Discussion of *Particle Markov chain Monte Carlo methods* by Andrieu et al.

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The authors present an elegant theory for novel methodology which makes Bayesian inference practical on implicit models. I will use their example, a sophisticated financial model involving a continuous time stochastic volatility process driven by Lévy noise, to compare their methodology with a state-of-the-art non-Bayesian approach. I applied iterated filtering (Ionides et al., 2006, 2009) implemented via the `mif` function in the R package `pomp` (King et al., 2008).

Fig. 1 shows some results from applying the iterated filtering algorithm with 1000 particles to the simulation study described by the authors in section 3.2. If θ denotes the parameter vector of interest, the algorithm generates a sequence of parameter estimates $\hat{\theta}_1, \hat{\theta}_2, \ldots$ converging to the maximum likelihood estimate $\hat{\theta}$. As a diagnostic, the log-likelihood of $\hat{\theta}_i$ is plotted against i (Fig. 1(a)). We see the sequence of log-likelihoods rapidly converges. On simulation studies like this, a quick check for successful maximization is to observe that the maximized log-likelihood typically exceeds the log-likelihood at the true parameter value by approximately half the number of estimated parameters (Fig. 1(a)). One can also check for successful local maximization by sliced likelihood plots (Fig. 1(b-e)), in which the likelihood surface is explored along one of the parameters, keeping the other parameters fixed at the estimated local maximum. The likelihood surface is seen to be flat as λ varies, consistent with the authors' observation that some parameter combinations are weakly identified in this model. A profile likelihood analysis could aid the investigation of the identifiability issue. Due to the quick convergence of iterated filtering with a relatively small number of particles, many profile likelihood plots can be generated at the computational expense of, say, one MCMC run of length 50,000.

The decision about whether one wishes to carry out a Bayesian analysis should depend on whether one wishes to impose a prior distribution on unknown parameters. Here, I have shown that likelihood-based non-Bayesian methodology provides a computationally viable alternative to the authors' Bayesian approach for complex dynamic models.

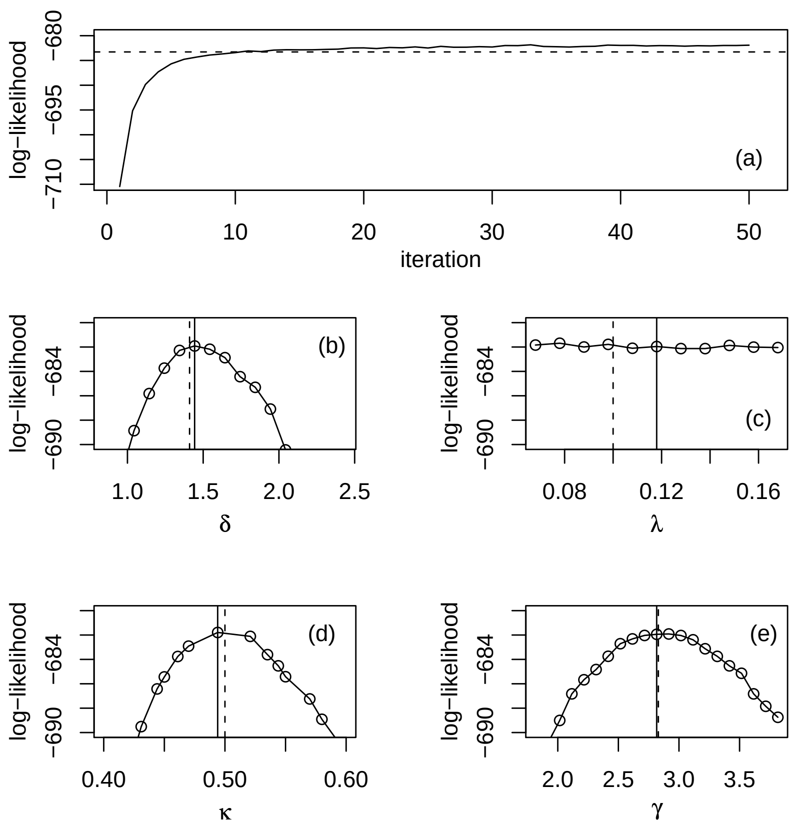

Figure 1: Diagnostic plots for iterated filtering. (a) likelihood at each iteration, evaluated by sequential Monte Carlo; the broken straight line marks the likelihood at the truth. (b) - (e) likelihood surface for each parameter sliced through the maximum; points show parameter values where the likelihoods were evaluated; solid straight lines mark the maximum likelihood estimate; broken straight lines mark the true parameter value.

References

Ionides, E. L., Bhadra, A., and King, A. A. (2009). Iterated filtering. Submitted for publication, and available online at http://arxiv.org/abs/0902.0347.

Ionides, E. L., Bretó, C., and King, A. A. (2006). Inference for nonlinear dynamical systems. *Proceedings of the National Academy of Sciences of the USA*, 103:18438–18443.

King, A. A., Ionides, E. L., and Bretó, C. M. (2008). *pomp: Statistical inference for partially observed Markov processes.* `http://cran.r-project.org/web/packages/pomp`.