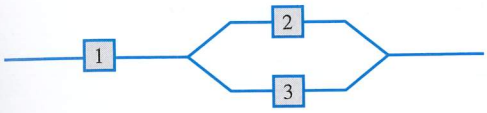
a. List all outcomes in the event A that all three vehicles go in the same direction.
b. List all outcomes in the event B that all three vehicles take different directions.
c. List all outcomes in the event C that exactly two of the three vehicles turn right.
d. List all outcomes in the event D that exactly two vehicles go in the same direction.
e. List outcomes in D', $C \cup D$, and $C \cap D$.

3. Three components are connected to form a system as shown in the accompanying diagram. Because the components in the 2–3 subsystem are connected in parallel, that subsystem will function if at least one of the two individual components functions. For the entire system to function, component 1 must function and so must the 2–3 subsystem.



The experiment consists of determining the condition of each component [S (success) for a functioning component and F (failure) for a nonfunctioning component].
a. Which outcomes are contained in the event A that exactly two out of the three components function?
b. Which outcomes are contained in the event B that at least two of the components function?
c. Which outcomes are contained in the event C that the system functions?
d. List outcomes in C', $A \cup C$, $A \cap C$, $B \cup C$, and $B \cap C$.

4. Each of a sample of four home mortgages is classified as fixed rate (F) or variable rate (V).
a. What are the 16 outcomes in $\mathcal{S}$?
b. Which outcomes are in the event that exactly three of the selected mortgages are fixed rate?
c. Which outcomes are in the event that all four mortgages are of the same type?
d. Which outcomes are in the event that at most one of the four is a variable-rate mortgage?
e. What is the union of the events in parts (c) and (d), and what is the intersection of these two events?
f. What are the union and intersection of the two events in parts (b) and (c)?

5. A family consisting of three persons—A, B, and C—goes to a medical clinic that always has a doctor at each of stations 1, 2, and 3. During a certain week, each member of the family visits the clinic once and is assigned at random to a station. The experiment consists of recording the station number for each member. One outcome is (1, 2, 1) for A to station 1, B to station 2, and C to station 1.
a. List the 27 outcomes in the sample space.
b. List all outcomes in the event that all three members go to the same station.
c. List all outcomes in the event that all members go to different stations.
d. List all outcomes in the event that no one goes to station 2.

6. A college library has five copies of a certain text on reserve. Two copies (1 and 2) are first printings, and the other three (3, 4, and 5) are second printings. A student examines these books in random order, stopping only when a second printing has been selected. One possible outcome is 5, and another is 213.
a. List the outcomes in $\mathcal{S}$.
b. Let A denote the event that exactly one book must be examined. What outcomes are in A?
c. Let B be the event that book 5 is the one selected. What outcomes are in B?
d. Let C be the event that book 1 is not examined. What outcomes are in C?

7. An academic department has just completed voting by secret ballot for a department head. The ballot box contains four slips with votes for candidate A and three slips with votes for candidate B. Suppose these slips are removed from the box one by one.
a. List all possible outcomes.
b. Suppose a running tally is kept as slips are removed. For what outcomes does A remain ahead of B throughout the tally?

8. An engineering construction firm is currently working on power plants at three different sites. Let A_i denote the event that the plant at site i is completed by the contract date. Use the operations of union, intersection, and complementation to describe each of the following events in terms of A_1, A_2, and A_3, draw a Venn diagram, and shade the region corresponding to each one.
a. At least one plant is completed by the contract date.
b. All plants are completed by the contract date.
c. Only the plant at site 1 is completed by the contract date.
d. Exactly one plant is completed by the contract date.
e. Either the plant at site 1 or both of the other two plants are completed by the contract date.

9. Use Venn diagrams to verify the following two relationships for any events A and B (these are called De Morgan's laws):
a. $(A \cup B)' = A' \cap B'$
b. $(A \cap B)' = A' \cup B'$

[*Hint:* In each part, draw a diagram corresponding to the left side and another corresponding to the right side.]

10. a. In Example 2.10, identify three events that are mutually exclusive.
b. Suppose there is no outcome common to all three of the events A, B, and C. Are these three events necessarily mutually exclusive? If your answer is yes, explain why; if your answer is no, give a counterexample using the experiment of Example 2.10.

14. Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

a. What is the probability that a randomly selected adult regularly consumes both coffee and soda?

b. What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

15. Consider the type of clothes dryer (gas or electric) purchased by each of five different customers at a certain store.

a. If the probability that at most one of these purchases an electric dryer is .428, what is the probability that at least two purchase an electric dryer?

b. If P(all five purchase gas) = .116 and P(all five purchase electric) = .005, what is the probability that at least one of each type is purchased?

16. An individual is presented with three different glasses of cola, labeled C, D, and P. He is asked to taste all three and then list them in order of preference. Suppose the same cola has actually been put into all three glasses.

a. What are the simple events in this ranking experiment, and what probability would you assign to each one?

b. What is the probability that C is ranked first?

c. What is the probability that C is ranked first and D is ranked last?

17. Let A denote the event that the next request for assistance from a statistical software consultant relates to the SPSS package, and let B be the event that the next request is for help with SAS. Suppose that $P(A) = .30$ and $P(B) = .50$.

a. Why is it not the case that $P(A) + P(B) = 1$?

b. Calculate $P(A')$.

c. Calculate $P(A \cup B)$.

d. Calculate $P(A' \cap B')$.

18. A wallet contains five \$10 bills, four \$5 bills, and six \$1 bills (nothing larger). If the bills are selected one by one in random order, what is the probability that at least two bills must be selected to obtain a first \$10 bill?

19. Human visual inspection of solder joints on printed circuit boards can be very subjective. Part of the problem stems from the numerous types of solder defects (e.g., pad nonwetting, knee visibility, voids) and even the degree to which a joint possesses one or more of these defects. Consequently, even highly trained inspectors can disagree on the disposition of a particular joint. In one batch of 10,000 joints, inspector A found 724 that were judged defective, inspector B found 751 such joints, and 1159 of the joints were judged defective by at least one of the inspectors. Suppose that one of the 10,000 joints is randomly selected.

a. What is the probability that the selected joint was judged to be defective by neither of the two inspectors?

b. What is the probability that the selected joint was judged to be defective by inspector B but not by inspector A?

20. A certain factory operates three different shifts. Over the last year, 200 accidents have occurred at the factory. Some of these can be attributed at least in part to unsafe working conditions, whereas the others are unrelated to working conditions. The accompanying table gives the percentage of accidents falling in each type of accident–shift category.

		Unsafe Conditions	Unrelated to Conditions
Shift	*Day*	10%	35%
	Swing	8%	20%
	Night	5%	22%

Suppose one of the 200 accident reports is randomly selected from a file of reports, and the shift and type of accident are determined.

a. What are the simple events?

b. What is the probability that the selected accident was attributed to unsafe conditions?

c. What is the probability that the selected accident did not occur on the day shift?

21. An insurance company offers four different deductible levels—none, low, medium, and high—for its homeowner's policyholders and three different levels—low, medium, and high—for its automobile policyholders. The accompanying table gives proportions for the various categories of policyholders who have both types of insurance. For example, the proportion of individuals with both low homeowner's deductible and low auto deductible is .06 (6% of all such individuals).

	Homeowner's			
Auto	N	L	M	H
L	.04	.06	.05	.03
M	.07	.10	.20	.10
H	.02	.03	.15	.15

Suppose an individual having both types of policies is randomly selected.

a. What is the probability that the individual has a medium auto deductible and a high homeowner's deductible?

b. What is the probability that the individual has a low auto deductible? A low homeowner's deductible?

c. What is the probability that the individual is in the same category for both auto and homeowner's deductibles?

d. Based on your answer in part (c), what is the probability that the two categories are different?

e. What is the probability that the individual has at least one low deductible level?

f. Using the answer in part (e), what is the probability that neither deductible level is low?

22. The route used by a certain motorist in commuting to work contains two intersections with traffic signals. The

probability that he must stop at the first signal is .4, the analogous probability for the second signal is .5, and the probability that he must stop at at least one of the two signals is .7. What is the probability that he must stop

a. At both signals?
b. At the first signal but not at the second one?
c. At exactly one signal?

23. The computers of six faculty members in a certain department are to be replaced. Two of the faculty members have selected laptop machines and the other four have chosen desktop machines. Suppose that only two of the setups can be done on a particular day, and the two computers to be set up are randomly selected from the six (implying 15 equally likely outcomes; if the computers are numbered 1, 2,..., 6, then one outcome consists of computers 1 and 2, another consists of computers 1 and 3, and so on).

a. What is the probability that both selected setups are for laptop computers?
b. What is the probability that both selected setups are desktop machines?
c. What is the probability that at least one selected setup is for a desktop computer?
d. What is the probability that at least one computer of each type is chosen for setup?

24. Show that if one event A is contained in another event B (i.e., A is a subset of B), then $P(A) \le P(B)$. [*Hint:* For such A and B, A and $B \cap A'$ are disjoint and $B = A \cup (B \cap A')$, as can be seen from a Venn diagram.] For general A and B, what does this imply about the relationship among $P(A \cap B)$, $P(A)$ and $P(A \cup B)$?

25. The three most popular options on a certain type of new car are a built-in GPS (A), a sunroof (B), and an automatic transmission (C). If 40% of all purchasers request A, 55% request B, 70% request C, 63% request A or B, 77% request A or C, 80% request B or C, and 85% request A or B or C, determine the probabilities of the following events. [*Hint:* "A or B" is the event that at least one of the two options is requested; try drawing a Venn diagram and labeling all regions.]

a. The next purchaser will request at least one of the three options.
b. The next purchaser will select none of the three options.
c. The next purchaser will request only an automatic transmission and not either of the other two options.
d. The next purchaser will select exactly one of these three options.

26. A certain system can experience three different types of defects. Let $A_i (i = 1,2,3)$ denote the event that the system has a defect of type i. Suppose that

$$P(A_1) = .12 \quad P(A_2) = .07 \quad P(A_3) = .05$$
$$P(A_1 \cup A_2) = .13 \quad P(A_1 \cup A_3) = .14$$
$$P(A_2 \cup A_3) = .10 \quad P(A_1 \cap A_2 \cap A_3) = .01$$

a. What is the probability that the system does not have a type 1 defect?
b. What is the probability that the system has both type 1 and type 2 defects?
c. What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?
d. What is the probability that the system has at most two of these defects?

27. An academic department with five faculty members—Anderson, Box, Cox, Cramer, and Fisher—must select two of its members to serve on a personnel review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representatives will be selected by putting the names on identical pieces of paper and then randomly selecting two.

a. What is the probability that both Anderson and Box will be selected? [*Hint:* List the equally likely outcomes.]
b. What is the probability that at least one of the two members whose name begins with C is selected?
c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years' teaching experience there?

28. In Exercise 5, suppose that any incoming individual is equally likely to be assigned to any of the three stations irrespective of where other individuals have been assigned. What is the probability that

a. All three family members are assigned to the same station?
b. At most two family members are assigned to the same station?
c. Every family member is assigned to a different station?

2.3 Counting Techniques

When the various outcomes of an experiment are equally likely (the same probability is assigned to each simple event), the task of computing probabilities reduces to counting. Letting N denote the number of outcomes in a sample space and $N(A)$ represent the number of outcomes contained in an event A,

$$P(A) = \frac{N(A)}{N} \tag{2.1}$$

EXERCISES Section 2.3 (29–44)

29. As of April 2006, roughly 50 million .com web domain names were registered (e.g., yahoo.com).
    a. How many domain names consisting of just two letters in sequence can be formed? How many domain names of length two are there if digits as well as letters are permitted as characters? [*Note:* A character length of three or more is now mandated.]
    b. How many domain names are there consisting of three letters in sequence? How many of this length are there if either letters or digits are permitted? [*Note:* All are currently taken.]
    c. Answer the questions posed in (b) for four-character sequences.
    d. As of April 2006, 97,786 of the four-character sequences using either letters or digits had not yet been claimed. If a four-character name is randomly selected, what is the probability that it is already owned?

30. A friend of mine is giving a dinner party. His current wine supply includes 8 bottles of zinfandel, 10 of merlot, and 12 of cabernet (he only drinks red wine), all from different wineries.
    a. If he wants to serve 3 bottles of zinfandel and serving order is important, how many ways are there to do this?
    b. If 6 bottles of wine are to be randomly selected from the 30 for serving, how many ways are there to do this?
    c. If 6 bottles are randomly selected, how many ways are there to obtain two bottles of each variety?
    d. If 6 bottles are randomly selected, what is the probability that this results in two bottles of each variety being chosen?
    e. If 6 bottles are randomly selected, what is the probability that all of them are the same variety?

31. The composer Beethoven wrote 9 symphonies, 5 piano concertos (music for piano and orchestra), and 32 piano sonatas (music for solo piano).
    a. How many ways are there to play first a Beethoven symphony and then a Beethoven piano concerto?
    b. The manager of a radio station decides that on each successive evening (7 days per week), a Beethoven symphony will be played followed by a Beethoven piano concerto followed by a Beethoven piano sonata. For how many years could this policy be continued before exactly the same program would have to be repeated?

32. An electronics store is offering a special price on a complete set of components (receiver, compact disc player, speakers, turntable). A purchaser is offered a choice of manufacturer for each component:

    Receiver: Kenwood, Onkyo, Pioneer, Sony, Sherwood
    Compact disc player: Onkyo, Pioneer, Sony, Technics
    Speakers: Boston, Infinity, Polk
    Turntable: Onkyo, Sony, Teac, Technics

    A switchboard display in the store allows a customer to hook together any selection of components (consisting of one of each type). Use the product rules to answer the following questions:
    a. In how many ways can one component of each type be selected?
    b. In how many ways can components be selected if both the receiver and the compact disc player are to be Sony?
    c. In how many ways can components be selected if none is to be Sony?
    d. In how many ways can a selection be made if at least one Sony component is to be included?
    e. If someone flips switches on the selection in a completely random fashion, what is the probability that the system selected contains at least one Sony component? Exactly one Sony component?

33. Again consider a Little League team that has 15 players on its roster.
    a. How many ways are there to select 9 players for the starting lineup?
    b. How many ways are there to select 9 players for the starting lineup and a batting order for the 9 starters?
    c. Suppose 5 of the 15 players are left-handed. How many ways are there to select 3 left-handed outfielders and have all 6 other positions occupied by right-handed players?

34. Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.
    a. How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?
    b. In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect?
    c. If a sample of 5 keyboards is randomly selected, what is the probability that at least 4 of these will have a mechanical defect?

35. A production facility employs 10 workers on the day shift, 8 workers on the swing shift, and 6 workers on the graveyard shift. A quality control consultant is to select 5 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 5 workers has the same chance of being selected as