Explain your reasoning. [*Hint:* Think about the consequences of a type I and type II error for each possibility.]

5. Before agreeing to purchase a large order of polyethylene sheaths for a particular type of high-pressure oil-filled submarine power cable, a company wants to see conclusive evidence that the true standard deviation of sheath thickness is less than .05 mm. What hypotheses should be tested, and why? In this context, what are the type I and type II errors?

6. Many older homes have electrical systems that use fuses rather than circuit breakers. A manufacturer of 40-amp fuses wants to make sure that the mean amperage at which its fuses burn out is in fact 40. If the mean amperage is lower than 40, customers will complain because the fuses require replacement too often. If the mean amperage is higher than 40, the manufacturer might be liable for damage to an electrical system due to fuse malfunction. To verify the amperage of the fuses, a sample of fuses is to be selected and inspected. If a hypothesis test were to be performed on the resulting data, what null and alternative hypotheses would be of interest to the manufacturer? Describe type I and type II errors in the context of this problem situation.

7. Water samples are taken from water used for cooling as it is being discharged from a power plant into a river. It has been determined that as long as the mean temperature of the discharged water is at most 150°F, there will be no negative effects on the river's ecosystem. To investigate whether the plant is in compliance with regulations that prohibit a mean discharge water temperature above 150°, 50 water samples will be taken at randomly selected times and the temperature of each sample recorded. The resulting data will be used to test the hypotheses H_0: $\mu = 150°$ versus H_a: $\mu > 150°$. In the context of this situation, describe type I and type II errors. Which type of error would you consider more serious? Explain.

8. A regular type of laminate is currently being used by a manufacturer of circuit boards. A special laminate has been developed to reduce warpage. The regular laminate will be used on one sample of specimens and the special laminate on another sample, and the amount of warpage will then be determined for each specimen. The manufacturer will then switch to the special laminate only if it can be demonstrated that the true average amount of warpage for that laminate is less than for the regular laminate. State the relevant hypotheses, and describe the type I and type II errors in the context of this situation.

9. Two different companies have applied to provide cable television service in a certain region. Let p denote the proportion of all potential subscribers who favor the first company over the second. Consider testing H_0: $p = .5$ versus H_a: $p \neq .5$ based on a random sample of 25 individuals. Let X denote the number in the sample who favor the first company and x represent the observed value of X.

a. Which of the following rejection regions is most appropriate and why?

$$R_1 = \{x: x \le 7 \text{ or } x \ge 18\},\ R_2 = \{x: x \le 8\},$$
$$R_3 = \{x: x \ge 17\}$$

b. In the context of this problem situation, describe what the type I and type II errors are.

c. What is the probability distribution of the test statistic X when H_0 is true? Use it to compute the probability of a type I error.

d. Compute the probability of a type II error for the selected region when $p = .3$, again when $p = .4$, and also for both $p = .6$ and $p = .7$.

e. Using the selected region, what would you conclude if 6 of the 25 queried favored company 1?

10. A mixture of pulverized fuel ash and Portland cement to be used for grouting should have a compressive strength of more than 1300 KN/m^2. The mixture will not be used unless experimental evidence indicates conclusively that the strength specification has been met. Suppose compressive strength for specimens of this mixture is normally distributed with $\sigma = 60$. Let μ denote the true average compressive strength.

a. What are the appropriate null and alternative hypotheses?

b. Let $\overline{X}$ denote the sample average compressive strength for $n = 20$ randomly selected specimens. Consider the test procedure with test statistic $\overline{X}$ and rejection region $\overline{x} \ge 1331.26$. What is the probability distribution of the test statistic when H_0 is true? What is the probability of a type I error for the test procedure?

c. What is the probability distribution of the test statistic when $\mu = 1350$? Using the test procedure of part (b), what is the probability that the mixture will be judged unsatisfactory when in fact $\mu = 1350$ (a type II error)?

d. How would you change the test procedure of part (b) to obtain a test with significance level .05? What impact would this change have on the error probability of part (c)?

e. Consider the standardized test statistic $Z = (\overline{X} - 1300)/(\sigma/\sqrt{n}) = (\overline{X} - 1300)/13.42$. What are the values of Z corresponding to the rejection region of part (b)?

11. The calibration of a scale is to be checked by weighing a 10-kg test specimen 25 times. Suppose that the results of different weighings are independent of one another and that the weight on each trial is normally distributed with $\sigma = .200$ kg. Let μ denote the true average weight reading on the scale.

a. What hypotheses should be tested?

b. Suppose the scale is to be recalibrated if either $\overline{x} \ge 10.1032$ or $\overline{x} \le 9.8968$. What is the probability that recalibration is carried out when it is actually unnecessary?

c. What is the probability that recalibration is judged unnecessary when in fact $\mu = 10.1$? When $\mu = 9.8$?

d. Let $z = (\overline{x} - 10)/(\sigma/\sqrt{n})$. For what value c is the rejection region of part (b) equivalent to the "two-tailed" region of *either* $z \ge c$ *or* $z \le -c$?

e. If the sample size were only 10 rather than 25, how should the procedure of part (d) be altered so that $\alpha = .05$?

f. Using the test of part (e), what would you conclude from the following sample data?

9.981	10.006	9.857	10.107	9.888
9.728	10.439	10.214	10.190	9.793

16. Let the test statistic T have a t distribution when H_0 is true. Give the significance level for each of the following situations:

a. H_a: $\mu > \mu_0$, df = 15, rejection region $t \geq 3.733$

b. H_a: $\mu < \mu_0$, $n = 24$, rejection region $t \leq -2.500$

c. H_a: $\mu \neq \mu_0$, $n = 31$, rejection region $t \geq 1.697$ or $t \leq -1.697$

17. Answer the following questions for the tire problem in Example 8.7.

a. If $\bar{x} = 30{,}960$ and a level $\alpha = .01$ test is used, what is the decision?

b. If a level .01 test is used, what is $\beta(30{,}500)$?

c. If a level .01 test is used and it is also required that $\beta(30{,}500) = .05$, what sample size n is necessary?

d. If $\bar{x} = 30{,}960$, what is the smallest α at which H_0 can be rejected (based on $n = 16$)?

18. Reconsider the paint-drying situation of Example 8.2, in which drying time for a test specimen is normally distributed with $\sigma = 9$. The hypotheses H_0: $\mu = 75$ versus H_a: $\mu < 75$ are to be tested using a random sample of $n = 25$ observations.

a. How many standard deviations (of $\bar{X}$) below the null value is $\bar{x} = 72.3$?

b. If $\bar{x} = 72.3$, what is the conclusion using $\alpha = .01$?

c. What is α for the test procedure that rejects H_0 when $z \leq -2.88$?

d. For the test procedure of part (c), what is $\beta(70)$?

e. If the test procedure of part (c) is used, what n is necessary to ensure that $\beta(70) = .01$?

f. If a level .01 test is used with $n = 100$, what is the probability of a type I error when $\mu = 76$?

19. The melting point of each of 16 samples of a certain brand of hydrogenated vegetable oil was determined, resulting in $\bar{x} = 94.32$. Assume that the distribution of the melting point is normal with $\sigma = 1.20$.

a. Test H_0: $\mu = 95$ versus H_a: $\mu \neq 95$ using a two-tailed level .01 test.

b. If a level .01 test is used, what is $\beta(94)$, the probability of a type II error when $\mu = 94$?

c. What value of n is necessary to ensure that $\beta(94) = .1$ when $\alpha = .01$?

20. Lightbulbs of a certain type are advertised as having an average lifetime of 750 hours. The price of these bulbs is very favorable, so a potential customer has decided to go ahead with a purchase arrangement unless it can be conclusively demonstrated that the true average lifetime is smaller than what is advertised. A random sample of 50 bulbs was selected, the lifetime of each bulb determined, and the appropriate hypotheses were tested using Minitab, resulting in the accompanying output.

Variable	N	Mean	StDev	SEMean	Z	P-Value
lifetime	50	738.44	38.20	5.40	−2.14	0.016

What conclusion would be appropriate for a significance level of .05? A significance level of .01? What significance level and conclusion would you recommend?

21. The true average diameter of ball bearings of a certain type is supposed to be .5 in. A one-sample t test will be carried out to see whether this is the case. What conclusion is appropriate in each of the following situations?

a. $n = 13$, $t = 1.6$, $\alpha = .05$

b. $n = 13$, $t = -1.6$, $\alpha = .05$

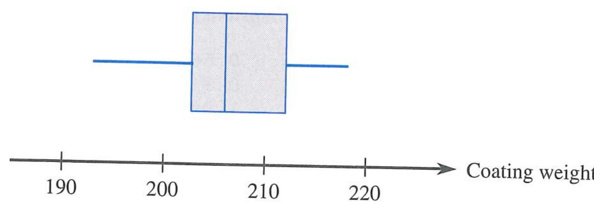
c. $n = 25$, $t = -2.6$, $\alpha = .01$

d. $n = 25$, $t = -3.9$

22. The article "The Foreman's View of Quality Control" (*Quality Engr.*, 1990: 257–280) described an investigation into the coating weights for large pipes resulting from a galvanized coating process. Production standards call for a true average weight of 200 lb per pipe. The accompanying descriptive summary and boxplot are from Minitab.

Variable	N	Mean	Median	TrMean	StDev	SEMean
ctg wt	30	206.73	206.00	206.81	6.35	1.16

Variable	Min	Max	Q1	Q3
ctg wt	193.00	218.00	202.75	212.00



a. What does the boxplot suggest about the status of the specification for true average coating weight?

b. A normal probability plot of the data was quite straight. Use the descriptive output to test the appropriate hypotheses.

23. Exercise 36 in Chapter 1 gave $n = 26$ observations on escape time (sec) for oil workers in a simulated exercise, from which the sample mean and sample standard deviation are 370.69 and 24.36, respectively. Suppose the investigators had believed *a priori* that true average escape time would be at most 6 min. Does the data contradict this prior belief? Assuming normality, test the appropriate hypotheses using a significance level of .05.

24. Reconsider the sample observations on stabilized viscosity of asphalt specimens introduced in Exercise 46 in Chapter 1 (2781, 2900, 3013, 2856, and 2888). Suppose that for a particular application it is required that true average viscosity be 3000. Does this requirement appear to have been satisfied? State and test the appropriate hypotheses.

25. The desired percentage of SiO_2 in a certain type of aluminous cement is 5.5. To test whether the true average percentage is 5.5 for a particular production facility, 16 independently obtained samples are analyzed. Suppose that the percentage of SiO_2 in a sample is normally distributed with $\sigma = .3$ and that $\bar{x} = 5.25$.

a. Does this indicate conclusively that the true average percentage differs from 5.5? Carry out the analysis using the sequence of steps suggested in the text.

b. If the true average percentage is $\mu = 5.6$ and a level $\alpha = .01$ test based on $n = 16$ is used, what is the probability of detecting this departure from H_0?

c. What value of n is required to satisfy $\alpha = .01$ and $\beta(5.6) = .01$?