1. a. We use the sample mean, $\bar{x}$, to estimate the population mean μ.

$$\hat{\mu} = \bar{x} = \frac{\sum x_i}{n} = \frac{219.8}{27} = 8.1407.$$

b. We use the sample median, $\tilde{x} = 7.7$ (the ~~median~~ middle observation when arranged in ascending order).

c. We use the sample standard deviation, $s = \sqrt{s^2} = \sqrt{\dfrac{1860.94 - \frac{(219.8)^2}{27}}{26}} = 1.660$

d. With "success" = observation greater than 10, x = # of successes $= 4$, and $\hat{p} = \frac{x}{n} = \frac{4}{27} = .1481$

e. We use the sample (std dev)/(mean), or $\frac{s}{\bar{x}} = \frac{1.660}{8.1407} = .2039$.

4. a. $E(\bar{X}-\bar{Y}) = E(\bar{X}) - E(\bar{Y}) = \mu_1 - \mu_2$;

$\bar{x} - \bar{y} = 8.141 - 8.575 = -.434.$

b. $V(\bar{X}-\bar{Y}) = V(\bar{X}) + V(\bar{Y}) = \sigma_{\bar{X}}^2 + \sigma_{\bar{Y}}^2 = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$

$\sigma_{\bar{X}-\bar{Y}} = \sqrt{V(\bar{X}-\bar{Y})} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$

The estimate would be $s_{\bar{X}-\bar{Y}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{1.66^2}{27} + \frac{2.104^2}{20}} = .5687$

c. $\frac{s_1}{s_2} = \frac{1.660}{2.104} = .7890$

d. $V(X-Y) = V(X) + V(Y) = \sigma_1^2 + \sigma_2^2 = 1.66^2 + 2.104^2 = 7.1824$

11. a. $E\left(\frac{X_1}{n_1} - \frac{X_2}{n_2}\right) = \frac{1}{n_1}E(X_1) - \frac{1}{n_2}E(X_2) = \frac{1}{n_1}(n_1 p_1) - \frac{1}{n_2}(n_2 p_2)$

$= p_1 - p_2$

b. $V\left(\frac{X_1}{n_1} - \frac{X_2}{n_2}\right) = V\left(\frac{X_1}{n_1}\right) + V\left(\frac{X_2}{n_2}\right) = \left(\frac{1}{n_1}\right)^2 V(X_1) + \left(\frac{1}{n_2}\right)^2 V(X_2)$

$= \frac{1}{n_1^2}(n_1 p_1 q_1) + \frac{1}{n_2^2}(n_2 p_2 q_2)$

c. With $\hat{p}_1 = \frac{X_1}{n_1}$, $\hat{q}_1 = 1 - \hat{p}_1$, $\hat{p}_2 = \frac{X_2}{n_2}$, $\hat{q}_2 = 1 - \hat{p}_2$, the estimated standard error is $\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$

d. $(\hat{p}_1 - \hat{p}_2) = \frac{127}{200} - \frac{176}{200} = .635 - .880 = -.245$

e. $\sqrt{\frac{(.635)(.365)}{200} + \frac{(.880)(.120)}{200}} = .041$

13. $\mu = \mathbb{E}(X) = \int_{-1}^{1} x \cdot \frac{1}{2}(1+\theta x)\,dx = \frac{x^2}{4} + \frac{\theta x^3}{6}\Big|_{-1}^{1}$

$$= \frac{1}{3}\theta$$

$\Rightarrow \quad \theta = 3\mu$

$\hat{\theta} = 3\bar{X} \quad \Rightarrow \mathbb{E}(\hat{\theta}) = \mathbb{E}(3\bar{X}) = 3\,\mathbb{E}(\bar{X}) = 3\mu = 3(\frac{1}{3})\theta = \theta$.

$\mathbb{E}(X) = \frac{1}{3}\theta$

The method of moment: $\mathbb{E}(X) = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$.

$$\Rightarrow \frac{1}{3}\theta = \bar{X} \quad \Rightarrow \theta = 3\bar{X}$$

Thus $\tilde{\theta} = 3\bar{X}$ is the method of moment estimator of θ.

2. **a.** With X = # of T's in the sample, the estimator is $\hat{p} = \frac{X}{n}$; $x = 10$, so $\hat{p} = \frac{10}{20} = .50$.

b. Here, X = # in sample without TI graphing calculator, and $x = 16$, so $\hat{p} = \frac{16}{20} = .80$.