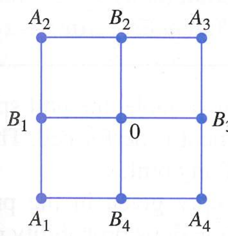
7. For each random variable defined here, describe the set of possible values for the variable, and state whether the variable is discrete.
   a. X = the number of unbroken eggs in a randomly chosen standard egg carton
   b. Y = the number of students on a class list for a particular course who are absent on the first day of classes
   c. U = the number of times a duffer has to swing at a golf ball before hitting it
   d. X = the length of a randomly selected rattlesnake
   e. Z = the sales tax percentage for a randomly selected amazon.com purchase
   f. Y = the pH of a randomly chosen soil sample
   g. X = the tension (psi) at which a randomly selected tennis racket has been strung
   h. X = the total number of times three tennis players must spin their rackets to obtain something other than *UUU* or *DDD* (to determine which two play next)

8. Each time a component is tested, the trial is a success (S) or failure (F). Suppose the component is tested repeatedly until a success occurs on three *consecutive* trials. Let Y denote the number of trials necessary to achieve this. List all outcomes corresponding to the five smallest possible values of Y, and state which Y value is associated with each one.

9. An individual named Claudius is located at the point 0 in the accompanying diagram.



Using an appropriate randomization device (such as a tetrahedral die, one having four sides), Claudius first moves to one of the four locations B_1, B_2, B_3, B_4. Once at one of these locations, another randomization device is used to decide whether Claudius next returns to 0 or next visits one of the other two adjacent points. This process then continues; after each move, another move to one of the (new) adjacent points is determined by tossing an appropriate die or coin.
   a. Let X = the number of moves that Claudius makes before first returning to 0. What are possible values of X? Is X discrete or continuous?
   b. If moves are allowed also along the diagonal paths connecting 0 to A_1, A_2, A_3, and A_4, respectively, answer the questions in part (a).

10. The number of pumps in use at both a six-pump station and a four-pump station will be determined. Give the possible values for each of the following random variables:
   a. T = the total number of pumps in use
   b. X = the difference between the numbers in use at stations 1 and 2
   c. U = the maximum number of pumps in use at either station
   d. Z = the number of stations having exactly two pumps in use

3.2 Probability Distributions for Discrete Random Variables

Probabilities assigned to various outcomes in $\mathcal{S}$ in turn determine probabilities associated with the values of any particular rv X. The *probability distribution of* X says how the total probability of 1 is distributed among (allocated to) the various possible X values. Suppose, for example, that a business has just purchased four laser printers, and let X be the number among these that require service during the warranty period. Possible X values are then 0, 1, 2, 3, and 4. The probability distribution will tell us how the probability of 1 is subdivided among these five possible values—how much probability is associated with the X value 0, how much is apportioned to the X value 1, and so on. We will use the following notation for the probabilities in the distribution:

$$p(0) = \text{the probability of the } X \text{ value } 0 = P(X = 0)$$
$$p(1) = \text{the probability of the } X \text{ value } 1 = P(X = 1)$$

and so on. In general, $p(x)$ will denote the probability assigned to the value x.

14. A contractor is required by a county planning department to submit one, two, three, four, or five forms (depending on the nature of the project) in applying for a building permit. Let Y = the number of forms required of the next applicant. The probability that y forms are required is known to be proportional to y—that is, $p(y) = ky$ for $y = 1, \dots, 5$.

a. What is the value of k? [*Hint:* $\sum_{y=1}^{5} p(y) = 1$.]

b. What is the probability that at most three forms are required?

c. What is the probability that between two and four forms (inclusive) are required?

d. Could $p(y) = y^2/50$ for $y = 1, \dots, 5$ be the pmf of Y?

15. Many manufacturers have quality control programs that include inspection of incoming materials for defects. Suppose a computer manufacturer receives computer boards in lots of five. Two boards are selected from each lot for inspection. We can represent possible outcomes of the selection process by pairs. For example, the pair (1, 2) represents the selection of boards 1 and 2 for inspection.

a. List the ten different possible outcomes.

b. Suppose that boards 1 and 2 are the only defective boards in a lot of five. Two boards are to be chosen at random. Define X to be the number of defective boards observed among those inspected. Find the probability distribution of X.

c. Let $F(x)$ denote the cdf of X. First determine $F(0) = P(X \leq 0)$, $F(1)$, and $F(2)$; then obtain $F(x)$ for all other x.

16. Some parts of California are particularly earthquake-prone. Suppose that in one metropolitan area, 25% of all homeowners are insured against earthquake damage. Four homeowners are to be selected at random; let X denote the number among the four who have earthquake insurance.

a. Find the probability distribution of X. [*Hint:* Let S denote a homeowner who has insurance and F one who does not. Then one possible outcome is *SFSS*, with probability (.25)(.75)(.25)(.25) and associated X value 3. There are 15 other outcomes.]

b. Draw the corresponding probability histogram.

c. What is the most likely value for X?

d. What is the probability that at least two of the four selected have earthquake insurance?

17. A new battery's voltage may be acceptable (A) or unacceptable (U). A certain flashlight requires two batteries, so batteries will be independently selected and tested until two acceptable ones have been found. Suppose that 90% of all batteries have acceptable voltages. Let Y denote the number of batteries that must be tested.

a. What is $p(2)$, that is, $P(Y = 2)$?

b. What is $p(3)$? [*Hint:* There are two different outcomes that result in $Y = 3$.]

c. To have $Y = 5$, what must be true of the fifth battery selected? List the four outcomes for which $Y = 5$ and then determine $p(5)$.

d. Use the pattern in your answers for parts (a)–(c) to obtain a general formula for $p(y)$.

18. Two fair six-sided dice are tossed independently. Let M = the maximum of the two tosses (so $M(1,5) = 5$, $M(3,3) = 3$, etc.).

a. What is the pmf of M? [*Hint:* First determine $p(1)$, then $p(2)$, and so on.]

b. Determine the cdf of M and graph it.

19. A library subscribes to two different weekly news magazines, each of which is supposed to arrive in Wednesday's mail. In actuality, each one may arrive on Wednesday, Thursday, Friday, or Saturday. Suppose the two arrive independently of one another, and for each one $P(\text{Wed.}) = .3$, $P(\text{Thurs.}) = .4$, $P(\text{Fri.}) = .2$, and $P(\text{Sat.}) = .1$. Let Y = the number of days beyond Wednesday that it takes for both magazines to arrive (so possible Y values are 0, 1, 2, or 3). Compute the pmf of Y. [*Hint:* There are 16 possible outcomes; $Y(W,W) = 0$, $Y(F,Th) = 2$, and so on.]

20. Three couples and two single individuals have been invited to an investment seminar and have agreed to attend. Suppose the probability that any particular couple or individual arrives late is .4 (a couple will travel together in the same vehicle, so either both people will be on time or else both will arrive late). Assume that different couples and individuals are on time or late independently of one another. Let X = the number of people who arrive late for the seminar.

a. Determine the probability mass function of X. [*Hint:* label the three couples #1, #2, and #3 and the two individuals #4 and #5.]

b. Obtain the cumulative distribution function of X, and use it to calculate $P(2 \leq X \leq 6)$.

21. Suppose that you read through this year's issues of the *New York Times* and record each number that appears in a news article—the income of a CEO, the number of cases of wine produced by a winery, the total charitable contribution of a politician during the previous tax year, the age of a celebrity, and so on. Now focus on the leading digit of each number, which could be 1, 2, . . . , 8, or 9. Your first thought might be that the leading digit X of a randomly selected number would be equally likely to be one of the nine possibilities (a discrete uniform distribution). However, much empirical evidence as well as some theoretical arguments suggest an alternative probability distribution called *Benford's law:*

$$p(x) = P(\text{1st digit is } x) = \log_{10}\left(\frac{x+1}{x}\right) \qquad x = 1, 2, \dots, 9$$

a. Without computing individual probabilities from this formula, show that it specifies a legitimate pmf.

b. Now compute the individual probabilities and compare to the corresponding discrete uniform distribution.

c. Obtain the cdf of X.

d. Using the cdf, what is the probability that the leading digit is at most 3? At least 5?

[*Note:* Benford's law is the basis for some auditing procedures used to detect fraud in financial reporting—for example, by the Internal Revenue Service.]

22. Refer to Exercise 13, and calculate and graph the cdf $F(x)$. Then use it to calculate the probabilities of the events given in parts (a)–(d) of that problem.

23. A consumer organization that evaluates new automobiles customarily reports the number of major defects in each car examined. Let X denote the number of major defects in a randomly selected car of a certain type. The cdf of X is as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ .06 & 0 \le x < 1 \\ .19 & 1 \le x < 2 \\ .39 & 2 \le x < 3 \\ .67 & 3 \le x < 4 \\ .92 & 4 \le x < 5 \\ .97 & 5 \le x < 6 \\ 1 & 6 \le x \end{cases}$$

Calculate the following probabilities directly from the cdf:

a. $p(2)$, that is, $P(X = 2)$ **b.** $P(X > 3)$
c. $P(2 \le X \le 5)$ **d.** $P(2 < X < 5)$

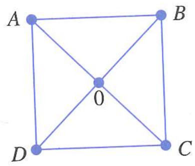
24. An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let X = the number of months between successive payments. The cdf of X is as follows:

$$F(x) = \begin{cases} 0 & x < 1 \\ .30 & 1 \le x < 3 \\ .40 & 3 \le x < 4 \\ .45 & 4 \le x < 6 \\ .60 & 6 \le x < 12 \\ 1 & 12 \le x \end{cases}$$

a. What is the pmf of X?
b. Using just the cdf, compute $P(3 \le X \le 6)$ and $P(4 \le X)$.

25. In Example 3.12, let Y = the number of girls born before the experiment terminates. With $p = P(B)$ and $1 - p = P(G)$, what is the pmf of Y? [*Hint:* First list the possible values of Y, starting with the smallest, and proceed until you see a general formula.]

26. Alvie Singer lives at 0 in the accompanying diagram and has four friends who live at A, B, C, and D. One day Alvie decides to go visiting, so he tosses a fair coin twice to decide which of the four to visit. Once at a friend's house, he will either return home or else proceed to one of the two adjacent houses (such as 0, A, or C when at B), with each of the three possibilities having probability $\frac{1}{3}$. In this way, Alvie continues to visit friends until he returns home.



a. Let X = the number of times that Alvie visits a friend. Derive the pmf of X.
b. Let Y = the number of straight-line segments that Alvie traverses (including those leading to and from 0). What is the pmf of Y?
c. Suppose that female friends live at A and C and male friends at B and D. If Z = the number of visits to female friends, what is the pmf of Z?

27. After all students have left the classroom, a statistics professor notices that four copies of the text were left under desks. At the beginning of the next lecture, the professor distributes the four books in a completely random fashion to each of the four students (1, 2, 3, and 4) who claim to have left books. One possible outcome is that 1 receives 2's book, 2 receives 4's book, 3 receives his or her own book, and 4 receives 1's book. This outcome can be abbreviated as (2, 4, 3, 1).

a. List the other 23 possible outcomes.
b. Let X denote the number of students who receive their own book. Determine the pmf of X.

28. Show that the cdf $F(x)$ is a nondecreasing function; that is, $x_1 < x_2$ implies that $F(x_1) \le F(x_2)$. Under what condition will $F(x_1) = F(x_2)$?

3.3 Expected Values

Consider a university having 15,000 students and let X = the number of courses for which a randomly selected student is registered. The pmf of X follows. Since $p(1) = .01$, we know that $(.01) \cdot (15{,}000) = 150$of the students are registered for one course, and similarly for the other x values.

x	1	2	3	4	5	6	7
$p(x)$	.01	.03	.13	.25	.39	.17	.02
Number registered	150	450	1950	3750	5850	2550	300

(3.6)

36. Let X be the damage incurred (in \$) in a certain type of accident during a given year. Possible X values are 0, 1000, 5000, and 10000, with probabilities .8, .1, .08, and .02, respectively. A particular company offers a \$500 deductible policy. If the company wishes its expected profit to be \$100, what premium amount should it charge?

37. The n candidates for a job have been ranked $1, 2, 3, \ldots, n$. Let $X =$ the rank of a randomly selected candidate, so that X has pmf

$$p(x) = \begin{cases} 1/n & x = 1, 2, 3, \ldots, n \\ 0 & \text{otherwise} \end{cases}$$

(this is called the *discrete uniform distribution*). Compute $E(X)$ and $V(X)$ using the shortcut formula. [*Hint:* The sum of the first n positive integers is $n(n+1)/2$, whereas the sum of their squares is $n(n+1)(2n+1)/6$.]

38. Let $X =$ the outcome when a fair die is rolled once. If before the die is rolled you are offered either (1/3.5) dollars or $h(X) = 1/X$ dollars, would you accept the guaranteed amount or would you gamble? [*Note:* It is not generally true that $1/E(X) = E(1/X)$.]

39. A chemical supply company currently has in stock 100 lb of a certain chemical, which it sells to customers in 5-lb batches. Let $X =$ the number of batches ordered by a randomly chosen customer, and suppose that X has pmf

x	1	2	3	4
$p(x)$	.2	.4	.3	.1

Compute $E(X)$ and $V(X)$. Then compute the expected number of pounds left after the next customer's order is shipped and the variance of the number of pounds left. [*Hint:* The number of pounds left is a linear function of X.]

40. **a.** Draw a line graph of the pmf of X in Exercise 35. Then determine the pmf of $-X$ and draw its line graph. From these two pictures, what can you say about $V(X)$ and $V(-X)$?

b. Use the proposition involving $V(aX + b)$ to establish a general relationship between $V(X)$ and $V(-X)$.

41. Use the definition in Expression (3.13) to prove that $V(aX + b) = a^2 \cdot \sigma_X^2$ [*Hint:* With $h(X) = aX + b$, $E[h(X)] = a\mu + b$ where $\mu = E(X)$.]

42. Suppose $E(X) = 5$ and $E[X(X-1)] = 27.5$. What is

a. $E(X^2)$? [*Hint:* $E[X(X-1)] = E[X^2 - X] = E(X^2) - E(X)$]?

b. $V(X)$?

c. The general relationship among the quantities $E(X)$, $E[X(X-1)]$, and $V(X)$?

43. Write a general rule for $E(X - c)$ where c is a constant. What happens when you let $c = \mu$, the expected value of X?

44. A result called **Chebyshev's inequality** states that for any probability distribution of an rv X and any number k that is at least 1, $P(|X - \mu| \geq k\sigma) \leq 1/k^2$. In words, the probability that the value of X lies at least k standard deviations from its mean is at most $1/k^2$.

a. What is the value of the upper bound for $k = 2$? $k = 3$? $k = 4$? $k = 5$? $k = 10$?

b. Compute μ and σ for the distribution of Exercise 13. Then evaluate $P(|X - \mu| \geq k\sigma)$ for the values of k given in part (a). What does this suggest about the upper bound relative to the corresponding probability?

c. Let X have possible values -1, 0, and 1, with probabilities $\frac{1}{18}$, $\frac{8}{9}$, and $\frac{1}{18}$, respectively. What is $P(|X - \mu| \geq 3\sigma)$, and how does it compare to the corresponding bound?

d. Give a distribution for which $P(|X - \mu| \geq 5\sigma) = .04$.

45. If $a \leq X \leq b$, show that $a \leq E(X) \leq b$.

3.4 The Binomial Probability Distribution

There are many experiments that conform either exactly or approximately to the following list of requirements:

1. The experiment consists of a sequence of n smaller experiments called *trials*, where n is fixed in advance of the experiment.
2. Each trial can result in one of the same two possible outcomes (dichotomous trials), which we generically denote by success (S) and failure (F).
3. The trials are independent, so that the outcome on any particular trial does not influence the outcome on any other trial.
4. The probability of success $P(S)$ is constant from trial to trial; we denote this probability by p.

DEFINITION

An experiment for which Conditions 1–4 are satisfied is called a **binomial experiment.**

c. The bookstore has 15 new copies and 15 used copies in stock. If 25 people come in one by one to purchase this text, what is the probability that all 25 will get the type of book they want from current stock? [*Hint:* Let $X =$ the number who want a new copy. For what values of X will all 25 get what they want?]

d. Suppose that new copies cost \$100 and used copies cost \$70. Assume the bookstore currently has 50 new copies and 50 used copies. What is the expected value of total revenue from the sale of the next 25 copies purchased? Be sure to indicate what rule of expected value you are using. [*Hint:* Let $h(X) =$ the revenue when X of the 25 purchasers want new copies. Express this as a linear function.]

53. Exercise 30 (Section 3.3) gave the pmf of Y, the number of traffic citations for a randomly selected individual insured by a particular company. What is the probability that among 15 randomly chosen such individuals

a. At least 10 have no citations?

b. Fewer than half have at least one citation?

c. The number that have at least one citation is between 5 and 10, inclusive?*

54. A particular type of tennis racket comes in a midsize version and an oversize version. Sixty percent of all customers at a certain store want the oversize version.

a. Among ten randomly selected customers who want this type of racket, what is the probability that at least six want the oversize version?

b. Among ten randomly selected customers, what is the probability that the number who want the oversize version is within 1 standard deviation of the mean value?

c. The store currently has seven rackets of each version. What is the probability that all of the next ten customers who want this racket can get the version they want from current stock?

55. Twenty percent of all telephones of a certain type are submitted for service while under warranty. Of these, 60% can be repaired, whereas the other 40% must be replaced with new units. If a company purchases ten of these telephones, what is the probability that exactly two will end up being replaced under warranty?

56. The College Board reports that 2% of the 2 million high school students who take the SAT each year receive special accommodations because of documented disabilities (*Los Angeles Times,* July 16, 2002). Consider a random sample of 25 students who have recently taken the test.

a. What is the probability that exactly 1 received a special accommodation?

b. What is the probability that at least 1 received a special accommodation?

c. What is the probability that at least 2 received a special accommodation?

d. What is the probability that the number among the 25 who received a special accommodation is within 2 standard deviations of the number you would expect to be accommodated?

e. Suppose that a student who does not receive a special accommodation is allowed 3 hours for the exam, whereas an accommodated student is allowed 4.5 hours. What would you expect the average time allowed the 25 selected students to be?

57. Suppose that 90% of all batteries from a certain supplier have acceptable voltages. A certain type of flashlight requires two type-D batteries, and the flashlight will work only if both its batteries have acceptable voltages. Among ten randomly selected flashlights, what is the probability that at least nine will work? What assumptions did you make in the course of answering the question posed?

58. A very large batch of components has arrived at a distributor. The batch can be characterized as acceptable only if the proportion of defective components is at most .10. The distributor decides to randomly select 10 components and to accept the batch only if the number of defective components in the sample is at most 2.

a. What is the probability that the batch will be accepted when the actual proportion of defectives is .01? .05? .10? .20? .25?

b. Let p denote the actual proportion of defectives in the batch. A graph of P(batch is accepted) as a function of p, with p on the horizontal axis and P(batch is accepted) on the vertical axis, is called the *operating characteristic curve* for the acceptance sampling plan. Use the results of part (a) to sketch this curve for $0 \le p \le 1$.

c. Repeat parts (a) and (b) with "1" replacing "2" in the acceptance sampling plan.

d. Repeat parts (a) and (b) with "15" replacing "10" in the acceptance sampling plan.

e. Which of the three sampling plans, that of part (a), (c), or (d), appears most satisfactory, and why?

59. An ordinance requiring that a smoke detector be installed in all previously constructed houses has been in effect in a particular city for 1 year. The fire department is concerned that many houses remain without detectors. Let $p =$ the true proportion of such houses having detectors, and suppose that a random sample of 25 homes is inspected. If the sample strongly indicates that fewer than 80% of all houses have a detector, the fire department will campaign for a mandatory inspection program. Because of the costliness of the program, the department prefers not to call for such inspections unless sample evidence strongly argues for their necessity. Let X denote the number of homes with detectors among the 25 sampled. Consider rejecting the claim that $p \ge .8$ if $x \le 15$.

a. What is the probability that the claim is rejected when the actual value of p is .8?

b. What is the probability of not rejecting the claim when $p = .7$? When $p = .6$?

c. How do the "error probabilities" of parts (a) and (b) change if the value 15 in the decision rule is replaced by 14?

* "Between a and b, inclusive" is equivalent to $(a \le X \le b)$.

60. A toll bridge charges \$1.00 for passenger cars and \$2.50 for other vehicles. Suppose that during daytime hours, 60% of all vehicles are passenger cars. If 25 vehicles cross the bridge during a particular daytime period, what is the resulting expected toll revenue? [*Hint:* Let X = the number of passenger cars; then the toll revenue $h(X)$ is a linear function of X.]

61. A student who is trying to write a paper for a course has a choice of two topics, A and B. If topic A is chosen, the student will order two books through interlibrary loan, whereas if topic B is chosen, the student will order four books. The student believes that a good paper necessitates receiving and using at least half the books ordered for either topic chosen. If the probability that a book ordered through interlibrary loan actually arrives in time is .9 and books arrive independently of one another, which topic should the student choose to maximize the probability of writing a good paper? What if the arrival probability is only .5 instead of .9?

62. a. For fixed n, are there values of p $(0 \le p \le 1)$ for which $V(X) = 0$? Explain why this is so.

b. For what value of p is $V(X)$ maximized? [*Hint:* Either graph $V(X)$ as a function of p or else take a derivative.]

63. a. Show that $b(x; n, 1 - p) = b(n - x; n, p)$.

b. Show that $B(x; n, 1 - p) = 1 - B(n - x - 1; n, p)$. [*Hint:* At most x S's is equivalent to at least $(n - x)$ F's.]

c. What do parts (a) and (b) imply about the necessity of including values of p greater than .5 in Appendix Table A.1?

64. Show that $E(X) = np$ when X is a binomial random variable. [*Hint:* First express $E(X)$ as a sum with lower limit $x = 1$. Then factor out np, let $y = x - 1$ so that the sum is from $y = 0$ to $y = n - 1$, and show that the sum equals 1.]

65. Customers at a gas station pay with a credit card (A), debit card (B), or cash (C). Assume that successive customers make independent choices, with $P(A) = .5$, $P(B) = .2$, and $P(C) = .3$.

a. Among the next 100 customers, what are the mean and variance of the number who pay with a debit card? Explain your reasoning.

b. Answer part (a) for the number among the 100 who don't pay with cash.

66. An airport limousine can accommodate up to four passengers on any one trip. The company will accept a maximum of six reservations for a trip, and a passenger must have a reservation. From previous records, 20% of all those making reservations do not appear for the trip. Answer the following questions, assuming independence wherever appropriate.

a. If six reservations are made, what is the probability that at least one individual with a reservation cannot be accommodated on the trip?

b. If six reservations are made, what is the expected number of available places when the limousine departs?

c. Suppose the probability distribution of the number of reservations made is given in the accompanying table.

Number of reservations	3	4	5	6
Probability	.1	.2	.3	.4

Let X denote the number of passengers on a randomly selected trip. Obtain the probability mass function of X.

67. Refer to Chebyshev's inequality given in Exercise 44. Calculate $P(|X - \mu| \ge k\sigma)$ for $k = 2$ and $k = 3$ when $X \sim \text{Bin}(20, .5)$, and compare to the corresponding upper bound. Repeat for $X \sim \text{Bin}(20, .75)$.

3.5 Hypergeometric and Negative Binomial Distributions

The hypergeometric and negative binomial distributions are both related to the binomial distribution. The binomial distribution is the approximate probability model for sampling without replacement from a finite dichotomous (S–F) population provided the sample size n is small relative to the population size N; the hypergeometric distribution is the exact probability model for the number of S's in the sample. The binomial rv X is the number of S's when the number n of trials is fixed, whereas the negative binomial distribution arises from fixing the number of S's desired and letting the number of trials be random.

The Hypergeometric Distribution

The assumptions leading to the hypergeometric distribution are as follows:

1. The population or set to be sampled consists of N individuals, objects, or elements (a *finite* population).

EXERCISES Section 3.6 (79–93)

79. Let X, the number of flaws on the surface of a randomly selected boiler of a certain type, have a Poisson distribution with parameter $\mu = 5$. Use Appendix Table A.2 to compute the following probabilities:

a. $P(X \le 8)$ **b.** $P(X = 8)$ **c.** $P(9 \le X)$
d. $P(5 \le X \le 8)$ **e.** $P(5 < X < 8)$

80. Let X be the number of material anomalies occurring in a particular region of an aircraft gas-turbine disk. The article "Methodology for Probabilistic Life Prediction of Multiple-Anomaly Materials" (*Amer. Inst. of Aeronautics and Astronautics J.*, 2006: 787–793) proposes a Poisson distribution for X. Suppose that $\mu = 4$.

a. Compute both $P(X \le 4)$ and $P(X < 4)$.
b. Compute $P(4 \le X \le 8)$.
c. Compute $P(8 \le X)$.
d. What is the probability that the number of anomalies exceeds its mean value by no more than one standard deviation?

81. Suppose that the number of drivers who travel between a particular origin and destination during a designated time period has a Poisson distribution with parameter $\mu = 20$ (suggested in the article "Dynamic Ride Sharing: Theory and Practice," *J. of Transp. Engr.*, 1997: 308–312). What is the probability that the number of drivers will

a. Be at most 10?
b. Exceed 20?
c. Be between 10 and 20, inclusive? Be strictly between 10 and 20?
d. Be within 2 standard deviations of the mean value?

82. Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with parameter $\mu = .2$. (Suggested in "Average Sample Number for Semi-Curtailed Sampling Using the Poisson Distribution," *J. Quality Technology*, 1983: 126–129.)

a. What is the probability that a disk has exactly one missing pulse?
b. What is the probability that a disk has at least two missing pulses?
c. If two disks are independently selected, what is the probability that neither contains a missing pulse?

83. An article in the *Los Angeles Times* (Dec. 3, 1993) reports that 1 in 200 people carry the defective gene that causes inherited colon cancer. In a sample of 1000 individuals, what is the approximate distribution of the number who carry this gene? Use this distribution to calculate the approximate probability that

a. Between 5 and 8 (inclusive) carry the gene.
b. At least 8 carry the gene.

84. Suppose that only .10% of all computers of a certain type experience CPU failure during the warranty period. Consider a sample of 10,000 computers.

a. What are the expected value and standard deviation of the number of computers in the sample that have the defect?
b. What is the (approximate) probability that more than 10 sampled computers have the defect?
c. What is the (approximate) probability that no sampled computers have the defect?

85. Suppose small aircraft arrive at a certain airport according to a Poisson process with rate $\alpha = 8$ per hour, so that the number of arrivals during a time period of t hours is a Poisson rv with parameter $\mu = 8t$.

a. What is the probability that exactly 6 small aircraft arrive during a 1-hour period? At least 6? At least 10?
b. What are the expected value and standard deviation of the number of small aircraft that arrive during a 90-min period?
c. What is the probability that at least 20 small aircraft arrive during a 2.5-hour period? That at most 10 arrive during this period?

86. The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with a rate parameter of five per hour.

a. What is the probability that exactly four arrivals occur during a particular hour?
b. What is the probability that at least four people arrive during a particular hour?
c. How many people do you expect to arrive during a 45-min period?

87. The number of requests for assistance received by a towing service is a Poisson process with rate $\alpha = 4$ per hour.

a. Compute the probability that exactly ten requests are received during a particular 2-hour period.
b. If the operators of the towing service take a 30-min break for lunch, what is the probability that they do not miss any calls for assistance?
c. How many calls would you expect during their break?

88. In proof testing of circuit boards, the probability that any particular diode will fail is .01. Suppose a circuit board contains 200 diodes.

a. How many diodes would you expect to fail, and what is the standard deviation of the number that are expected to fail?
b. What is the (approximate) probability that at least four diodes will fail on a randomly selected board?
c. If five boards are shipped to a particular customer, how likely is it that at least four of them will work properly? (A board works properly only if all its diodes work.)

89. The article "Reliability-Based Service-Life Assessment of Aging Concrete Structures" (*J. Structural Engr.*, 1993: 1600–1621) suggests that a Poisson process can be used to represent the occurrence of structural loads over time. Suppose the mean time between occurrences of loads is .5 year.