

30. a. $t_{.025,10} = 2.228$, b. $t_{.025,15} = 2.131$,

c. $t_{.005,15} = 2.947$, d. $t_{.005,4} = 4.604$,

e. $t_{.01,24} = 2.492$, f. $t_{.005,37} \approx 2.719$. (we look at $df=36$ since $df=37$ isn't available)

32. We have $n = 20$, $\bar{x} = 1584$, $s = 607$; the critical value is $t_{\alpha/2, n-1}$
 $t_{.005, 20-1} = t_{.005, 19} = 2.861$

Thus, the 99% CI for μ is

$$1584 \pm 2.861 \frac{607}{\sqrt{20}} = 1584 \pm 388.3 = (1195.7, 1972.3)$$

We are 99% confident that the true average number of cycles required to break this type of condom is between 1195.7 cycles and 1972.3 cycles.

34. $n = 14$, $\bar{x} = 8.48$, $s = .79$ $t_{\alpha, n-1} = t_{.05, 13} = 1.771$

a. A 95% lower confidence bound: $8.48 - 1.771 \left(\frac{.79}{\sqrt{14}} \right) = 8.48 - .37 = 8.11$. With 95% confidence, the value of the true mean proportional limit stress of all such joints is greater than 8.11 MPa. We must assume that the sample observations were taken from a normally distributed population.

b. A 95% lower prediction bound: $8.48 - 1.771 (.79) \sqrt{1 + \frac{1}{14}} = 7.03$. If this bound is calculated for sample after sample, in the long run 95% of these bound will provide a lower bound for the corresponding future values of the proportional limit stress of a single joint of this type.

35. $n = 15$, $\bar{x} = 25.0$, $s = 3.5$; $t_{\alpha/2, n-1} = t_{.025, 14} = 2.145$

a. A 95% CI for the mean: $25.0 \pm 2.145 \frac{3.5}{\sqrt{15}} = (23.06, 26.94)$.

With 95% confidence, the true mean failure strain is between 23.06% to 26.94%

b. A 95% prediction interval: $25.0 \pm 2.145 \cdot 3.5 \cdot \left(\sqrt{1 + \frac{1}{15}} \right) = (17.25, 32.75)$

The prediction interval is about 4 times wider than the confidence interval

16. a. $\alpha = P(T_{15} \geq 3.733) = 0.001$.

b. $df = n - 1 = 23$, $\alpha = P(T_{23} \leq -2.500) = 0.01$.

c. $df = n - 1 = 30$, $\alpha = P(T_{30} \geq 1.697 \text{ or } T_{30} \leq -1.697) = 0.05 + 0.05 = 0.1$.

21. With $H_0: \mu = .5$ vs. $H_a: \mu \neq .5$, we reject H_0 if $t > t_{\alpha/2, n-1}$ or $t < -t_{\alpha/2, n-1}$.

a. $1.6 < t_{.025, 12} = 2.179$, so don't reject H_0 .

b. $-1.6 > -t_{.025, 12} = -2.179$, so don't reject H_0 .

c. $-2.6 > -t_{.005, 24} = -2.797$, so don't reject H_0 .

d. $-3.9 < -t_{v, 24}$ for all $v \geq 0.0005$, so we reject H_0 for all $\alpha \geq 0.001$.

23. $H_0: \mu = 360$ v. $H_a: \mu > 360$; $t = \frac{\bar{x} - 360}{s/\sqrt{n}}$;
reject H_0 if $t > t_{.95, 25} = 1.708$; $t = \frac{370.69 - 360}{24.36/\sqrt{26}} = 2.24 > 1.708$.
Thus H_0 should be rejected. There appears to be contradiction of the prior belief.

24. $H_0: \mu = 3000$ vs. $H_a: \mu \neq 3000$

We reject H_0 if $t > t_{\alpha/2, 4}$ or $t < -t_{\alpha/2, 4}$. Let us take $\alpha = 0.05$.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2887.6 - 3000}{84.0256/\sqrt{5}} = -2.9912 < -t_{.025, 4} = -2.776.$$

So we reject H_0 for $\alpha = 0.05$.

The requirement doesn't seem to have been satisfied at a significance level of 0.05.

31. a. $t_{.05,10} = 1.812$, b. $t_{.05,15} = 1.753$,

c. $t_{.01,15} = 2.602$, d. $t_{.01,4} = 3.747$,

e. $t_{.02,24} \approx 2.064$, (we look at $\alpha=.025$ since $\alpha=.02$ isn't available)

f. $t_{.01,37} \approx 2.434$. (we look at $df=36$ since $df=37$ isn't available)

This is a slightly liberal choice. The conservative choice from table A.5 would be $t_{.01,24} = 2.492$. In fact, table A.8 gives a much better conservative choice, which is $t_{.019,24} \approx 2.2$ (check the column for $df = 24$). The exact critical value obtained from software is 2.1715.

20. With $H_0: \mu = 750$ and $H_a: \mu < 750$ and a significance level of .05, we reject H_0 if $z < -1.645$; in this example, $z = -2.14 < -1.645$, so we reject the null hypothesis and do not continue with the purchase. At a significance level of .01, we reject H_0 if $z < -2.33$; $z = -2.14 > -2.33$, so we don't reject the null hypothesis and thus continue with the purchase.

None of these two significance levels can be preferred in general over the other. Setting the significance level is up to the potential customer. It depends on how much "false evidence" in favor of "poor quality" of lightbulbs they are willing to accept. You should never choose the significance level in a way such that it favors a particular conclusion.