A Renewal Look of Switching Rules in MIL-STD-105D

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Abstract

Let $\{L_k, k \geq 1\}$ be a sequence of independent Bernoulli random variables with $P[L_k = 1] = a_k = 1 - P[L_k = 0]$, where a_k and $1 - a_k$ are respectively the probabilities of acceptance and rejection of the k^{th} lot in a sampling inspection and a_k varies with the sampling plan. A sampling system, MIL-STD-105D, used in quality control consists of three sampling plans with different acceptance probabilities alternatingly used for lot inspection. The decision to switch from one plan to another is based on the history of $\{L_k\}$ and a set of stopping rules. We derive the performance measure AOQ of this sampling system from a renewal process where AOQ involves the moments of various stopping times. The renewal approach is computationally simpler than that of the Markov chain generally used in evaluating AOQ for an infinite sequence of lots. Additionally, it provides a formula for AOQ for a finite sequence of lots.

Key words: Renewal process, stopping time, quality control, acceptance sampling, Average Outgoing Quality.

I. INTRODUCTION

Consider a sequence of lots of N items each. A sampling plan is carried out in each lot by inspecting n items ($n \leq N$), and the lot is accepted if the number of defectives in the sample does not exceed a preset acceptance number. This sampling plan is therefore characterized by a pair of numbers, (n, A) representing the sample size and the acceptance number respectively. One can tighten or relax the lot inspection by varying the sample size and the acceptance number. The widely used MIL-STD-105D in quality control is a system of such sampling plans. The system consists of three levels of inspections, normal, tightened and reduced, which are alternatingly used on a sequence of lots. The decision to switch from a current sampling plan to another one depends on the history of the acceptance records. The stopping rule for the currently used sampling plan is defined in terms of the observed number as well as the pattern of occurrences of the accepted lots under the current sampling plan.

In this paper we formulate this continuous sampling procedure as an alternating renewal process in which the time between two renewals (of a sampling plan) is a sum of several stopping times. A stopping time is defined to be the number of lots inspected under a current plan before switching to a different one. The renewal process formulation provides a convenient mathematical basis for investigating the performance of such sampling system, for instance in computing the performance measure "Average Outgoing Quality" (AOQ) which is defined as the long run proportion of defectives that slipped through the sampling inspection (precise definition is given in Section III). In a renewal process, AOQ can be clearly defined and easily computed. Conceivably, this approach would help clarify some of the ambiguity in applying MIL-STD-105D with respect to the interpretation of "averages" (see Section III for details).

The performance of MIL-STD-105D was evaluated by Stephens and Larsen (1967) who used asymptotic properties of Markov chains to derive formulas for several performance measures which were later tabulated by Schilling and Sheesley (1978). The tables were included in the 1981 version of the MIL-STD-105D which was adopted as a voluntary American National Standard by the American National Standards Institute as ANSI-ASQC-Z1.4. This standard has also been adopted as an international standard, ISO 2859, by the International Organization for Standardization. Other contributors to the development of the performance measures include Dodge (1965) and Burnett (1967); see Schilling and Sheesley (1978) for a historical development. For further analysis based on Markov chains see Koyama (1979). The Standard has also been utilized for medical applications (Jennison and Tarnbull, 1983).

Theoretically, the AOQ curves obtained by the Markov chains and renewal processes should be identical. However, a small discrepancy between the two results exists because of the approximation used in Schilling-Sheesley and also the way they computed the average (see Section IV). The renewal method is computationally simpler than that of the Markov chain; for instance, the manipulation of a rather large transition probability matrix can be avoided. It also provides a way of evaluating the sampling system for a finite number of lots. We have used the renewal approach in studying the Continuous Sampling plans in MIL-STD-1235 (Yang, 1983). The mathematics required for 105D is however more complicated than that of MIL-STD-1235. In the former, the switchings are among the three sampling plans applied to individual items packed in lots, while in the latter the switchings are between the two sampling plans applied to individual items without having to consider the lots.

After presenting the notation and basic definitions in Section II, we discuss in Section

III several possible definitions of AOQ's for finite and infinite number of lots. The generating functions of the stopping times for each of the component sampling plans and for an inspection cycle are derived in Section IV. The distributions of the number of rejected lots are derived in Section V. Formulas for AOQ in finite number of lots and the asymptotic AOQ are given in Section VI together with a numerical illustration. Concluding remarks are given in Section VII.

II. NOTATION, BACKGROUND AND DEFINITIONS

Consider items which are produced at a uniform rate in a factory. Let Y_u, for $u = 1, 2, \ldots,$ indicate whether the u^{th} item is defective or non-defective; set $Y_u = 1$ if it is defective and $Y_u = 0$ otherwise. The process average used in the quality control literature is the limit of $\Sigma_{u=1}^{m} Y_u/m$ as m tends to infinity. We assume that the Y_u's are an i.i.d. sequence of random variables with

$$P\{Y_u = 1\} = 1 - P\{Y_u = 0\} = p, \quad \text{for } 0 < p < 1.$$

Then the p value is called the process average. In reality the Y_u's are not observable and p is unknown. If a sampling inspection plan is imposed on the items, then the attributes of those Y's included in the sample, say $\{Y_1, \ldots, Y_n\}$, can be determined.

In procurement, a number called AQL (acceptable quality level) is specified in the contract for the producer. It is set to be the largest p value that the consumer is willing to accept as an indication of the quality of the product. The acceptance sampling plan provided by MIL-STD-105D assures that the p value does not violate the specified AQL in the sense of having AOQ $\leq$ AQL.

Denote by $(n_1, A_1), (n_2, A_2)$ and (n_3, A_3) the sample size and the acceptance number of the normal, tightened and reduced inspection plans respectly. Associated with each component sampling plan is a lot acceptance probability which is a function of p, n, N and A. The acceptance probabilities of the normal, tightened, and reduced plans are denoted by $P_{a1}(p), P_{a2}(p)$ and $P_{a3}(p)$ respectively. They will be specified in Section III.

MIL-STD-105D starts with a normal inspection plan on a sequence of lots of N items each. It is convenient to think that we are sampling from an infinite sequence $L_1, L_2, \ldots$ of lots. A switch from the normal to a reduced inspection occurs if 10 consecutive lots are accepted*, and a switch to tightened inspection occurs if two out of five consecutive lots are rejected. A switch out of the normal inspection occurs when either one of the two events is observed. The normal inspection is reinstated either when five consecutive lots are accepted during the tightened inspection or when a single lot is rejected during the reduced inspection. The following diagram illustrates the directions of the switchings, where N^*, R and T denote normal, reduced and tightened inspections respectively,

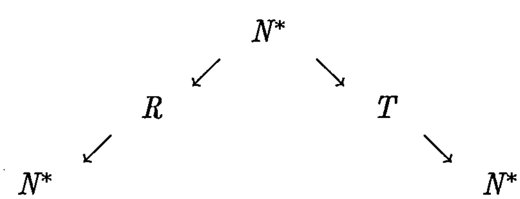

Fig. 1

We call a return to normal inspection from a normal inspection an inspection or a renewal cycle. A cycle can be completed either with a path $N^* \rightarrow R \rightarrow N^*$ or with a path $N^* \rightarrow T \rightarrow N^*$.

*To keep the presentation clear, we consider first the case that only 10 consecutive acceptances are required for switching. The extension to the case in which switching from normal to reduced inspection requires 10 consecutive acceptances and that the total number of defectives found in these 10 lots be less than a specified number will be given in Section VI.

For deriving AOQ and AOQ (t) we need to compute the cycle length, i.e. the number of lots inspected in a cycle. The problem can be broken down by computing the number of lots inspected, deriving $N^* \rightarrow R$, $R \rightarrow N^*$, $N^* \rightarrow T$ and $T \rightarrow N^*$ respected. The length of each segment depends on the lot acceptance probabilities P_{a1}, P_{a2} and P_{a3}.

For this purpose, we need the following set of variables.

Let

$A =$ the event of accepting 10 consecutive lots for the first time,

$A_k =$ the event that the kth lot being the 10th of a 10-consecutive acceptances, for $k = 1, 2, \ldots,$

$B =$ the event of rejecting 2 out of five consecutive lots for the first time,

$B_k =$ the event that the kth lot being the second reject in five consecutive lots inspected,

$C =$ the event that A occurs before B,

$V =$ number of lots inspected before switching out of the normal inspection,

$\tau_R =$ number of lots inspected during a normal inspection before switching to a reduced inspection (i.e., when A occurs before B),

$\tau =$ number of lots inspected until the first occurrence of the event A (irrespective of the occurrences of B),

$\gamma_T =$ number of lots inspected during a normal inspection before switching to a tightened inspection (when B occurs before A),

$\gamma =$ number of lots inspected until the first occurrence of the event B (regardless of the occurrence of A),

$\xi =$ number of lots inspected during a reduced inspection before reverting to a normal inspection,

$\rho =$ number of lots inspected during a tightened inspection before reverting to a normal inspection,

$W =$ number of lots inspected in one inspection cycle,

$Z_R =$ number of slip-through defective items in an $N^* \to R \to N^*$ inspection cycle,

$Z_T =$ number of slip-through defective items in an $N^* \to T \to N^*$ inspection cycle.

III. DEFINITIONS OF AVERAGE OUTGOING QUALITY (AOQ)

We are given an infinite sequence of Y's which are labelled as $Y_1, \ldots, Y_N, Y_{N+1}, \ldots,$ $Y_{2N}, Y_{2N+1}, \ldots$. The j^{th} segment $\{Y_{(j-1)N+1}, \ldots, Y_{jN}\}$ represents the defectiveness of the j^{th} lot. Let $D_1, D_2, \ldots$ denote the sequence of number of defectives found in the respective lot when applying the sampling system.

Under a tightened plan, the lot acceptance probability is given by

$$P_{a2}(p) = P[D \leq A_2]$$

where D is a binomial random variables $B(n_2, p)$.

Under a reduced plan, it is

$$P_{a3}(p) = P[D \leq A_3] + P[A_3 < D < R_e],$$

where D has a binomial distribution $B(n_3, P)$ and $R_e(< N)$ is another assigned number in the plan.

Under a normal plan, there are several alternatives for calculating the lot acceptance probability. One version is

$$P_{a1}(p) = P[D \leq A_1]$$

where D has a binomial distribution $B(n_1, p)$.

Other versions are to take two or multiple samples sequentially from the same lot to determine the lot acceptance and the acceptance probability (see Schilling, 1982).

The definition of AOQ given in ANSI/ASQC Z1.4-1981 is as follows:

"The AOQ is the average quality of outgoing product including all accepted lots or batches, plus all lots or batches which are not accepted after such lots or batches have been effectively 100 percent inspected and all nonconforming units replaced by conforming units."

Since the intended sequence of lots for sampling inspection is not specified, there are many different ways to interpret this definition, for example

(i) AOQ of a single lot given that we know which component plan is used on that lot,

(ii) AOQ of the j^{th} single lot under the sampling system,

(iii) AOQ of an inspection cycle,

(iv) AOQ of the first t lots inspected,

(v) AOQ of infinitely many lots inspected.

As to which AOQ to use, and to compare it with AQL, it depends, among other things, on the number of lots inspected under the sampling system.

In Case (i) the AOQ (p) for a single lot is simply

$$(1 - (n_i/N))P_{ai}(p) \cdot p, \text{ for a specified } i.$$

This is a conditional AOQ given that we know which component plan is used on the lot.

In Case (ii) we need to first determine the probability that i^{th} sampling plan will be used on the j^{th} lot and then calculate AOQ as a weighted average:

$$(1) \quad AOQ(p) = p\Sigma_{i=1}^{3}(1-\frac{n_i}{N})P_{a_i}(p)P[i^{th} \text{ component plan will be used on the } j^{th} \text{ lot}],$$

where n_i and P_{a_i} denote the sample size and the acceptance probability of the normal, tightened and reduced plans, for $i = 1, 2, 3$.

In Case (iii),

$$AOQ(p) = E(Z_R + Z_T)/N(EW)$$

In Case (iv), we denote the AOQ as

$$AOQ(t,p) = \text{ Expected number of slip-through defectives in the first } t \text{ inspections}/tN.$$

In Case (v),

$$AOQ(p) = \lim_{t->\infty} AOQ(t,p).$$

The expressions in Cases (i) and (ii) are for evaluations of the sampling system on a single lot, whereas the performance measures in Cases (iv) and (v) are for the sampling system that applies to either a finite number or an infinite number of lots. Since the two AOQ's defined in Cases (iii) and (v) are mathematically identical, the AOQ used for an inspection cycle in Case (iii) can be considered as an asymptotic measure of performance.

The rest of the paper will be devoted to the calculation of $AOQ(p)$ and $AOQ(t,p)$ in Cases (iii)-(v).

IV. DISTRIBUTION OF THE STOPPING TIMES

Our goal is to derive analytical expressions for the $AOQ(p)$ and $AOQ(t,p)$. For this we need to derive the distributions of $\gamma, \gamma_T, \tau, \tau_R, W$ and V. The computation of the distributions of other variables listed in Section II is straightforward. To simplify the notation, we set

(2) $$P_{a1}(p) = \lambda, \qquad P_{a2}(p) = \mu, \qquad P_{a3}(p) = \theta.$$

IV.1. <u>Distributions of γ and τ</u>

Set

(3)
$$g_m = P\{\gamma = m\} \text{ and}$$
$$b_m = P\{\text{the event B occurs at the } m^{th} \text{ inspection}\}.$$

Suppose that the first rejected lot occurs at the S_1^{th} inspection. Then S_1 has a geometric distribution with the probability generating function (pgf),

(4) $$G_{S_1}(s) = E[s^{S_1}] = \frac{(1-\lambda)s}{1-s\lambda}, \qquad |s| \leq 1.$$

The pgf of γ can be obtained by using a renewal argument as follows. Evaluate the probability g_m by conditioning on S_1,

$$g_m = E[P\{\gamma = m | S_1\}].$$

For $m \geq 6$, we obtain a recurrence formula for g_m,

(5) $$g_m = 4(1-\lambda)^2\lambda^{m-2} + \lambda^4\Sigma_{j=5}^{m-1}\lambda^{m-j-1}(1-\lambda)g_{j-4}.$$

For $2 \leq m \leq 5$,

(6) $$g_m = (m-1)(1-\lambda)^2\lambda^{m-2},$$

and g_0 and g_1 equal zero. Making use of the g_m's in (5) and (6) we obtain the *pgf* of γ

$$G_\gamma(s) = \frac{(1-\lambda)^2\left[\left(\Sigma_{j=0}^{3}(j+1)\lambda^j s^{j+2}\right)(1-\lambda s) + 4\lambda^4 s^6\right]}{1 - s\lambda - \lambda^4(1-\lambda)s^5}. \tag{7}$$

For related study of γ in connection with theory of runs see Li (1980), Huntington (1976).

The distribution of τ is the same as that studied in Yang (1983, see eq. A.1) for Continuous Sampling Plans in which τ is "the first success run of length i". To apply eq. (A.1), we set $i = 10$ and $q = \lambda$ and obtain the probability generating function of our τ as

$$G_\tau(s) = E[s^\tau] = \frac{\lambda^{10}s^{10}(1-\lambda s)}{1 - s + (1-\lambda)\lambda^{10}s^{11}}, \quad |s| \leq 1. \tag{8}$$

IV.2. Distributions of γ_T, τ_R, and V

The techniques used in this section can be found in Feller (1968, Vol. 1, Chapter 8). For $k = 1, 2, \ldots$, set

$$b_k = P\{B_k\}, \quad h_k = P\{A_k\}, \quad c_k = b_k + h_k$$

where B_k and A_k are the events defined in Section II. Denote the generating function of $\{b_k\}$ by $H_\gamma(s)$, and that of $\{h_k\}$ by $H_\tau(s)$. For every fixed k, the events A_k and B_k are mutually exclusive and therefore,

$$P\{\text{either } A_k \text{ or } B_k \text{ occurs}\} = b_k + h_k = c_k.$$

Consequently, the generating function of the sequence $\{c_k\}$ is

$$H_V(s) = H_\gamma(s) + H_\tau(s) - 1. \tag{9}$$

Since $V = \min(\gamma, \tau)$, the probability generating function of V is given by

$$\begin{aligned} G_V(s) &= 1 - \frac{1}{H_V(s)}, \quad |s| \leq 1 \\ &= 1 - \frac{(1 - G_\tau(s))(1 - G_\gamma(s))}{1 - G_\tau(s)G_\gamma(s)}, \quad |s| < 1. \end{aligned} \tag{10}$$

From (10) we obtain the expected duration of a normal inspection,

(11) $$E(V) = G'_{\mathrm{v}}(1) = E\tau E\gamma/(E\tau + E\gamma).$$

Given the *pgf*'s of γ and τ in (7) and (8) we can readily write down the *pgf*'s of τ_R and γ_T which are

(12) $$G_{\tau_R}(s) = \frac{G_\tau(s)\{1 - G_\gamma(s)\}}{1 - G_\tau(s)G_\gamma(s)}\ , \quad |s| < 1,$$

and

(13) $$G_{\gamma_T}(s) = \frac{G_\gamma(s)\{1 - G_\tau(s)\}}{1 - G_\tau(s)G_\gamma(s)}\ , \quad |s| < 1.$$

IV.3. Distributions of ξ and ρ

The ρ has the same *pgf* as that of τ in (8) except that the 10 (acceptances) is changed to 5 and that the acceptance probability λ is replaced by μ, (see (2)) the acceptance probability of the tightened plan. For easy reference, we present it below.

(13) $$G_\rho(s) = \frac{\mu^5 s^5 (1 - \mu s)}{1 - s + (1 - \mu)\mu^5 s^6}\ , \quad |s| \le 1.$$

The variable ξ has a geometric distribution with *pgf*

(14) $$G_\xi(s) = \frac{(1 - \theta)s}{1 - \theta s}\ ,$$

where θ is the acceptance probability under the reduced inspection plan.

IV.4. Distribution of W

To determine the *pgf* of W, the number of lots inspected during a full inspection cycle, we partition the set of all possible ways of completing an inspection cycle into two subsets according to whether A occurs before B or B occurs before A. Let

$$\delta = I[A \text{ occurs before } B] = I[\tau < \gamma].$$

Then

$$W = \tau_R + \xi, \qquad \text{if } \delta = 1$$

and

$$W = \tau_T + \rho, \qquad \text{if } \delta = 0.$$

It follows that

$$G_W(s) = E\left[\delta s^{\tau_R+\xi}\right] + E\left[(1-\delta)s^{\gamma_T+\rho}\right] \tag{15}$$

$$= G_{\tau_R}(s)G_\xi(s) + G_{\gamma_T}(s)G_\rho(s), \quad |s| < 1.$$

Since τ_R and ξ are independent, likewise γ_T and ρ, we have

$$EW = G_{\tau_R}(1)E\xi + E\tau_R + E\gamma_T + G_{\gamma_T}(1)E\rho. \tag{16}$$

V. COUNTING REJECTED LOTS

To quantify the number of slip-through defectives in an inspection cycle, we follow the convention that a rejected lot is subject to a 100% inspection and every defective item found is replaced by a nondefective one. Therefore the slip-through defectives are contained in the accepted lots only. For this we need to determine the expected number of accepted lots in an inspection cycle. For computational convenience, it will be derived as the difference of the expected number of lots inspected and rejected. The expected number of rejected lots needs to be calculated separately for each of the three component inspection plans.

Let S_j, for $j = 1, 2, \ldots$, be the time at which the j^{th} rejected lot is observed (for convenience, we identify the time as the lot number). Then the normal inspection continues as long as

$$S_1 \leq 10 \quad \text{and} \quad 5 \leq S_j - S_{j-1} \leq 10, \quad \text{for} \quad j = 2, 3, 4, \ldots,$$

where $S_0 = 0$. The random variables $S_j - S_{j-1}$ are independent with a common geometric distribution and parameter λ.

Let K_R be the number of lots rejected during a normal inspection before switching to a reduced inspection, i.e., we evaluate K_R on the event $[\tau < \gamma]$. We have

$$(17)\qquad \begin{aligned} P[K_R = 0] &= P[\text{first 10 lots are accepted}] = \lambda^{10}, \\ P[K_R = k] &= P[S_1 \le 10]P[5 \le S_j - S_{j-1} \le 10]^{k-1}P[\text{10 lots} \\ &\qquad\qquad \text{following } S_k \text{ are accepted}] \\ &= (1-\lambda^{10})(\lambda^5 - \lambda^{10})^{k-1}\lambda^{10}, \text{ for } k = 1, 2, \ldots . \end{aligned}$$

It follows that

$$(18)\qquad EK_R\delta = \frac{\lambda^{10}(1-\lambda^{10})}{(1-\lambda^5+\lambda^{10})^2},$$

where $\delta = I[\tau < \gamma]$.

Note that both τ_R and K_R are defined on the event $[\tau < \gamma]$, and they are not independent. It would be natural to compute $P[K_R = k]$ via the conditional probability as

$$(19)\qquad P[K_R = k] = \Sigma_\ell P[K_R = k|\tau_R = \ell]P[\tau_R = \ell].$$

But for evaluating $EK_RI[\tau < \gamma]$, (17) is much easier to use than (19). Similarly, the probability distribution of the number of rejected lots K_T (if normal is switched to a tightened inspection) is given by

$$(20)\qquad \begin{aligned} P\{K_T = k\} &= P\{S_1 \le 10\}P\{5 \le S_j - S_{j-1} \le 10 \\ &\qquad \text{for } j = 1, 2, \ldots, k-1\}^{k-2}P\{S_k - S_{k-1} \le 4\} \\ &= (1-\lambda^{10})(\lambda^5 - \lambda^{10})^{k-2}(1-\lambda^4), \text{ for } k = 2, 3, \ldots . \end{aligned}$$

The number of lots rejected during the tightened inspection, $\tilde{K}$, has a probability distribution

(21) $$P[\tilde{K} = 0] = \mu^5,$$

$$P\{\tilde{K} = k\} = P\{S_j - S_{j-1} \leq 5, \text{ for } j = 1, 2, \ldots, k, \text{ and } S_{k+1} - S_k \geq 6\}$$
$$= (1 - \mu^5)^k \mu^5, k = 1, 2, \ldots .$$

VI. FORMULAS FOR AOQ (p) and AOQ (t, p) IN CASES (iii) - (v)

The switching rule for the reduced plan involves three numbers, n_3, A_3 and R_e. A lot is accepted if the number of defectives D found in the sample $\leq A_3$ and it is rejected if $D \geq R_e$. If $A_3 < D < R_e$, the lot is accepted, however, when this happens the reduced plan will be inserted to the normal plan. Recall that ξ is the first lot under reduced plan with $D \geq A_3$, then ξ^{th} lot will be accepted with probability

$$\psi = P[A_3 < D < R_e | D > A_3].$$

From the results in Sections IV and V, we find that the expected number of slip-through defectives Z_R in an inspection cycle using the path $N^* \rightarrow R \rightarrow N^*$ is given by

(22) $$EZ_R = p\{(E\tau_R - EK_R)(N - n_1) + (E\xi - 1 + \psi)(N - n_3)G_{\tau_R}(1)\}.$$

Similarly, the expected number of slip-through defectives in an inspection cycle using the path $N^* \rightarrow T \rightarrow N^*$ is

(23) $$EZ_T = p\{(E\gamma_T - EK_T)(N - n_1) + (E\rho - E\tilde{K})(N - n_2)G_{\gamma_T}(1)\}.$$

Then, AOQ (p) is the ratio

(24) $$AOQ(p) = E(Z_R + Z_T)/NEW.$$

The required moments for computing AOQ (p) are listed in Table 1.

For a finite sequence of t lots, the $AOQ(t,p)$ can be studied in exactly the same way as in Yang, (1983, eq. (8)), an approximation formula for $AOQ(t,p)$ can be obtained as

$$AOQ^*(t,p) = AOQ(p) + \frac{E(Z_R + Z_T)}{2tN}\left[\frac{\sigma_W^2 + EW}{(EW)^2} - 1\right] . \tag{25}$$

The moments in the second term of the right side of (25) can be computed without difficulty, though tediously, from the pgf's given in Sections IV and V. The second moment of W can be easily put into a computer-ready formula which is long and will not be given here.

The formulas in (24) and (25) are for the switching rules described in Section II. We now extend these formulas to the case where switching from a normal to a reduced plan requires the simultaneous occurrence of ten consecutive acceptances and that the total number of defectives in these ten samples be no more than a specified number, say ℓ_R.

Consider a direct path $N^* \to R$ (without going through T). At τ_R, ten consecutive acceptances have occurred. Let D_j, for $j = 1, \ldots, 10$, denote the number of defectives found in these ten samples. Let

$$\phi = P\left[\Sigma_{j=1}^{10} D_j \le \ell_R | D_j \le Ac, \text{ for } j = 1, \ldots, 10\right] . \tag{26}$$

Given the occurrence of τ_R, the probability of switching to a reduced plan in the next inspection is ϕ and the probability of starting from scratch with a normal plan in the next inspection is $1-\phi$. The number of inspections needed to complete the direct path $N^* \to R$ is

$$\tau_R^* = \Sigma_{j-1}^{M} \tau_R(j), \tag{27}$$

where M has a geometric distribution with pgf

$$G_M(s) = \frac{s\phi}{1 - s(1-\phi)} , \quad |s| \le 1,.$$

and $\tau_R(j)$ are i.i.d. random variables with $pgf\, G_{\tau_R}(s)$ given in (12). It follows that τ_R^* has a pgf

$$G_{\tau_R^*}(s) = G_M(G_{\tau_R}(s)), \quad |s| < 1. \tag{28}$$

Replacing τ_R by τ_R^* in eqs. (16) and (22) gives EZ_R^* and EW^* corresponding to the expected number of slip-through defectives in the inspection cycle $N^* \to R \to N^*$ and the total number of inspections in an inspection cycle respectively. Therefore, the new $AOQ(p)$ is

$$AOQ(p) = E(Z_R^* + Z_T)/NEW^*. \tag{29}$$

The AOQL is the maximum

$$AOQL = \max_{0 \le p \le 1} AOQ(p) \quad \text{and} \quad AOQL(t) = \max_{0 \le p \le 1} AOQ(p,t). \tag{30}$$

Figure 2 gives a numerical example of the $AOQ(p)$ and $AOQL$. The upper AOQ curve is computed with formula (24) which corresponds to the definition in Cases (iii) and (v) of Section III. The lower AOQ curve is computed with formula (1) where the probability P [ith component plan will be used on the jth lot] is replaced by its asymptotic probability P [ith component plan will be used on any lot]. In our formulation, this probability equals to $\frac{EV}{EW}, (EP)G_{\gamma_T}(1)/EW, (E\xi)G_{\tau_R}(1)/EW$, respectively for the normal, tightened and reduced plan. This definition of AOQ corresponds to the one used in Schilling-Sheesley (1978). Figure 2 exhibits a slight difference of the two definitions. In addition to the difference in definitions, their computation involves an approximation which may further contribute to the discrepancy.

Figure 3 is the probability of switching to a reduced plan first. It was computed from eq. (12) evaluated at $s = 1$ by using the L'Hospital rule.

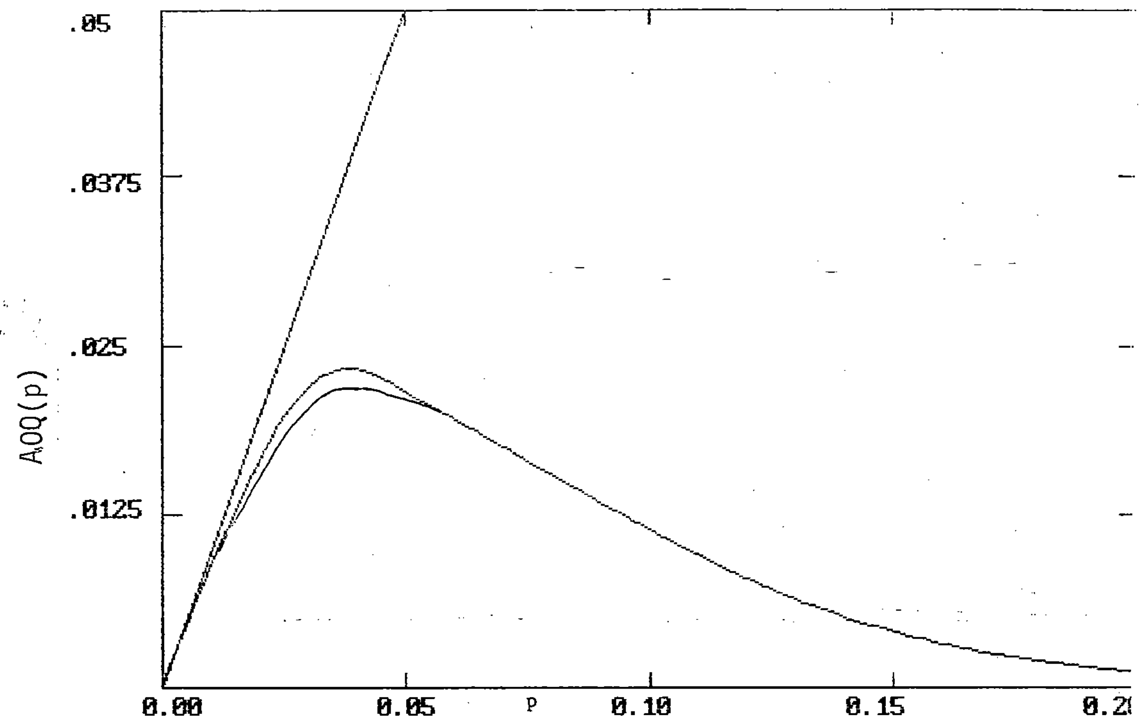

Figure 2 - Average Outgoing Quality of the Sampling System, with normal plan $(n_1 = 20, A_1 = 1)$, reduced plan $(n_3 = 8, A_3 = 0, R_e = 2)$, tightened plan $(n_2 = 32, A_2 = 1)$, upper AOQ curve is computed with eq. (24), lower curve is based on the definition of AOQ used by Schilling-Sheesley.

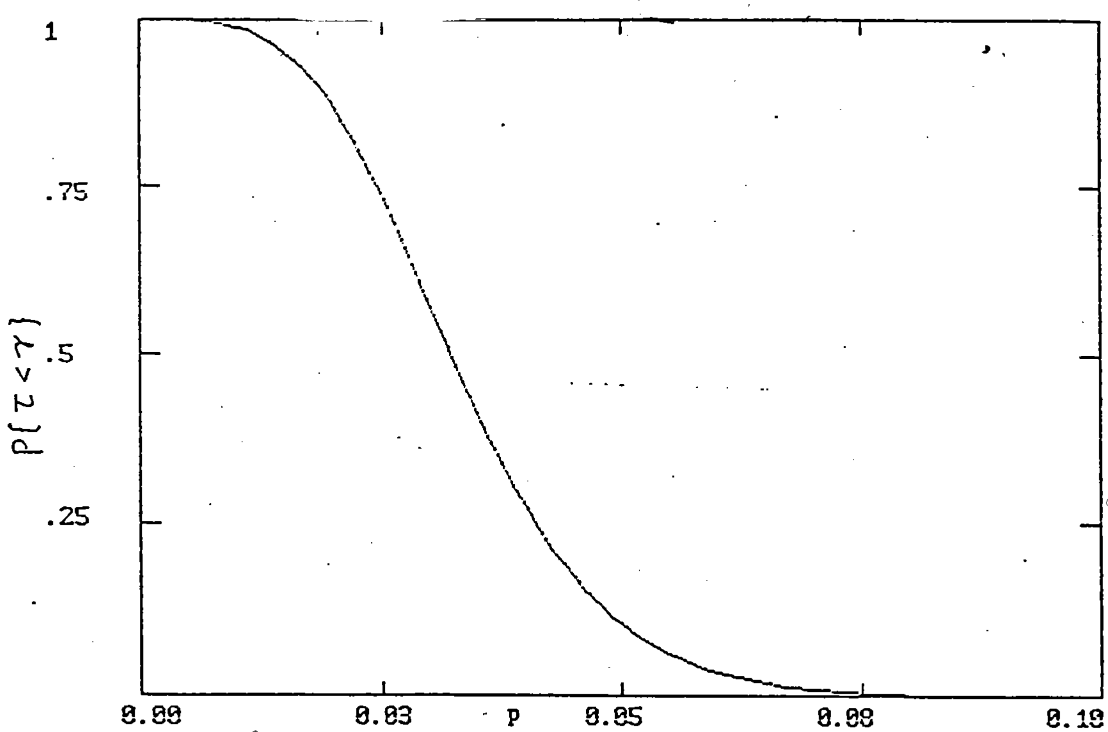

Figure 3 - Probability of the Event that Ten Consecutive Acceptances Occurs Before Observing Two Rejects in Five Consecutive Lots Under Normal Inspection

VII. CONCLUDING REMARKS

We studied the performance measure of the sampling system MIL-STD 105D, AOQ, by using alternating renewal processes. Our intention is to illustrate its utility rather than providing extensive tables for applications. This approach also clearly reveals a variety of ways of interpreting the AOQ, which of course affects the evaluation of the sampling system.

While the AOQ depends on the acceptance probabilities $P_{a_i}(p)$, for $i = 1, 2, 3$, it does not depend on how they are calculated. This means that our formula for AOQ is applicable to either a single or a multiple-sample version of the normal inspection plan.

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Table 1. Moments

$E\tau$	$(1-\lambda^{10})/(1-\lambda)\lambda^{10}$
$G''_{\tau}(1)$	$2(1-12\lambda^{10}+11\lambda^{11}+\lambda^{20}-\lambda^{21})/\left((1-\lambda)\lambda^{10}\right)^2$
$E\gamma$	$\left.\dfrac{[x(s)y'(s)-y(s)x'(s)]}{(x(s))^2}\right\rvert_{s=1}$ where $x(s)=1-\lambda s-\lambda^4(1-\lambda)s^5$. $y(s)=(1-\lambda)^2\left[\left(\Sigma_{j=0}^{3}(j+1)\lambda^j s^{j+2}\right)(1-\lambda s)+4\lambda^4 s^6\right]$, $x'(s)$, $y'(s)$, $x''(s)$, $y''(s)$ are derivatives with respect to s.
$G''_{\gamma}(1)$	$\left.\dfrac{[x(s)y''(s)-y(s)x''(s)]}{(x(s))^2}-\dfrac{2E\gamma x'(s)}{x(s)}\right\rvert_{s=1}$
$E\tau_R$	$\left[(E\tau)G''_{\gamma}(1)+2(E\tau)^2E\gamma-G''_{\tau}(1)E\gamma\right]/2[E\tau+E\gamma]^2$
$G_{\tau_R}(1)$	$[1+(E\tau/E\gamma)]^{-1}$
E_{γ_T}	interchange τ and γ in $E\tau_R$
$G_{\gamma_T}(1)$	$1-G_{\tau_R}(1)$
$E\xi$	$(1-\theta)^{-1}$
$E\rho$	$(1-\mu^5)/(1-\mu)\mu^5$
EW	$G_{\tau_R}(1)E\xi+E\tau_R+E\gamma_T+G_{\gamma_T}(1)E\rho$
EK_R	$\lambda^{10}(1-\lambda^{10})/(1-\lambda^5+\lambda^{10})^2$
EK_R	$(1-\lambda^4)(1-\lambda^{10})/(1-\lambda^5+\lambda^{10})^2$
$E\tilde{K}$	$(1-\mu^5)/\mu^5$
EZ_R	$p[(E\tau_R-EK_R)(N-n_1)+(E\xi-1+\psi)(N-n_2)G_{\tau_R}(1)]$
EZ_T	$p\left[(E\gamma_T-EK_T)(N-n_1)+(E\rho-E\tilde{K})(N-n_3)G_{\gamma_T}(1)\right]$

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