \le 18)$
e. $P(|X - 15| \le 3)$

33. Mopeds (small motorcycles with an engine capacity below 50 cm^3) are very popular in Europe because of their mobility, ease of operation, and low cost. The article "Procedure to Verify the Maximum Speed of Automatic Transmission Mopeds in Periodic Motor Vehicle Inspections" (*J. of Automobile Engr.*, 2008: 1615–1623) described a rolling bench test for determining maximum vehicle speed. A normal distribution with mean value 46.8 km/h and standard deviation 1.75 km/h is postulated. Consider randomly selecting a single such moped.

a. What is the probability that maximum speed is at most 50 km/h?
b. What is the probability that maximum speed is at least 48 km/h?
c. What is the probability that maximum speed differs from the mean value by at most 1.5 standard deviations?

34. The article "Reliability of Domestic-Waste Biofilm Reactors" (*J. of Envir. Engr.*, 1995: 785–790) suggests that substrate concentration (mg/cm^3) of influent to a reactor is normally distributed with $\mu = .30$ and $\sigma = .06$.

a. What is the probability that the concentration exceeds .25?
b. What is the probability that the concentration is at most .10?
c. How would you characterize the largest 5% of all concentration values?

35. Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$, as suggested in the article "Simulating a Harvester-Forwarder Softwood Thinning" (*Forest Products J.*, May 1997: 36–41).

a. What is the probability that the diameter of a randomly selected tree will be at least 10 in.? Will exceed 10 in.?
b. What is the probability that the diameter of a randomly selected tree will exceed 20 in.?
c. What is the probability that the diameter of a randomly selected tree will be between 5 and 10 in.?
d. What value c is such that the interval $(8.8 - c, 8.8 + c)$ includes 98% of all diameter values?
e. If four trees are independently selected, what is the probability that at least one has a diameter exceeding 10 in.?

36. Spray drift is a constant concern for pesticide applicators and agricultural producers. The inverse relationship between droplet size and drift potential is well known. The paper "Effects of 2,4-D Formulation and Quinclorac on Spray Droplet Size and Deposition" (*Weed Technology*, 2005: 1030–1036) investigated the effects of herbicide formulation on spray atomization. A figure in the paper suggested the normal distribution with mean 1050 μm and standard deviation 150 μm was a reasonable model for droplet size for water (the "control treatment") sprayed through a 760 ml/min nozzle.

a. What is the probability that the size of a single droplet is less than 1500 μm? At least 1000 μm?

b. What is the probability that the size of a single droplet is between 1000 and 1500 μm?

c. How would you characterize the smallest 2% of all droplets?

d. If the sizes of five independently selected droplets are measured, what is the probability that at least one exceeds 1500 μm?

37. Suppose that blood chloride concentration (mmol/L) has a normal distribution with mean 104 and standard deviation 5 (information in the article "Mathematical Model of Chloride Concentration in Human Blood," *J. of Med. Engr. and Tech.*, 2006: 25–30, including a normal probability plot as described in Section 4.6, supports this assumption).

a. What is the probability that chloride concentration equals 105? Is less than 105? Is at most 105?

b. What is the probability that chloride concentration differs from the mean by more than 1 standard deviation? Does this probability depend on the values of μ and σ?

c. How would you characterize the most extreme .1% of chloride concentration values?

38. There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3 cm and standard deviation .1 cm. The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation .02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?

39. **a.** If a normal distribution has $\mu = 30$ and $\sigma = 5$, what is the 91st percentile of the distribution?

b. What is the 6th percentile of the distribution?

c. The width of a line etched on an integrated circuit chip is normally distributed with mean 3.000 μm and standard deviation .140. What width value separates the widest 10% of all such lines from the other 90%?

40. The article "Monte Carlo Simulation—Tool for Better Understanding of LRFD" (*J. of Structural Engr.*, 1993: 1586–1599) suggests that yield strength (ksi) for A36 grade steel is normally distributed with $\mu = 43$ and $\sigma = 4.5$.

a. What is the probability that yield strength is at most 40? Greater than 60?

b. What yield strength value separates the strongest 75% from the others?

41. The automatic opening device of a military cargo parachute has been designed to open when the parachute is 200 m above the ground. Suppose opening altitude actually has a normal distribution with mean value 200 m and standard deviation 30 m. Equipment damage will occur if the parachute opens at an altitude of less than 100 m. What is the probability that there is equipment damage to the payload of at least one of five independently dropped parachutes?

42. The temperature reading from a thermocouple placed in a constant-temperature medium is normally distributed with mean μ, the actual temperature of the medium, and standard deviation σ. What would the value of σ have to be to ensure that 95% of all readings are within .1° of μ?

43. The distribution of resistance for resistors of a certain type is known to be normal, with 10% of all resistors having a resistance exceeding 10.256 ohms and 5% having a resistance smaller than 9.671 ohms. What are the mean value and standard deviation of the resistance distribution?

44. If bolt thread length is normally distributed, what is the probability that the thread length of a randomly selected bolt is

a. Within 1.5 SDs of its mean value?

b. Farther than 2.5 SDs from its mean value?

c. Between 1 and 2 SDs from its mean value?

45. A machine that produces ball bearings has initially been set so that the true average diameter of the bearings it produces is .500 in. A bearing is acceptable if its diameter is within .004 in. of this target value. Suppose, however, that the setting has changed during the course of production, so that the bearings have normally distributed diameters with mean value .499 in. and standard deviation .002 in. What percentage of the bearings produced will not be acceptable?

46. The Rockwell hardness of a metal is determined by impressing a hardened point into the surface of the metal and then measuring the depth of penetration of the point. Suppose the Rockwell hardness of a particular alloy is normally distributed with mean 70 and standard deviation 3. (Rockwell hardness is measured on a continuous scale.)

a. If a specimen is acceptable only if its hardness is between 67 and 75, what is the probability that a randomly chosen specimen has an acceptable hardness?

b. If the acceptable range of hardness is $(70 - c, 70 + c)$, for what value of c would 95% of all specimens have acceptable hardness?

c. If the acceptable range is as in part (a) and the hardness of each of ten randomly selected specimens is independently determined, what is the expected number of acceptable specimens among the ten?

d. What is the probability that at most eight of ten independently selected specimens have a hardness of less than 73.84?

DEFINITION

Let ν be a positive integer. Then a random variable X is said to have a **chi-squared distribution** with parameter ν if the pdf of X is the gamma density with $\alpha = \nu/2$ and $\beta = 2$. The pdf of a chi-squared rv is thus

$$f(x;\nu) = \begin{cases} \dfrac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{(\nu/2)-1}e^{-x/2} & x \ge 0 \\ 0 & x < 0 \end{cases} \tag{4.10}$$

The parameter ν is called the **number of degrees of freedom** (df) of X. The symbol χ^2 is often used in place of "chi-squared."

EXERCISES Section 4.4 (59–71)

59. Let X = the time between two successive arrivals at the drive-up window of a local bank. If X has an exponential distribution with $\lambda = 1$ (which is identical to a standard gamma distribution with $\alpha = 1$), compute the following:

- **a.** The expected time between two successive arrivals
- **b.** The standard deviation of the time between successive arrivals
- **c.** $P(X \le 4)$ **d.** $P(2 \le X \le 5)$

60. Let X denote the distance (m) that an animal moves from its birth site to the first territorial vacancy it encounters. Suppose that for banner-tailed kangaroo rats, X has an exponential distribution with parameter $\lambda = .01386$ (as suggested in the article **"Competition and Dispersal from Multiple Nests,"** ***Ecology*, 1997: 873–883**).

- **a.** What is the probability that the distance is at most 100 m? At most 200 m? Between 100 and 200 m?
- **b.** What is the probability that distance exceeds the mean distance by more than 2 standard deviations?
- **c.** What is the value of the median distance?

61. Data collected at Toronto Pearson International Airport suggests that an exponential distribution with mean value 2.725 hours is a good model for rainfall duration (***Urban Stormwater Management Planning with Analytical Probabilistic Models*, 2000, p. 69**).

- **a.** What is the probability that the duration of a particular rainfall event at this location is at least 2 hours? At most 3 hours? Between 2 and 3 hours?
- **b.** What is the probability that rainfall duration exceeds the mean value by more than 2 standard deviations? What is the probability that it is less than the mean value by more than one standard deviation?

62. The article **"Microwave Observations of Daily Antarctic Sea-Ice Edge Expansion and Contribution Rates"** (***IEEE Geosci. and Remote Sensing Letters*, 2006: 54–58**) states that "The distribution of the daily sea-ice advance/retreat from each sensor is similar and is approximately double exponential." The proposed double exponential distribution has density function $f(x) = .5\lambda e^{-\lambda|x|}$ for $-\infty < x < \infty$. The standard deviation is given as 40.9 km.

- **a.** What is the value of the parameter λ?
- **b.** What is the probability that the extent of daily sea-ice change is within 1 standard deviation of the mean value?

63. A consumer is trying to decide between two long-distance calling plans. The first one charges a flat rate of 10¢ per minute, whereas the second charges a flat rate of 99¢ for calls up to 20 minutes in duration and then 10¢ for each additional minute exceeding 20 (assume that calls lasting a noninteger number of minutes are charged proportionately to a whole-minute's charge). Suppose the consumer's distribution of call duration is exponential with parameter λ.

- **a.** Explain intuitively how the choice of calling plan should depend on what the expected call duration is.
- **b.** Which plan is better if expected call duration is 10 minutes? 15 minutes? [*Hint*: Let $h_1(x)$ denote the cost for the first plan when call duration is x minutes and let $h_2(x)$ be the cost function for the second plan. Give expressions for these two cost functions, and then determine the expected cost for each plan.]

64. Evaluate the following:

- **a.** $\Gamma(6)$ **b.** $\Gamma(5/2)$
- **c.** $F(4; 5)$ (the incomplete gamma function) and $F(5; 4)$
- **d.** $P(X \le 5)$ when X has a standard gamma distribution with $\alpha = 7$.
- **e.** $P(3 < X < 8)$ when X has the distribution specified in (d).

65. Let X denote the data transfer time (ms) in a grid computing system (the time required for data transfer between a "worker" computer and a "master" computer. Suppose that X has a gamma distribution with mean value 37.5 ms and standard deviation 21.6 (suggested by the article **"Computation Time of Grid Computing with Data Transfer Times that Follow a Gamma Distribution,"** ***Proceedings of the First International Conference on Semantics, Knowledge, and Grid, 2005*****)**.

a. What are the values of α and β?

b. What is the probability that data transfer time exceeds 50 ms?

c. What is the probability that data transfer time is between 50 and 75 ms?

66. The two-parameter gamma distribution can be generalized by introducing a third parameter γ, called a *threshold* or *location* parameter: replace x in (4.8) by $x - \gamma$ and $x \geq 0$ by $x \geq \gamma$. This amounts to shifting the density curves in Figure 4.27 so that they begin their ascent or descent at γ rather than 0. The article **"Bivariate Flood Frequency Analysis with Historical Information Based on Copulas"** **(*J. of Hydrologic Engr.*, 2013: 1018–1030)** employs this distribution to model X = 3-day flood volume (10^8 m^3). Suppose that values of the parameters are $\alpha = 12$, $\beta = 7$, $\gamma = 40$ (very close to estimates in the cited article based on past data).

a. What are the mean value and standard deviation of X?

b. What is the probability that flood volume is between 100 and 150?

c. What is the probability that flood volume exceeds its mean value by more than one standard deviation?

d. What is the 95th percentile of the flood volume distribution?

67. Suppose that when a transistor of a certain type is subjected to an accelerated life test, the lifetime X (in weeks) has a gamma distribution with mean 24 weeks and standard deviation 12 weeks.

a. What is the probability that a transistor will last between 12 and 24 weeks?

b. What is the probability that a transistor will last at most 24 weeks? Is the median of the lifetime distribution less than 24? Why or why not?

c. What is the 99th percentile of the lifetime distribution?

d. Suppose the test will actually be terminated after t weeks. What value of t is such that only .5% of all transistors would still be operating at termination?

68. The special case of the gamma distribution in which α is a positive integer n is called an Erlang distribution. If we replace β by $1/\lambda$ in Expression (4.8), the Erlang pdf is

$$f(x; \lambda, n) = \begin{cases} \dfrac{\lambda(\lambda x)^{n-1}e^{-\lambda x}}{(n-1)!} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

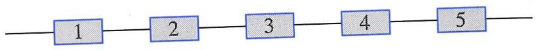
It can be shown that if the times between successive events are independent, each with an exponential distribution with parameter λ, then the total time X that elapses before all of the next n events occur has pdf $f(x; \lambda, n)$.

a. What is the expected value of X? If the time (in minutes) between arrivals of successive customers is exponentially distributed with $\lambda = .5$, how much time can be expected to elapse before the tenth customer arrives?

b. If customer interarrival time is exponentially distributed with $\lambda = .5$, what is the probability that the tenth customer (after the one who has just arrived) will arrive within the next 30 min?

c. The event $\{X \leq t\}$ occurs iff at least n events occur in the next t units of time. Use the fact that the number of events occurring in an interval of length t has a Poisson distribution with parameter λt to write an expression (involving Poisson probabilities) for the Erlang cdf $F(t; \lambda, n) = P(X \leq t)$.

69. A system consists of five identical components connected in series as shown:



As soon as one component fails, the entire system will fail. Suppose each component has a lifetime that is exponentially distributed with $\lambda = .01$ and that components fail independently of one another. Define events A_i = {ith component lasts at least t hours}, $i = 1, \ldots, 5$, so that the A_is are independent events. Let X = the time at which the system fails—that is, the shortest (minimum) lifetime among the five components.

a. The event $\{X \geq t\}$ is equivalent to what event involving $A_1, \ldots, A_5$?

b. Using the independence of the A_i's, compute $P(X \geq t)$. Then obtain $F(t) = P(X \leq t)$ and the pdf of X. What type of distribution does X have?

c. Suppose there are n components, each having exponential lifetime with parameter λ. What type of distribution does X have?

70. If X has an exponential distribution with parameter λ, derive a general expression for the $(100p)$th percentile of the distribution. Then specialize to obtain the median.

71. **a.** The event $\{X^2 \leq y\}$ is equivalent to what event involving X itself?

b. If X has a standard normal distribution, use part (a) to write the integral that equals $P(X^2 \leq y)$. Then differentiate this with respect to y to obtain the pdf of X^2 [the square of a $N(0, 1)$ variable]. Finally, show that X^2 has a chi-squared distribution with $\nu = 1$ df [see (4.10)]. [*Hint*: Use the following identity.]

$$\frac{d}{dy}\left\{\int_{a(y)}^{b(y)} f(x)\,dx\right\} = f[b(y)] \cdot b'(y) - f[a(y)] \cdot a'(y)$$

c. What is the probability that lifetime is at least 200? Greater than 200?

78. The article **"On Assessing the Accuracy of Offshore Wind Turbine Reliability-Based Design Loads from the Environmental Contour Method" (*Intl. J. of Offshore and Polar Engr.*, 2005: 132–140)** proposes the Weibull distribution with $\alpha = 1.817$ and $\beta = .863$ as a model for 1-hour significant wave height (m) at a certain site.

a. What is the probability that wave height is at most .5 m?

b. What is the probability that wave height exceeds its mean value by more than one standard deviation?

c. What is the median of the wave-height distribution?

d. For $0 < p < 1$, give a general expression for the 100pth percentile of the wave-height distribution.

79. Nonpoint source loads are chemical masses that travel to the main stem of a river and its tributaries in flows that are distributed over relatively long stream reaches, in contrast to those that enter at well-defined and regulated points. The article **"Assessing Uncertainty in Mass Balance Calculation of River Nonpoint Source Loads" (*J. of Envir. Engr.*, 2008: 247–258)** suggested that for a certain time period and location, X = nonpoint source load of total dissolved solids could be modeled with a lognormal distribution having mean value 10,281 kg/day/km and a coefficient of variation $CV = .40$ ($CV = \sigma_X/\mu_X$).

a. What are the mean value and standard deviation of $\ln(X)$?

b. What is the probability that X is at most 15,000 kg/day/km?

c. What is the probability that X exceeds its mean value, and why is this probability not .5?

d. Is 17,000 the 95th percentile of the distribution?

80. **a.** Use Equation (4.13) to write a formula for the median $\tilde{\mu}$ of the lognormal distribution. What is the median for the load distribution of Exercise 79?

b. Recalling that z_α is our notation for the $100(1-\alpha)$ percentile of the standard normal distribution, write an expression for the $100(1-\alpha)$ percentile of the lognormal distribution. In Exercise 79, what value will load exceed only 1% of the time?

81. Sales delay is the elapsed time between the manufacture of a product and its sale. According to the article **"Warranty Claims Data Analysis Considering Sales Delay" (*Quality and Reliability Engr. Intl.*, 2013: 113–123),** it is quite common for investigators to model sales delay using a lognormal distribution. For a particular product, the cited article proposes this distribution with parameter values $\mu = 2.05$ and $\sigma^2 = .06$ (here the unit for delay is months).

a. What are the variance and standard deviation of delay time?

b. What is the probability that delay time exceeds 12 months?

c. What is the probability that delay time is within one standard deviation of its mean value?

d. What is the median of the delay time distribution?

e. What is the 99th percentile of the delay time distribution?

f. Among 10 randomly selected such items, how many would you expect to have a delay time exceeding 8 months?

82. As in the case of the Weibull and Gamma distributions, the lognormal distribution can be modified by the introduction of a third parameter γ such that the pdf is shifted to be positive only for $x > \gamma$. The article cited in Exercise 4.39 suggested that a shifted lognormal distribution with shift (i.e., threshold) = 1.0, mean value = 2.16, and standard deviation = 1.03 would be an appropriate model for the rv X = *maximum-to-average depth ratio* of a corrosion defect in pressurized steel.

a. What are the values of μ and σ for the proposed distribution?

b. What is the probability that depth ratio exceeds 2?

c. What is the median of the depth ratio distribution?

d. What is the 99th percentile of the depth ratio distribution?

83. What condition on α and β is necessary for the standard beta pdf to be symmetric?

84. Suppose the proportion X of surface area in a randomly selected quadrat that is covered by a certain plant has a standard beta distribution with $\alpha = 5$ and $\beta = 2$.

a. Compute $E(X)$ and $V(X)$.

b. Compute $P(X \le .2)$.

c. Compute $P(.2 \le X \le .4)$.

d. What is the expected proportion of the sampling region not covered by the plant?

85. Let X have a standard beta density with parameters α and β.

a. Verify the formula for $E(X)$ given in the section.

b. Compute $E[(1 - X)^m]$. If X represents the proportion of a substance consisting of a particular ingredient, what is the expected proportion that does not consist of this ingredient?

86. Stress is applied to a 20-in. steel bar that is clamped in a fixed position at each end. Let Y = the distance from the left end at which the bar snaps. Suppose $Y/20$ has a standard beta distribution with $E(Y) = 10$ and $V(Y) = \frac{100}{7}$.

a. What are the parameters of the relevant standard beta distribution?

b. Compute $P(8 \le Y \le 12)$.

c. Compute the probability that the bar snaps more than 2 in. from where you expect it to.

a. Obtain the pdf $f(x)$ and sketch its graph.

b. Compute $P(.5 \le X \le 2)$.

c. Compute $E(X)$.

102. The breakdown voltage of a randomly chosen diode of a certain type is known to be normally distributed with mean value 40 V and standard deviation 1.5 V.

a. What is the probability that the voltage of a single diode is between 39 and 42?

b. What value is such that only 15% of all diodes have voltages exceeding that value?

c. If four diodes are independently selected, what is the probability that at least one has a voltage exceeding 42?

103. The article "Computer Assisted Net Weight Control" (*Quality Progress,* 1983: 22–25) suggests a normal distribution with mean 137.2 oz and standard deviation 1.6 oz for the actual contents of jars of a certain type. The stated contents was 135 oz.

a. What is the probability that a single jar contains more than the stated contents?

b. Among ten randomly selected jars, what is the probability that at least eight contain more than the stated contents?

c. Assuming that the mean remains at 137.2, to what value would the standard deviation have to be changed so that 95% of all jars contain more than the stated contents?

104. When circuit boards used in the manufacture of compact disc players are tested, the long-run percentage of defectives is 5%. Suppose that a batch of 250 boards has been received and that the condition of any particular board is independent of that of any other board.

a. What is the approximate probability that at least 10% of the boards in the batch are defective?

b. What is the approximate probability that there are exactly 10 defectives in the batch?

105. The article "Characterization of Room Temperature Damping in Aluminum-Indium Alloys" (*Metallurgical Trans.,* 1993: 1611–1619) suggests that Al matrix grain size (μm) for an alloy consisting of 2% indium could be modeled with a normal distribution with a mean value 96 and standard deviation 14.

a. What is the probability that grain size exceeds 100?

b. What is the probability that grain size is between 50 and 80?

c. What interval (a, b) includes the central 90% of all grain sizes (so that 5% are below a and 5% are above b)?

106. The reaction time (in seconds) to a certain stimulus is a continuous random variable with pdf

$$f(x) = \begin{cases} \dfrac{3}{2} \cdot \dfrac{1}{x^2} & 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

a. Obtain the cdf.

b. What is the probability that reaction time is at most 2.5 sec? Between 1.5 and 2.5 sec?

c. Compute the expected reaction time.

d. Compute the standard deviation of reaction time.

e. If an individual takes more than 1.5 sec to react, a light comes on and stays on either until one further second has elapsed or until the person reacts (whichever happens first). Determine the expected amount of time that the light remains lit. [*Hint*: Let $h(X)$ = the time that the light is on as a function of reaction time X.]

107. Let X denote the temperature at which a certain chemical reaction takes place. Suppose that X has pdf

$$f(x) = \begin{cases} \dfrac{1}{9}(4 - x^2) & -1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a. Sketch the graph of $f(x)$.

b. Determine the cdf and sketch it.

c. Is 0 the median temperature at which the reaction takes place? If not, is the median temperature smaller or larger than 0?

d. Suppose this reaction is independently carried out once in each of ten different labs and that the pdf of reaction time in each lab is as given. Let Y = the number among the ten labs at which the temperature exceeds 1. What kind of distribution does Y have? (Give the names and values of any parameters.)

108. The article "Determination of the MTF of Positive Photoresists Using the Monte Carlo Method" (*Photographic Sci. and Engr.,* 1983: 254–260) proposes the exponential distribution with parameter $\lambda = .93$ as a model for the distribution of a photon's free path length (μm) under certain circumstances. Suppose this is the correct model.

a. What is the expected path length, and what is the standard deviation of path length?

b. What is the probability that path length exceeds 3.0? What is the probability that path length is between 1.0 and 3.0?

c. What value is exceeded by only 10% of all path lengths?

109. The article "The Prediction of Corrosion by Statistical Analysis of Corrosion Profiles" (*Corrosion Science,* 1985: 305–315) suggests the following cdf for the depth X of the deepest pit in an experiment involving the exposure of carbon manganese steel to acidified seawater.

$$F(x; \alpha, \beta) = e^{-e^{-(x-\alpha)/\beta}} \qquad -\infty < x < \infty$$

The authors propose the values $\alpha = 150$ and $\beta = 90$. Assume this to be the correct model.

a. What is the probability that the depth of the deepest pit is at most 150? At most 300? Between 150 and 300?

b. Below what value will the depth of the maximum pit be observed in 90% of all such experiments?

c. What is the density function of X?

d. The density function can be shown to be unimodal (a single peak). Above what value on the measurement axis does this peak occur? (This value is the mode.)

Bonus problem:

Let X have an exponential distribution with parameter λ (> 0) and c be a positive constant.

(a) Find $P(X \leq x \mid X > c)$ for all possible values of x. This is the "conditional CDF" of X given $X > c$.

(b) Now find the "conditional PDF" of X given $X > c$ from part a. Verify that it is a valid PDF. This is called a 'Shifted exponential distribution'.

(c) Now find $E (X \mid X > c)$, i.e. the expectation of the conditional distribution (or conditional expectation) of X given $X > c$ from part b.

(d) How can you obtain the answer of part c without doing any integration? Write a brief explanation.