We can be highly confident that at least 95% of all lumber specimens have modulus of elasticity values between 8,564.9 and 20,500.1.

The 95% CI for μ is (13,437.3, 15,627.7), and the 95% prediction interval for the modulus of elasticity of a single lumber specimen is (10,017.0, 19,048.0). Both the prediction interval and the tolerance interval are substantially wider than the confidence interval. ■

Intervals Based on Nonnormal Population Distributions

The one-sample t CI for μ is robust to small or even moderate departures from normality unless n is quite small. By this we mean that if a critical value for 95% confidence, for example, is used in calculating the interval, the actual confidence level will be reasonably close to the nominal 95% level. If, however, n is small and the population distribution is highly nonnormal, then the actual confidence level may be considerably different from the one you think you are using when you obtain a particular critical value from the t table. It would certainly be distressing to believe that your confidence level is about 95% when in fact it was really more like 88%! The bootstrap technique, introduced in Section 7.1, has been found to be quite successful at estimating parameters in a wide variety of nonnormal situations.

In contrast to the confidence interval, the validity of the prediction and tolerance intervals described in this section is closely tied to the normality assumption. These latter intervals should not be used in the absence of compelling evidence for normality. The excellent reference *Statistical Intervals,* cited in the bibliography at the end of this chapter, discusses alternative procedures of this sort for various other situations.

EXERCISES Section 7.3 (28–41)

28. Determine the values of the following quantities:

a. $t_{.1,15}$ b. $t_{.05,15}$ c. $t_{.05,25}$ d. $t_{.05,40}$ e. $t_{.005,40}$

29. Determine the t critical value(s) that will capture the desired t-curve area in each of the following cases:

a. Central area = .95, df = 10
b. Central area = .95, df = 20
c. Central area = .99, df = 20
d. Central area = .99, df = 50
e. Upper-tail area = .01, df = 25
f. Lower-tail area = .025, df = 5

30. Determine the t critical value for a two-sided confidence interval in each of the following situations:

a. Confidence level = 95%, df = 10
b. Confidence level = 95%, df = 15
c. Confidence level = 99%, df = 15
d. Confidence level = 99%, $n = 5$
e. Confidence level = 98%, df = 24
f. Confidence level = 99%, $n = 38$

31. Determine the t critical value for a lower or an upper confidence bound for each of the situations described in Exercise 30.

32. According to the article "Fatigue Testing of Condoms" (*Polymer Testing,* 2009: 567–571), "tests currently used for condoms are surrogates for the challenges they face in use," including a test for holes, an inflation test, a package seal test, and tests of dimensions and lubricant quality (all fertile territory for the use of statistical methodology!). The investigators developed a new test that adds cyclic strain to a level well below breakage and determines the number of cycles to break. A sample of 20 condoms of one particular type resulted in a sample mean number of 1584 and a sample standard deviation of 607. Calculate and interpret a confidence interval at the 99% confidence level for the true average number of cycles to break. [*Note*: The article presented the results of hypothesis tests based on the t distribution; the validity of these depends on assuming normal population distributions.]

33. The article "Measuring and Understanding the Aging of Kraft Insulating Paper in Power Transformers" (*IEEE Electrical Insul. Mag.,* 1996: 28–34) contained the following observations on degree of polymerization for paper specimens for which viscosity times concentration fell in a certain middle range:

418	421	421	422	425	427	431
434	437	439	446	447	448	453
454	463	465				

- **a.** Construct a boxplot of the data and comment on any interesting features.
- **b.** Is it plausible that the given sample observations were selected from a normal distribution?
- **c.** Calculate a two-sided 95% confidence interval for true average degree of polymerization (as did the authors of the article). Does the interval suggest that 440 is a plausible value for true average degree of polymerization? What about 450?

34. A sample of 14 joint specimens of a particular type gave a sample mean proportional limit stress of 8.48 MPa and a sample standard deviation of .79 MPa ("Characterization of Bearing Strength Factors in Pegged Timber Connections," *J. of Structural Engr.,* 1997: 326–332).

- **a.** Calculate and interpret a 95% lower confidence bound for the true average proportional limit stress of all such joints. What, if any, assumptions did you make about the distribution of proportional limit stress?
- **b.** Calculate and interpret a 95% lower prediction bound for the proportional limit stress of a single joint of this type.

35. Silicone implant augmentation rhinoplasty is used to correct congenital nose deformities. The success of the procedure depends on various biomechanical properties of the human nasal periosteum and fascia. The article "Biomechanics in Augmentation Rhinoplasty" (*J. of Med. Engr. and Tech.,* 2005: 14–17) reported that for a sample of 15 (newly deceased) adults, the mean failure strain (%) was 25.0, and the standard deviation was 3.5.

- **a.** Assuming a normal distribution for failure strain, estimate true average strain in a way that conveys information about precision and reliability.
- **b.** Predict the strain for a single adult in a way that conveys information about precision and reliability. How does the prediction compare to the estimate calculated in part (a)?

36. The $n = 26$ observations on escape time given in Exercise 36 of Chapter 1 give a sample mean and sample standard deviation of 370.69 and 24.36, respectively.

- **a.** Calculate an upper confidence bound for population mean escape time using a confidence level of 95%.
- **b.** Calculate an upper prediction bound for the escape time of a single additional worker using a prediction level of 95%. How does this bound compare with the confidence bound of part (a)?
- **c.** Suppose that two additional workers will be chosen to participate in the simulated escape exercise. Denote their escape times by X_{27} and X_{28}, and let $\overline{X}_{\text{new}}$ denote the average of these two values. Modify the formula for a PI for a single x value to obtain a PI for $\overline{X}_{\text{new}}$, and calculate a 95% two-sided interval based on the given escape data.

37. A study of the ability of individuals to walk in a straight line ("Can We Really Walk Straight?" *Amer. J. of Physical Anthro.,* 1992: 19–27) reported the accompanying data on cadence (strides per second) for a sample of $n = 20$ randomly selected healthy men.

.95	.85	.92	.95	.93	.86	1.00	.92	.85	.81
.78	.93	.93	1.05	.93	1.06	1.06	.96	.81	.96

A normal probability plot gives substantial support to the assumption that the population distribution of cadence is approximately normal. A descriptive summary of the data from Minitab follows:

```
Variable N   Mean    Median  TrMean   StDev   SEMean
cadence  20  0.9255  0.9300  0.9261   0.0809  0.0181

Variable       Min      Max      Q1       Q3
cadence     0.7800   1.0600   0.8525   0.9600
```

- **a.** Calculate and interpret a 95% confidence interval for population mean cadence.
- **b.** Calculate and interpret a 95% prediction interval for the cadence of a single individual randomly selected from this population.
- **c.** Calculate an interval that includes at least 99% of the cadences in the population distribution using a confidence level of 95%.

38. A sample of 25 pieces of laminate used in the manufacture of circuit boards was selected, and the amount of warpage (in.) under particular conditions was determined for each piece, resulting in a sample mean warpage of .0635 and a sample standard deviation of .0065.

- **a.** Calculate a prediction for the amount of warpage of a single piece of laminate in a way that provides information about precision and reliability.
- **b.** Calculate an interval for which you can have a high degree of confidence that at least 95% of all pieces of laminate result in amounts of warpage that are between the two limits of the interval.

39. Exercise 72 of Chapter 1 gave the following observations on a receptor binding measure (adjusted distribution volume) for a sample of 13 healthy individuals: 23, 39, 40, 41, 43, 47, 51, 58, 63, 66, 67, 69, 72.

- **a.** Is it plausible that the population distribution from which this sample was selected is normal?
- **b.** Calculate an interval for which you can be 95% confident that at least 95% of all healthy individuals in the population have adjusted distribution volumes lying between the limits of the interval.
- **c.** Predict the adjusted distribution volume of a single healthy individual by calculating a 95% prediction interval. How does this interval's width compare to the width of the interval calculated in part (b)?

40. Exercise 13 of Chapter 1 presented a sample of $n = 153$ observations on ultimate tensile strength, and Exercise 17 of the previous section gave summary quantities and requested a large-sample confidence interval. Because the sample size is large, no assumptions about the population distribution are required for the validity of the CI.

- **a.** Is any assumption about the tensile-strength distribution required prior to calculating a lower prediction bound for

16. Let the test statistic T have a t distribution when H_0 is true. Give the significance level for each of the following situations:

a. H_a: $\mu > \mu_0$, df $= 15$, rejection region $t \geq 3.733$

b. H_a: $\mu < \mu_0$, $n = 24$, rejection region $t \leq -2.500$

c. H_a: $\mu \neq \mu_0$, $n = 31$, rejection region $t \geq 1.697$ or $t \leq -1.697$

17. Answer the following questions for the tire problem in Example 8.7.

a. If $\bar{x} = 30{,}960$ and a level $\alpha = .01$ test is used, what is the decision?

b. If a level .01 test is used, what is $\beta(30{,}500)$?

c. If a level .01 test is used and it is also required that $\beta(30{,}500) = .05$, what sample size n is necessary?

d. If $\bar{x} = 30{,}960$, what is the smallest α at which H_0 can be rejected (based on $n = 16$)?

18. Reconsider the paint-drying situation of Example 8.2, in which drying time for a test specimen is normally distributed with $\sigma = 9$. The hypotheses H_0: $\mu = 75$ versus H_a: $\mu < 75$ are to be tested using a random sample of $n = 25$ observations.

a. How many standard deviations (of $\bar{X}$) below the null value is $\bar{x} = 72.3$?

b. If $\bar{x} = 72.3$, what is the conclusion using $\alpha = .01$?

c. What is α for the test procedure that rejects H_0 when $z \leq -2.88$?

d. For the test procedure of part (c), what is $\beta(70)$?

e. If the test procedure of part (c) is used, what n is necessary to ensure that $\beta(70) = .01$?

f. If a level .01 test is used with $n = 100$, what is the probability of a type I error when $\mu = 76$?

19. The melting point of each of 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in $\bar{x} = 94.32$. Assume that the distribution of the melting point is normal with $\sigma = 1.20$.

a. Test H_0: $\mu = 95$ versus H_a: $\mu \neq 95$ using a two-tailed level .01 test.

b. If a level .01 test is used, what is $\beta(94)$, the probability of a type II error when $\mu = 94$?

c. What value of n is necessary to ensure that $\beta(94) = .1$ when $\alpha = .01$?

20. Lightbulbs of a certain type are advertised as having an average lifetime of 750 hours. The price of these bulbs is very favorable, so a potential customer has decided to go ahead with a purchase arrangement unless it can be conclusively demonstrated that the true average lifetime is smaller than what is advertised. A random sample of 50 bulbs was selected, the lifetime of each bulb determined, and the appropriate hypotheses were tested using Minitab, resulting in the accompanying output.

```
Variable   N    Mean  StDev  SEMean      Z  P-Value
lifetime  50  738.44  38.20    5.40  -2.14    0.016
```

What conclusion would be appropriate for a significance level of .05? A significance level of .01? What significance level and conclusion would you recommend?

21. The true average diameter of ball bearings of a certain type is supposed to be .5 in. A one-sample t test will be carried out to see whether this is the case. What conclusion is appropriate in each of the following situations?

a. $n = 13$, $t = 1.6$, $\alpha = .05$

b. $n = 13$, $t = -1.6$, $\alpha = .05$

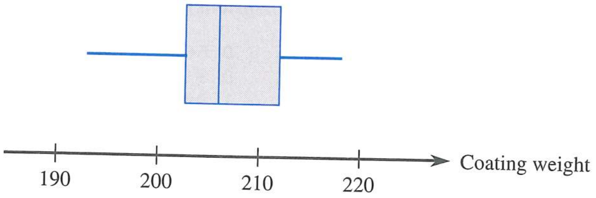
c. $n = 25$, $t = -2.6$, $\alpha = .01$

d. $n = 25$, $t = -3.9$

22. The article "The Foreman's View of Quality Control" (*Quality Engr.,* 1990: 257–280) described an investigation into the coating weights for large pipes resulting from a galvanized coating process. Production standards call for a true average weight of 200 lb per pipe. The accompanying descriptive summary and boxplot are from Minitab.

```
Variable  N    Mean  Median  TrMean  StDev  SEMean
ctg wt   30  206.73  206.00  206.81   6.35    1.16

Variable      Min       Max       Q1       Q3
ctg wt     193.00    218.00   202.75   212.00
```



a. What does the boxplot suggest about the status of the specification for true average coating weight?

b. A normal probability plot of the data was quite straight. Use the descriptive output to test the appropriate hypotheses.

23. Exercise 36 in Chapter 1 gave $n = 26$ observations on escape time (sec) for oil workers in a simulated exercise, from which the sample mean and sample standard deviation are 370.69 and 24.36, respectively. Suppose the investigators had believed *a priori* that true average escape time would be at most 6 min. Does the data contradict this prior belief? Assuming normality, test the appropriate hypotheses using a significance level of .05.

24. Reconsider the sample observations on stabilized viscosity of asphalt specimens introduced in Exercise 46 in Chapter 1 (2781, 2900, 3013, 2856, and 2888). Suppose that for a particular application it is required that true average viscosity be 3000. Does this requirement appear to have been satisfied? State and test the appropriate hypotheses.

25. The desired percentage of SiO_2 in a certain type of aluminous cement is 5.5. To test whether the true average percentage is 5.5 for a particular production facility, 16 independently obtained samples are analyzed. Suppose that the percentage of SiO_2 in a sample is normally distributed with $\sigma = .3$ and that $\bar{x} = 5.25$.

a. Does this indicate conclusively that the true average percentage differs from 5.5? Carry out the analysis using the sequence of steps suggested in the text.

b. If the true average percentage is $\mu = 5.6$ and a level $\alpha = .01$ test based on $n = 16$ is used, what is the probability of detecting this departure from H_0?

c. What value of n is required to satisfy $\alpha = .01$ and $\beta(5.6) = .01$?