1. a. $z_{\alpha/2} = 2.81$ implies that $\alpha/2 = 1 - \Phi(2.81) = .0025$, so $\alpha = .005$ and the confidence level is $100(1-\alpha)\% = 99.5\%$

b. $z_{\alpha/2} = 1.44$ implies that $\alpha = 2[1 - \Phi(1.44)] = .15$, and the confidence level is $100(1-\alpha)\% = 85\%$.

c. 99.7% confidence implies that $\alpha = .003$, $\alpha/2 = .0015$, and $z_{.0015} = 2.96$ (Look for cumulative area equal to $1 - .0015 = .9985$ in the main body of table A.3) Or, just use $z \approx 3$ by the empirical rule.

d. 75% confidence implies $\alpha = .25$, $\alpha/2 = .125$, and $z_{.125} = 1.15$

5. a. $4.85 \pm (1.96)(.75)/\sqrt{20} = 4.85 \pm .33 = (4.52, 5.18)$.

b. $z_{1-\alpha/2} = z_{0.99} = 2.33$, so the CI is $4.56 \pm (2.33)(.75)/\sqrt{16} = (4.12, 5)$.

c. $n = [2(1.96)(.75)/.4]^2 = 54.02 \nearrow 55$.

d. Width: $w = 2(.2) = 0.4$, so $n = [2(2.58)(.75)/.4]^2 = 93.61 \nearrow 94$.

8. a. With probability $1-\alpha$, $-z_{\alpha_1} \le (\bar{X} - \mu) \div (\frac{\sigma}{\sqrt{n}}) \le z_{\alpha_2}$. These inequalities can be manipulated exactly as was done in the text to isolate μ; the result is $(\bar{X} - z_{\alpha_2}\frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha_1}\frac{\sigma}{\sqrt{n}})$.

b. The usual 95% confidence interval has length $3.92\frac{\sigma}{\sqrt{n}}$, while this interval will have length $(z_{\alpha_1} + z_{\alpha_2})\frac{\sigma}{\sqrt{n}}$. With $z_{\alpha_1} = z_{.0125} = 2.24$ and $z_{\alpha_2} = z_{.0375} = 1.78$, the length is $(2.24 + 1.78)\frac{\sigma}{\sqrt{n}} = 4.02\frac{\sigma}{\sqrt{n}}$, which is longer.

13. a. $\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} = 654.16 \pm 1.96\frac{164.43}{\sqrt{50}} = (608.58, 699.74)$. We are 95% confident that the true average CO_2 level in this population of homes with gas cooking appliances is between 608.58 ppm and 699.74 ppm

b. $w = 50 = \frac{2(1.96)(1.75)}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{2(1.96)(1.75)}{50}$

$\Rightarrow n = 188.24$, which round up to 189.

* We assume that the CO_2 level is following any distribution with finite mean and variance

With sample size 50. $\bar{x}$ follow normal distribution with CLT.

7. If $L = 2z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ and we increase the sample size by a factor of 4, the new length is $L' = 2z_{\alpha/2}\frac{\sigma}{\sqrt{4n}} = \left[2z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]\left(\frac{1}{2}\right) = \frac{L}{2}$. Thus halving the length requires n to be increased fourfold. If $n' = 25n$, then $L' = \frac{L}{5}$, so the length is decreased by a factor of 5.