Isotonic Selection Procedures for Tukey's Lambda Distributions*

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Abstract

The problem of selecting all populations better than a control or standard with an ordering prior is considered for Tukey's generalized symmetric lambda distributions. Some isotonic procedures based on sample medians are proposed and their properties are investigated. It is shown that the isotonic procedure is better than some classical procedures in terms of reducing the expected number of bad populations in the selected subset. Tables of associated constants for the proposed procedure are provided.

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KEY WORDS: Isotonic Selection Procedures; Tukey's Generalized Lambda Disributions; Ordering Prior; Distribution of Sample Medians; Comparison with a Control

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1. Introduction

The problem of selecting a subset containing all populations better than a control or standard has been considered by many authors under different formulations (for references, see Gupta and Panchapakesan(1979)). However, most of them assume that there is no knowledge about the correct ordering among unknown parameters. But in practice, there are cases where the experimenter may know the correct ordering even though the true values of the parameters are unknown. For example, in the pharmacological studies, a higher amount of acetaminophen in the pain reliever will result in a quicker effect on relieving fever. In this situation, when the experimenter considers the time taken to reduce the temperature to a certain degree as a measurement of the effect, the experimenter knows the correct ordering among several pain relievers with different amounts of acetaminophen even though the true values of the effects are unknown. For this case, it is reasonable to assume an ordering prior. Selection procedures under the assumption of ordering priors are, in general, concerned with isotonic inference. Gupta and Yang(1984) have considered isotonic selection procedures for the case of normal populations. They have also considered some isotonic procedures under the assumption of partial ordering. Gupta and Huang(1984) have studied isotonic procedures for the case of binomial populations and Huang(1984) has considered a nonparametric isotonic selection procedure. Gupta and Leu(1986) have proposed and studied isotonic selection procedures for unknown guarantee lifetimes in the case of two-parameter exponential populations. Recently, Liang(1988) has studied isotonic rules for selecting good truncated exponential populations. Also, Liang and Panchapakesan (1988) have studied a Bayesian approach to isotonic selection.

In this paper, we investigate isotonic procedures for the family of Tukey's generalized symmetric lambda distributions (hereafter called symmetric lambda distributions). It is well known that the family of lambda distributions can be used to approximate many univariate continuous distributions. For further discussions, reference should be made to Ramberg et al. (1979) or Sohn(1985).

In Section 2, we formulate the problem of selecting populations better than a control or standard and propose some isotonic selection procedures.

In Section 3, we discuss the approximate evaluation of constants used in the proposed procedures. The approximations are useful because of difficulties involved in obtaining the exact distribution of the sum of sample medians.

In Section 4, comparisons are made between proposed procedures and some other procedures in terms of expected number of bad populations included in the selected subset.

2. Preliminaries

Let $\pi_0, \pi_1, ..., \pi_k$ be k+1 independent lambda populations, where π_0 can be regarded as a control or standard population. Let the random variable X_i be the observable characteristic of π_i, which follows a symmetric lambda distribution $F_i = F(\cdot|\theta_i, \beta, \gamma)$, which is defined by its inverse function as follows:

$$x = F_i^{-1}(p) = \theta_i + \frac{1}{\beta}\{p^\gamma - (1-p)^\gamma\}, \quad x \in \mathbb{R}^1, \quad 0 < p < 1.$$

Here θ_i is an unknown location parameter that we are interested in, β and γ are common known scale and shape parameters. The variance of X_i is given by

$$var(X_i) = \frac{2}{\beta^2}\{\frac{1}{2\gamma+1} - \frac{[\Gamma(\gamma+1)]^2}{\Gamma(2\gamma+2)}\}, \quad i = 0, 1, \ldots, k.$$

Without loss of generality, we assume that $var(X_i)$ =1, i=0,1,...,k. Let X_{ij}, $j = 1, ..., n$ be n independent random samples from π_i, i=1,2,...,k, respectively. The value of θ_0 associated with π_0 may or may not be known. A population π_i is said to be "good" ("bad") if $\theta_i \geq (<)\theta_0$. Assume that we have a simple ordering prior of $\theta_1, ..., \theta_k$. Without loss of generality, let $\theta_1 \leq \theta_2 \leq ... \leq \theta_k$. Our goal is to select a subset which includes all good populations with requirement that the minimum probability of a correct selection (CS) be at least equal to a preassigned number $P^*(0 < P^* < 1)$. Note that a correct selection is the selection of any subset which includes all good populations.

Let $\Omega = \{\underline{\theta} = (\theta_0, \theta_1, ..., \theta_k) | -\infty < \theta_1 \leq \theta_2 \leq ... \leq \theta_k < \infty, \ -\infty < \theta_0 < \infty\}$ be the parameter space, where $\Omega \subseteq \Re^{k+1}$. Also let us define

$$\Omega_0 = \{\underline{\theta} \in \Omega | \theta_k < \theta_0\},$$

$$\Omega_i = \{\underline{\theta} \in \Omega | \theta_{k-i} < \theta_0 \leq \theta_{k-i+1}\}, i = 1, 2, ..., k-1,$$

and

$$\Omega_k = \{\underline{\theta} \in \Omega | \theta_0 \leq \theta_1\}.$$

Then the sets Ω_i, $i = 0, 1, ..., k$ are mutually disjoint sets and $\Omega = \bigcup_{i=0}^{k} \Omega_i$.

Now we give some definitions.

Definition 2.1. A selection procedure R is called isotonic if and only if whenever it selects π_i with parameter θ_i, it also selects π_j when $\theta_i \leq \theta_j$.

Definition 2.2. A real-valued function f defined on a poset (S,$\lesssim$), where $\lesssim$ denotes a binary partial order on S, is called isotonic if f preserves the partial order on S.

Definition 2.3. Let g be a given function on $(S, \lesssim)$ and let W be a given positive function on $(S, \lesssim)$. An isotonic function g^* on $(S, \lesssim)$ is called an isotonic regression of g with weights W if it minimizes the sum $\sum_{x \in S} [g(x) - g^*(x)]^2 W(x)$ over a class of all isotonic functions on S.

From Barlow et al. (1972), it is known that there exists one and only one isotonic regression of a given g with weights W on S when S is simply ordered. Also the isotonic estimator of θ_i can be found by using the max-min formulas given by Ayer et al. (1955) as follows.

The generalized lambda distribution is used to approximate a large class of distributions including those with heavy tails. As such, the sample median is a robust estimator of θ unlike the sample mean. This is the primary motivation for our procedure based on sample medians. Furthermore, the computation of procedures based on sample means are mathematically hard for the generalized lambda distributions.

Let $\tilde{X}_i$ be the sample median of π_i based on n independent random samples $X_{i1}, \ldots, X_{in}, i = 1, 2, \ldots, k$, respectively. For convenience, let $n = 2m + 1, m$ being a nonnegative integer. Also let C^2 denote the common known variance of $\tilde{X}_i$. Let us define a finite set $S = \{\theta_1, \ldots, \theta_k | \theta_1 \leq \ldots \leq \theta_k\}$ and let $W(\theta_i) \equiv w_i = n, i = 1, 2, \ldots, k$, respectively. Then by the max-min formulas, the isotonic regression of g with the weights $W(\theta_i) = w_i$ is g^*, where

$$g^*(\theta_i) = \max_{1 \leq s \leq 1} \min_{s \leq t \leq k} \left\{ \frac{\tilde{X}_s + \ldots + \tilde{X}_t}{t - s + 1} \right\}. \tag{2.1}$$

Hence the isotonic estimator $\hat{X}_{i:k}$ of θ_i is

$$\hat{X}_{i:k} = \max_{1 \leq s \leq i} \hat{\tilde{X}}_{s:k}, \tag{2.2}$$

$$\hat{\tilde{X}}_{s:k} = min\left\{\tilde{X}_s, \frac{\tilde{X}_s + \tilde{X}_{s+1}}{2}, \ldots, \frac{\tilde{X}_s + \ldots + \tilde{X}_k}{k-s+1}\right\}, \tag{2.3}$$

for $i = 1, 2, \ldots, k$, respectively.

Here we confine ourselves to the class of isotonic procedures which also satisfy the P^*-condition, *i. e.*, for an isotonic rule R,

$$\inf_{\underline{\theta} \in \Omega} P_{\underline{\theta}}(CS|R) \geq P^*. \tag{2.4}$$

2.1. Proposed Rules R_1 and R_2

(A) θ_0 known

In this case, no samples need to be taken from the control population π_0. The proposed rule R_1 is as follows:

Procedure R_1: The procedure consists of k steps where steps $i = 1, 2, \ldots, k$ are defined as follows: For $i = 1, 2, \ldots, k-1$,

Step i. Select the subset $\{\pi_i, \ldots, \pi_k\}$ and stop if

$$\hat{X}_{i:k} \geq \theta_0 - C\ d_{i:k}^{(1)},$$

otherwise reject π_i and go to Step $i+1$, and

Step k. Select π_k if

$$\hat{X}_{k:k} \geq \theta_0 - C\ d_{k:k}^{(1)},$$

otherwise reject π_k and decide that none of k populations are good.

Here $d^{(1)}_{i:k}, i = 1, 2, \ldots, k$ are chosen to be the smallest non-negative constants so that the procedure R_1 is isotonic and meets the P^*-condition. Now for any $\underline{\theta} \in \Omega_i, 1 \leq i \leq k$,

$$\hat{\tilde{Z}}_{i:k} = min\left\{\tilde{Z}_i, \frac{\tilde{Z}_i + \tilde{Z}_{i+1}}{2}, \ldots, \frac{\tilde{Z}_i + \ldots + \tilde{Z}_k}{k-i+1}\right\}, \tag{2.5}$$

where

$$\tilde{Z}_i = \frac{\tilde{X}_i - \theta_i}{C}, i = 1, 2, \ldots, k, \text{ respectively.}$$

Then the following theorem holds.

Theorem 2.1. For given $P^*(0 < P^* < 1)$ and $\underline{\theta} \in \Omega_i$,

$$\inf_{\underline{\theta} \in \Omega_i} P_{\underline{\theta}}(CS|R_1) = Pr\{\hat{\tilde{Z}}_{1:i} \geq -d^{(1)}_{k-i+l:k}\}, i = 1, 2, \ldots, k. \tag{2.6}$$

From Theorem 2.1, we have the following corollary.

Corollary 2.1. For given $P^*(0 < P^* < 1), d^{(1)}_{k-i+1:k}$ is the solution of the equation

$$Pr(\hat{\tilde{Z}}_{1:i} \geq -z) = P^* \tag{2.7}$$

and satisfies the P^*-condition for the rule R_1.

The evaluation of the constants $d^{(1)}_{k-i+1:k}$ will be discussed in the next section.

Remarks: Since $\hat{\tilde{Z}}_{k-i+1:k}$ has the same distribution as $\hat{\tilde{Z}}_{1:i}, d^{(1)}_{k-i+1:k} = d^{(1)}_{1:i}, i = 1, 2, \ldots, k$, respectively. Also it can be seen that $d^{(1)}_{1:i}$ is nondecreasing in i.

(B) θ_0 unknown

Since θ_0 is unknown, n independent observations $X_{01}, \ldots, X_{0n}$ from the control population

π_0 are taken. Let $\tilde{X}_0$ denote the median of the above samples. Then the selection procedure R_2 is defined as follows:

Procedure R_2: Steps $i = 1, 2, \ldots, k$ are defined as follows: For $i = 1, 2, \ldots, k-1$,

Step i. Select the subset $\{\pi_i, \ldots, \pi_k\}$ and stop if

$$\hat{X}_{i:k} \geq \tilde{X}_0 - Cd^{(2)}_{i:k},$$

otherwise reject π_i and go to Step $i+1$, and

Step k. Select π_k only and stop if

$$\hat{X}_{k:k} \geq \tilde{X}_0 - Cd^{(2)}_{k:k},$$

otherwise reject π_k and decide that none of them are good populations.

Now similar to Theorem 2.1 and Corollary 2.1, the following theorem and its corollary hold.

Theorem 2.2. For given $P^*(0 < P^* < 1)$ and $\underline{\theta} \in \Omega_i$,

$$\inf_{\underline{\theta} \in \Omega_i} P_{\underline{\theta}}(CS|R_2) = Pr\{\hat{\tilde{Z}}_{1:i} \geq \tilde{Z}_0 - d^{(2)}_{k-i+1:k}\}, i = 1, 2, \ldots, k, \tag{2.8}$$

where $\tilde{Z}_0 = (\tilde{X}_0 - \theta_0)/C$.

Corollary 2.2. For given $P^*(0 < P^* < 1), d^{(2)}_{k-i+1:k}$, which is the solution of the equation

$$Pr\{\hat{\tilde{Z}}_{1:k} \geq \tilde{Z}_0 - z\} = P^*, \tag{2.9}$$

satisfies the P^*-condition for the rule R_2.

The evaluation of the constants $d^{(2)}_{k-i+1:k}$ will be discussed in the following section.

Remark: It can be seen that for $i = 1, 2, \ldots, k, d^{(2)}_{k-i+1:k} = d^{(2)}_{1:i}$ and $d^{(2)}_{1:i}$ is nondecreasing in i.

3. The Evaluation of Constants $d^{(1)}_{k-i+1:k}$ and $d^{(2)}_{k-i+1:k}$.

Since the evaluation of the constants $d^{(2)}_{k-i+1:k}$ is similar to that of $d^{(1)}_{k-i+1:k}$, we will discuss here only the evaluation of the constants $d^{(1)}_{k-i+1:k}$. Now to solve the equation (2.8) the following lemmas are needed. The first lemma, due to Gupta and Yang (1984), is cited without proof.

Lemma 3.1. Suppose $U_1, U_2, \ldots,$ are iid random variables whose distribution is not concentrated on a half-axis. Let $T_0 = 0, T_j = U_1 + \ldots + U_j, j = 1, 2, \ldots,$ respectively and let $U_i = Q_i - x$, where $E(Q_i) = 0$, for $i = 1, 2, \ldots,$ respectively. Let $V_j = \min_{1 \le r \le j} \frac{1}{r} T_r$. Then

$$Pr\{V_{\ell+1} \ge v\} = \frac{1}{\ell+1} \sum_{j=0}^{\ell} Pr\{V_j \ge v\} Pr\{T_{\ell-j+1} \ge 0\}, \ell = 0, 1, \ldots, \qquad (3.1)$$

where $Pr\{V_0 \ge v\} \equiv 1$ for all v.

To use Lemma 3.1, it is necessary to evaluate the quantity $Pr\{T_{\ell-j+1} \ge 0\}$, where T_j denotes the sum of j iid sample medians from symmetric lambda populations. To find the exact and closed forms of distribution of T_j is very difficult. Since the lambda family of distributions can be used to approximate many theoretical distributions very well, the distribution of T_j can also be approximated by a lambda distribution. Ramberg et al. (1979) and Sohn (1985) have studied the approximation of a distribution by using a lambda distribution based on the second and fourth central moments. Thus it is necessary

to compute the standardized second and fourth central moments of the sum of j sample medians from j iid symmetric lambda distributions with mean 0 and variance 1. To this end, we have the following lemma.

Lemma 3.2. Let μ_r be the rth central moment of the sum of j sample medians from j iid lambda distributions based on a common sample size $n = 2m + 1, m \geq 0$, where β and γ are common scale and shape parameters, respectively. Then

$$\mu_2 = \frac{2j\Gamma(2m+2)\{\Gamma(m+1)\Gamma(m+2\gamma+1) - [\Gamma(m+1+\gamma)]^2\}}{\beta[\Gamma(m+1)]^2\Gamma(2m+2\gamma+2)}, \tag{3.2}$$

and

$$\begin{aligned}\mu_4 = {} & \frac{12j(j-1)}{\beta^4}\left\{\frac{\Gamma(2m+2)}{[\Gamma(m+1)]^2}\right\}^2\left\{\frac{\Gamma(m+1)\Gamma(m+2\gamma+1) - [\Gamma(m+2\gamma+1)]^2}{\Gamma(2m+2\gamma+2)}\right\}^2 + \\ & + \frac{2j\Gamma(2m+2)}{\beta^4[\Gamma(m+1)]^2\Gamma(2m+2+4\gamma)}\{\Gamma(m+1)\Gamma(m+1+4\gamma) - \\ & - 4\Gamma(m+1+\gamma)\Gamma(m+1+3\gamma) + 3[\Gamma(m+1+2\gamma)]^2\},\end{aligned} \tag{3.3}$$

where $\Gamma(\cdot)$ is the gamma function.

Now based on Lemma 3.1 and Lemma 3.2, values of $d^{(1)}_{k-i+1:k}$ are computed by using recursion formula (3.1). They are given in Table 1 for $m = 0(1)3(2)9, 10, P^* = 0.75, 0.90, 0.95, 0.99$ when the lambda populations have common scale and shape parameters $(\beta, \gamma) = (-0.0466, -0.0246), (-0.1686, -0.0802)$ and $(-0.2307, -0.1045)$. It should be pointed out that each lambda distribution with scale and shape parameters $(\beta, \gamma) = (-0.0466, -0.0246), (-0.1686, -0.0802)$ and $(-0.2307, -0.1045)$ has kurtosis 4.6, 6.0 and 7.0, respectively. These lambda distributions thus have heavier tail than a standard logistic distribution. Results of Lorenzen and McDonald (1981) for a selection procedure

for logistic distributions indicate that their median procedure is fairly as efficient as their mean procedure. Hence, in our case, the median selection rule can be expected to be fairly as efficient as the mean selection rule. Finally, values of $d^{(2)}_{k-i+1;k}$ can be computed similarly.

4. Expected Number of Bad Populations in the Selected Subset.

Suppose θ_0 is known and thus, without loss of generality let $\theta_0 = 0$. Let B be the (random) number of bad populations in the subset selected. Then the expected number of bad populations, denoted by $E_{\underline{\theta}}(B|R)$, can be used as a measure of the efficiency of the rule R. Now, for the rule R_1, for any $j, 0 \leq j \leq k$,

$$\sup_{\underline{\theta}\in\Omega_{k-j}} E_{\underline{\theta}}(B|R_1) = \sup_{\underline{\theta}\in\Omega_{k-j}} \sum_{r=1}^{j} P_{\underline{\theta}}\{\bigcup_{i=1}^{r}(\hat{X}_{i:k} \geq -Cd^{(1)}_{i:k})\}$$

$$= \sum_{r=1}^{j} Pr\{\bigcup_{i=1}^{r}(\hat{\hat{Z}}_{i:j} \geq -d^{(1)}_{i:k})\}. \tag{4.1}$$

Now let us consider an alternative rule R_3 which uses a fixed constant d_3 and selects a subset and satisfies the P^*-condition. This rule R_3 is

R_3: Select π_i if and only if $\hat{X}_{i:k} \geq -Cd_3$ for $i = 1, \ldots, k$, where $d_3(\geq 0)$ is chosen so as to satisfy the P^*-condition. Then one can see that $d_3 = d^{(1)}_{1:k}$ and also

$$\sup_{\underline{\theta}\in\Omega_{k-j}} E_{\underline{\theta}}(B|R_3) = \sum_{r=1}^{j} Pr\{\bigcup_{r=1}^{j}(\hat{\hat{Z}}_{i:j} \geq -d_3)\}. \tag{4.2}$$

The following theorem is straightforward.

Theorem 4.1. For any $j, 0 \leq j \leq k$,

$$\sup_{\underline{\theta}\in\Omega_{k-j}} E_{\underline{\theta}}(B|R_1) \leq \sup_{\underline{\theta}\in\Omega_{k-j}} E_{\underline{\theta}}(B|R_3). \tag{4.3}$$

From the above theorem, the rule R_1 is uniformly better than the rule R_3 in terms of the expected number of bad populations in the selected subset.

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Table 1. Values of $d_{1:k}^{(1)}$ for the case of symmetric lambda populations with common scale and shape parameters

$$(\beta, \gamma) = (-0.0466, -0.0246), \quad \text{Kurtosis} = 4.6$$

m	k	P^*	0.75	0.90	0.95	0.99
0	1		0.5920	1.1949	1.6141	2.5688
	2		0.7382	1.2879	1.6796	2.5929
	3		0.7836	1.3087	1.6899	2.5938
	4		0.8029	1.3148	1.6918	2.5939
	5		0.8123	1.3167	1.6922	2.5939
	6		0.8174	1.3174	1.6922	2.5939
1	1		0.3860	0.7614	1.0081	1.5278
	2		0.4745	0.8109	1.0393	1.5358
	3		0.5005	0.8209	1.0433	1.5360
	4		0.5111	0.8236	1.0439	1.5360
	5		0.5161	0.8242	1.0440	1.5360
	6		0.5187	0.8244	1.0440	1.5360
2	1		0.3077	0.6008	0.7885	1.1670
	2		0.3758	0.6368	0.8100	1.1747
	3		0.3954	0.6437	0.8126	1.1748
	4		0.4032	0.6454	0.8130	1.1748
	5		0.4069	0.6459	0.8130	1.1748
	6		0.4088	0.6460	0.8130	1.1748
3	1		0.2634	0.5115	0.6682	0.9804
	2		0.3259	0.5408	0.6853	0.9839
	3		0.3369	0.5463	0.6873	0.9839
	4		0.3433	0.5477	0.6875	0.9839
	5		0.3463	0.5480	0.6875	0.9839
	6		0.3478	0.5481	0.6875	0.9839
5	1		0.2127	0.4107	0.5339	0.7743
	2		0.2579	0.4331	0.5466	0.7767
	3		0.2706	0.4372	0.5480	0.7767
	4		0.2756	0.4382	0.5482	0.7767
	5		0.2780	0.4384	0.5482	0.7767
	6		0.2791	0.4385	0.5482	0.7767

Table 1. (continued)

m	k	P^*	0.75	0.90	0.95	0.99
7	1		0.1832	0.3527	0.4573	0.6795
	2		0.2217	0.3715	0.4679	0.6614
	3		0.2325	0.3749	0.4690	0.6614
	4		0.2367	0.3757	0.4691	0.6614
	5		0.2387	0.3759	0.4691	0.6614
	6		0.2397	0.3759	0.4691	0.6614
9	1		0.1633	0.3138	0.4064	0.5840
	2		0.1974	0.3303	0.4155	0.5856
	3		0.2069	0.3333	0.4165	0.5856
	4		0.2107	0.3340	0.4166	0.5856
	5		0.2124	0.3342	0.4166	0.5856
	6		0.2132	0.3342	0.4166	0.5856
10	1		0.1555	0.2987	0.3866	0.5548
	2		0.1879	0.3143	0.3952	0.5563
	3		0.1970	0.3171	0.3961	0.5563
	4		0.2005	0.3178	0.3962	0.5563
	5		0.2021	0.3179	0.3962	0.5563
	6		0.2029	0.3180	0.3962	0.5563
$(\beta, \gamma) = (-.1686, -.0802)$, Kurtosis $= 6.0$						
0	1		0.5591	1.1526	1.5863	2.6451
	2		0.7055	1.2573	1.6683	2.6867
	3		0.7537	1.2834	1.6837	2.6897
	4		0.7751	1.2920	1.6874	2.6897
	5		0.7860	1.2951	1.6883	2.6897
	6		0.7920	1.2963	1.6885	2.6897
1	1		0.3619	0.7218	0.9650	1.4973
	2		0.4480	0.7731	0.9991	1.5078
	3		0.4740	0.7839	1.0040	1.5080
	4		0.4847	0.7869	1.0048	1.5080
	5		0.4899	0.7878	1.0049	1.5080
	6		0.4927	0.7881	1.0049	1.5080

Table 1. (continued)

m	k	P^*	0.75	0.90	0.95	0.99
2	1		0.2883	0.5671	0.7490	1.1286
	2		0.3537	0.6032	0.7714	1.1340
	3		0.3729	0.6103	0.7742	1.1341
	4		0.3806	0.6121	0.7747	1.1341
	5		0.3843	0.6127	0.7747	1.1341
	6		0.3862	0.6128	0.7747	1.1341
3	1		0.2468	0.4819	0.6324	0.9384
	2		0.3014	0.5107	0.6497	0.9422
	3		0.3171	0.5162	0.6518	0.9422
	4		0.3234	0.5176	0.6520	0.9422
	5		0.3264	0.5180	0.6521	0.9422
	6		0.3279	0.5181	0.6521	0.9422
5	1		0.1992	0.3861	0.5035	0.7356
	2		0.2421	0.4078	0.5160	0.7380
	3		0.2543	0.4118	0.5174	0.7380
	4		0.2591	0.4128	0.5176	0.7380
	5		0.2614	0.4131	0.5176	0.7380
	6		0.2625	0.4132	0.5176	0.7380
7	1		0.1716	0.3312	0.4305	0.6242
	2		0.2080	0.3493	0.4407	0.6261
	3		0.2183	0.3526	0.4419	0.6261
	4		0.2223	0.3534	0.4420	0.6261
	5		0.2242	0.3536	0.4420	0.6261
	6		0.2251	0.3536	0.4420	0.6261
9	1		0.1530	0.2946	0.3822	0.5515
	2		0.1852	0.3103	0.3910	0.5531
	3		0.1942	0.3132	0.3919	0.5531
	4		0.1977	0.3139	0.3920	0.5531
	5		0.1994	0.3141	0.3921	0.5531
	6		0.2002	0.3131	0.3921	0.5531

Table 1. (continued)

m	k	P^*	0.75	0.90	0.95	0.99
10	1		0.1457	0.2803	0.3634	0.5235
	2		0.1762	0.2952	0.3717	0.5250
	3		0.1848	0.2979	0.3726	0.5250
	4		0.1882	0.2985	0.3727	0.5250
	5		0.1897	0.2987	0.3727	0.5250
	6		0.1905	0.2987	0.3727	0.5250
$(\beta, \gamma) = (-.2307, -.1045)$, Kurtosis $= 7.0$						
0	1		0.5437	1.1317	1.5708	2.6758
	2		0.6894	1.2413	1.6607	2.7282
	3		0.7387	1.2702	1.6792	2.7331
	4		0.7610	1.2802	1.6840	2.7331
	5		0.7727	1.2840	1.6854	2.7331
	6		0.7793	1.2856	1.6857	2.7331
1	1		0.3507	0.7031	0.9441	1.4808
	2		0.4355	0.7750	0.9795	1.4925
	3		0.4614	0.7663	0.9848	1.4929
	4		0.4723	0.7694	0.9857	1.4929
	5		0.4776	0.7704	0.9859	1.4929
	6		0.4803	0.7708	0.9859	1.4929
2	1		0.2794	0.5513	0.7303	1.1081
	2		0.3435	0.5874	0.7530	1.1139
	3		0.3624	0.5946	0.7560	1.1140
	4		0.3701	0.5965	0.7564	1.1140
	5		0.3738	0.5970	0.7565	1.1140
	6		0.3756	0.5972	0.7565	1.1140
3	1		0.2391	0.4681	0.6156	0.9181
	2		0.2925	0.4967	0.6329	0.9221
	3		0.3079	0.5022	0.6351	0.9221
	4		0.3141	0.5036	0.6354	0.9221
	5		0.3171	0.5040	0.6354	0.9221
	6		0.3186	0.5041	0.6354	0.9221

Table 1. (continued)

m	k	P^*	0.75	0.90	0.95	0.99
5	1		0.1930	0.3747	0.4893	0.7172
	2		0.2349	0.3961	0.5017	0.7197
	3		0.2468	0.4001	0.5031	0.7197
	4		0.2515	0.4010	0.5033	0.7197
	5		0.2537	0.4013	0.5033	0.7197
	6		0.2548	0.4014	0.5033	0.7197
7	1		0.1662	0.3213	0.4181	0.6076
	2		0.2017	0.3390	0.4281	0.6095
	3		0.2117	0.3423	0.4293	0.6095
	4		0.2157	0.3431	0.4294	0.6095
	5		0.2175	0.3433	0.4294	0.6095
	6		0.2184	0.3433	0.4294	0.6095
9	1		0.1482	0.2857	0.3709	0.5363
	2		0.1795	0.3011	0.3796	0.5379
	3		0.1883	0.3039	0.3806	0.5379
	4		0.1918	0.3046	0.3807	0.5379
	5		0.1934	0.3048	0.3807	0.5379
	6		0.1942	0.3048	0.3807	0.5379
10	1		0.1411	0.2718	0.3526	0.5090
	2		0.1786	0.2864	0.3508	0.5104
	3		0.1792	0.2890	0.3617	0.5104
	4		0.1825	0.2896	0.3618	0.5104
	5		0.1840	0.2898	0.3618	0.5104
	6		0.1847	0.2898	0.3618	0.5104

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The problem of selecting all populations better than a control or standard with an ordering prior is considered for Tukey's generalized symmetric lambda distributions. Some isotonic procedures based on sample medians are proposed and their properties are investigated. It is shown that the isotonic procedure is better than some classical procedures in terms of reducing the expected number of bad populations in the selected subset. Tables of associated constants for the proposed procedure are provided.

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