# A STATISTICAL ANALYSIS OF SHALLOW PATCHING COSTS IN INDIANA FOR 1982-3

bу

Virgil L. Anderson<sup>1</sup> Kumares C. Sinha<sup>2</sup> and Stephen M. O'Brien<sup>3</sup>

Technical Report #84-41

Department of Statistics Purdue University

October 1984

<sup>&</sup>lt;sup>1</sup> Professor of Statistics, Purdue University

<sup>&</sup>lt;sup>2</sup> Professor of Civil Engineering, Purdue University

<sup>&</sup>lt;sup>3</sup> Graduate Student, Civil Engineering, Purdue University

#### ABSTRACT

This paper presents one phase of the routine maintenance productivity analysis (with no quality evaluation ) for the Indiana Department of Highways (IDOH). It investigates many factors and independent variables influencing cost of shallow patching in the 37 subdistricts of IDOH for 1982-3. The detailed statistical analysis includes analysis of variance, regression analysis, covariance analysis, normality tests and confidence intervals on the subdistricts means, unadjusted and adjusted for independent variables, including labor and material costs.

The statistical analyses were interesting in that labor costs were most influential on shallow patching costs but material costs were not. The engineers explain this result by concluding that labor costs are variable depending on work practice, weather, road conditions and material workability, but material costs are fixed and depend only on the amount purchased.

#### I. BACKGROUND

During the past few years, cost of maintaining roads in the United States has been continually increasing while funds for maintenance operations have not kept pace with inflation. Today if productivity of routine maintenance projects could be increased by one percent (1%), a savings of over \$150 million per year could be obtained. [Reference: American's Highways: Accelerating the Search for Innovation, TRB Special Report 202, pg. 99]. Purdue University, through the funding of the Indiana Department of Highways (IDOH) has launched an extensive research program in the quest of improving routine maintenance productivity.

The first phase of the program was to identify potential areas of cost and energy savings. It was found that considerable cost and energy savings could be achieved through better assignment of equipment in different activities (See Appendix 1 for the abstract of the paper pertaining to this).

The second phase of the program was to compile the IDOH's data base and transform the large amount of material found in the data base into concise, easy to read reports so that first and second line supervisors can assign the proper mix of personnel, equipment, materials and procedures to routine maintenance tasks. (See Appendix 2 for the abstract of the paper).

A management information system (MIS) was developed for IDOH. This system uses for input the crew day cards completed in the field and produced an output of concise reports and bar charts that second line supervisors can use to establish how their subdistricts are doing with respect to all the subdistricts in the state. The program is run for

each activity. It was found that crack sealing and shallow patching are the two highest cost pavement maintenance activities performed on a routine basis. Therefore, attention has been focused on these two activities.

(Abstracts in Appendix 3 and 4 describe papers written on this subject).

The ongoing phase of research deals with determining the factors that influence routine maintenance productivity (but no quality evaluation) for shallow patching. Data that is used in the MIS program is being statistically analyzed to see which factors influence production. (The Abstract in Appendix 5 has information on this topic). The analysis using the dependent variable, unit cost of shallow patching (cost/ton of mix applied), over two month periods from July 1982 through June 1983 for each of 37 subdistricts in Indiana, is given in this paper. The following 16 covariates were measured simultaneously with the dependent variable (over each 2 month period);

- Frequency of application (how often shallow patching is performed in the two month period)
- Average accomplishment per day or tons of mix placed per day for shallow patching
- Average crew size per day (number of people used for the 2 months/day)
- 4. Average hours worked per accomplishment
- 5. fm Bituminous Hot Mix
- 6. pm Bituminous Hot Mix
- 7. fm Bituminous Cold Mix
- 8. pm Bituminous Cold Mix
- 9. fm Salvage Bituminous Mix
- 10. pm Salvage Bituminous Mix

- 11. fm Asphalt and Tars
- 12. pm Asphalt and Tars
- 13. fm Aggregate (coarse)
- 14. pm Aggregate (coarse)
- 15. fm Seal/Cover Aggregate (sand)
- 16. pm Seal/Cover Aggregate (sand)

#### where:

fm = Fraction of time the material was used

pm = Average amount of <u>material</u> used per accomplishment or <u>production</u> unit

The following steps are used to analyze the data in the next section of this paper:

- 1. A two way analysis of variance on Subdistricts and Months using  ${\sf GLM}$ .
- 2. A covariance analysis including the significant effects of the analysis of variance and the sixteen covariates.
- 3. A correlation analysis of the sixteen covariates and the interaction term of subdistricts by months.
- A test on the equality of the regression coefficients of all significant covariates across the subdistricts.
- 5. Determination of the "best" model for these data.
- Test for normality of the residuals from the "best" model.
- 7. Compare distributions of the means of the dependent variable over subdistricts when no adjustment is made using the covariates versus the means of the dependent variable over subdistricts when the "best" model is used.

## II. ANALYSIS OF THE DATA

## 1. Analysis of Variance

This part of the analysis was run to determine whether or not there was an interaction of the subdistricts by months. If there was no interaction nor month's effect present, the months could be considered random and a test for normality of the data using the Shapiro-Wilk W-test (Anderson and McLean, p. 26, 1974) could be used on the six observations for each subdistrict. There would be 37 tests (using only six observations per test) which would give a very good indication of whether the cost variable was normally distributed or not. In addition to the test of normality, a test of homogeneity of the variances across subdistricts could be run using any of the statistical packages.

These two assumptions (normality and homogeneity of variances) are necessary for the analysis of variance and/or regression analysis to be run on data.

Since there was only one observation per cell in this two-way classification (subdistricts and months) of the data, the Tukey one degree of freedom for nonadditivity (Anderson and McLean, p. 45, 1974) procedure was run. The program to use this procedure in the SAS (Statistical Analysis System) package is given in Appendix 6. This program makes the interaction term a covariate or an independent variable, which in turn requires the general linear model (GLM) to be used in SAS for the analysis of variance and Type III Sum of Squares is used for interpretation.

The results of this analysis ar	The	results	of	this	ana	lysis	are
---------------------------------	-----	---------	----	------	-----	-------	-----

Source	<u>df</u>	MS	F
Subdistricts (s)	36	1747.2	6.7**
Months (M)	5	2756.8	10.6**
S*M (X)	1	1205.7	4.6*
Residual	179	259.3	

Since the S\*M and Months are significant, there is no generally accepted way to test for normality nor homogeneity of variances in this case and we must include S M and S\*M (or X) in the model with the 16 covariates.

#### 2. Analysis using Covariates

Let the interaction S\*M be represented by X in the covariate model as follows:

$$Y = S M X X1...X16$$

The analysis of covariance that was run on GLM in SAS gave the following results:

Source	<u>df</u>	MS
M	5	2.09
S	36	3.47
Χ	1	<b>.</b> 67
X1	1	.06
X2	1	.17
Х3	1	8.94
X4*	1	20,582.11*
X5*	1	190.07*
Х6	1	2.18
X7*	1	183.08*
Х8	1	.19
Х9	1	6.12
X10	1	.30
X11	1	2.45
X12*	1	31.71*
X13*	1	114.89*
X14	1	7.96
X15	1	5.99
X16	1	4.19
Residual	160	2.28

<sup>\*</sup> Significant beyond  $\alpha$  = .05 \*\* Significant beyond  $\alpha$  = .01

where \* indicates the most significant covariates that should be considered in the "best" model, that model which appears to be the one to consider for adjusting the subdistrict means.

This analysis indicates that one need not consider Months nor M\*S (which is X) in adjusting the subdistricts' means when X4, X5, X7, X12, and X13 are used for adjustment.

The next step in the overall analysis was to find out whether or not a model with fewer than five covariates could be used effectively. To accomplish this the MAXR (maximum regression) stepwise program in SAS was used. This program allows one to look at <u>nearly</u> all possible regressions efficiently. That is, the computer time is reduced considerably while losing very little infomation on the subsets. In this case the  ${\rm C_p}$  value (Draper and Smith, p. 299, 1981) which compares the residual sum of squares for that model less than the full versus the residual variance calculated from the full model. In all instances the  ${\rm C_p}$  value decreased until the full model (all five covariates) was obtained. In addition, the residual or the error mean square decreased all the way to the full model. Hence these results indicated all five covariates should be used in the adjusting equation if all asssumptions are met.

## Correlation Analysis

The highest correlation with Y is X4 and the next highest is X2, but X4 and X2 are correlated with an r = .76. As a result, X4 went in the model and X2 did not. One other correlation, X5 with X7 is .95; however, both of these variables stayed in the equation even though the

correlations of X5 with Y is only .05 and X7 with Y is only .04. It is true that the correlations of X4 with X5 is only .06 and with X7 is only .04, indicating near orthogonality.

In a similar result, the correlations of X4 with X12 is .01 and with X13 is .06. Looking at all the correlations with Y and the intercorrelations between all pairs of X's, there is no problem of multicollinearity (too high correlations between X's that would make the computing results suspect because the determinant of the correlation matrix would be too close to zero). A reference for multicollinearity is Draper and Smith, p. 258, 1981.

In general, then, the results look satisfactory using the model:

$$Y = S \times X4 \times X5 \times X7 \times X12 \times X13$$

if all assumptions for a covariance analysis are met.

#### 4. Analysis for Homogeneous Slopes

One of the most critical assumptions in a covariance analysis (the X's are used to adjust the subdistricts' means) is the test that the relationship between the X's and Y for all S is the same. Here the linear regression is ased and the following model is appropriate to test each of the five X's:

$$Y = S \quad X \quad S*X$$

If the interaction term, S\*X, is non-significant for a given X, then the slopes or the linear regression coefficients are considered equal for all S.

The following lists the results for X4, X5, X7, X12, and X13:

a. For X4, S\*X4 was most non-significant ( $\alpha$  = .81) and the plots were very parallel, indicating X4 should be used in the equation.

- b. For X5, X7, and X12, the interaction terms were all significant  $(\alpha < .01)$ , hence X5, X7, and X12 should not be used in the equation.
- c. For X13, S\*X13 was not significant ( $\alpha$  = .22), but the plots showed 14 subdistricts with all zeroes for X13 for the 6 months and the other 23 subdistricts plots were very heterogeneous. All of this suggests that the zeroes must have a large effect on showing homogeneity of the slopes. Hence one must conclude that the slopes are <u>not</u> equal even though the statistical test indicated equality, and X13 should not be used.

#### 5. Determination of the "best" model for these data

The upshot of the tests given above that only X4 should be used as a covariate. In this case it is very good because X4 accounts for over 98% of the variation in Y.

The final model to be used for adjustment of the subdistricts' means, then, is

$$Y = S X4,$$

because the assumption of equal slopes across all subdistricts has been met for X4 only.

#### 6. Test for Normality of Residuals

The test of normality of the residuals may be made next for the model (adjusted for X4)

$$Y = S X4.$$

To do this it is necessary to use a regression procedure, where all terms in the model must have a single degree of freedom. To handle this problem, one method is to use indicator or dummy variables (Draper and Smith, p. 71, 1981). The technique is to use the 37

subdistricts as follows (remember X, X1,.	,X16 have been t	usea):
---	------------------	--------

	Dummy Variable (36)			
Subdistrict	X17	X18 .		
1 2 3	0 1 0	0 0 1. 0		

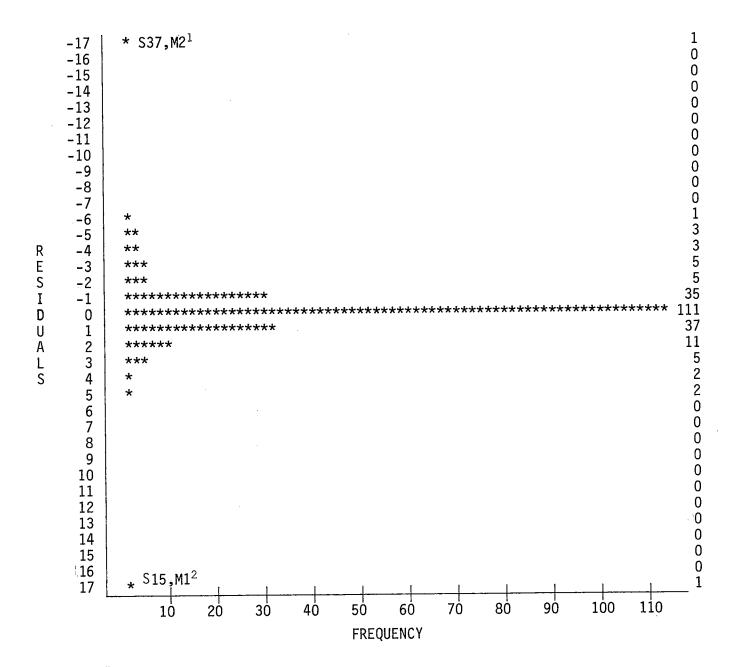
The independent variables X4, X17, X18,...,X52, were put in the regression program with Y, and the residuals were printed out. In addition, SAS tests these residuals for normality using the Kolmogorov-Smirnov test (Ostle and Mensing, p. 352, 1975), for N > 50. In this case N = 222. Since there are 38 parameters to estimate, there are 184 degrees of freedom for the residuals. The Kolomgorov-Smirnov D statistic was .2136. Using Appendix 16 in Ostle and Mensing, 1975, the critical D value for  $\alpha$  = .01 is  $\frac{1.63}{\sqrt{n}}$ , and for  $\alpha$  = .05 is  $\frac{1.36}{\sqrt{n}}$ 

It is suggested that n = 184 (degrees of freedom) replace 222 to obtain a conservative test

$$D_{.05} = \frac{1.36}{\sqrt{184}} \stackrel{\circ}{=} .100$$

$$D_{.01} = \frac{1.63}{\sqrt{184}} = .120$$

Since D observed is .2136, the test for normality is rejected and the residuals were plotted giving the following results:



#### where:

The symmetry is excellent, but 2 of the points are very different. The residual for Subdistrict 15, Month 1 is too high and for Subdistrict 37, Month 2 is too low causing enormous kurtosis. There seems to be something odd about these two points when adjusted for labor (X4), but there was no cause given by the engineers.

The distribution of the residuals for the model (unadjusted)

Y = S

has a Kolmogorov-Smirnov

D = .0682

with an approximate D $_{.05}$  (critical)  $\stackrel{\sim}{=}$  .100. Hence one accepts the hypothesis that the distribution is normal. The plot looks very good.

With all the above information available a comparison of the distribution of the means, unadjusted and adjusted for X4, is given in the following section.

7. Comparison of the Distribution of the Subdistrict Y Means Unadjusted with the Distribution of the Y Means Adjusted for X4

Una	djusted	Means	Adjusted	Means
District	Numb <u>er</u>	Mean	District Number	<u>Mean</u>
most cost	16 2 13 5 4	141.53 130.18 122.51 121.37 118.89	15 5 1 36	99.05 98.84 98.37 97.90
	32	117.27	12	97.79
	17	116.55	1 <u>1</u>	97.61
	33 28 36 30 15 35 22 11 34 14 37 7 20 29 12 21	108.30 106.17 105.17 101.43 100.30 98.83 96.82 95.86 95.28 95.27 94.78 94.40 94.23 94.23 93.03 92.92	7 33 20 21 35 13 6 9 17 14 23 26 4 22	97.81 97.53 97.45 97.30 97.19 97.17 97.07 96.84 96.83 96.65 96.65 96.65 96.57 96.57
	3	92.13	29	96.48
	25	91.56	28	96.46
	19	91.37	27	96.37
	1	91.27	32	96.26
	6	90.13	34	96.04
	10	86.65	19	96.00
	26	85.51	2	95.99
	23	78.85	24	95.89
least cost	<pre> {</pre>	78.11 76.37 74.42 72.67 72.67 61.30	16 31 30 3 3	95.40 95.35 95.23 95.06 92.95

 $\overline{Y} \pm S_{\overline{Y}}: 96.70 \pm 18.22$ 

68% range: 78.48 to 114.92

 $96.70 \pm 1.12$ 

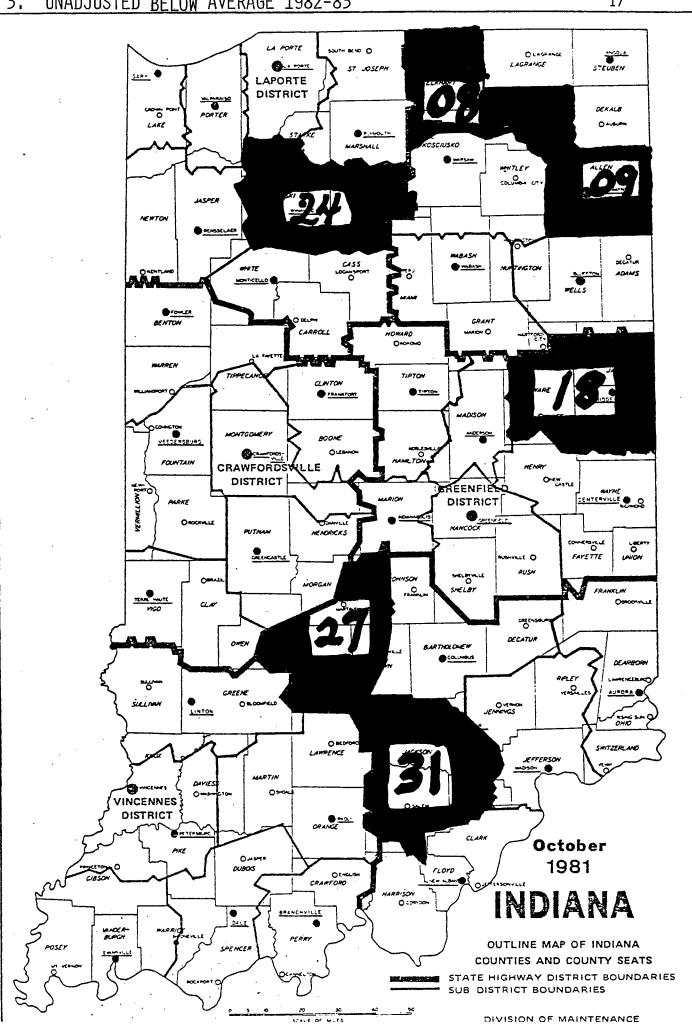
95.58 to 97.82

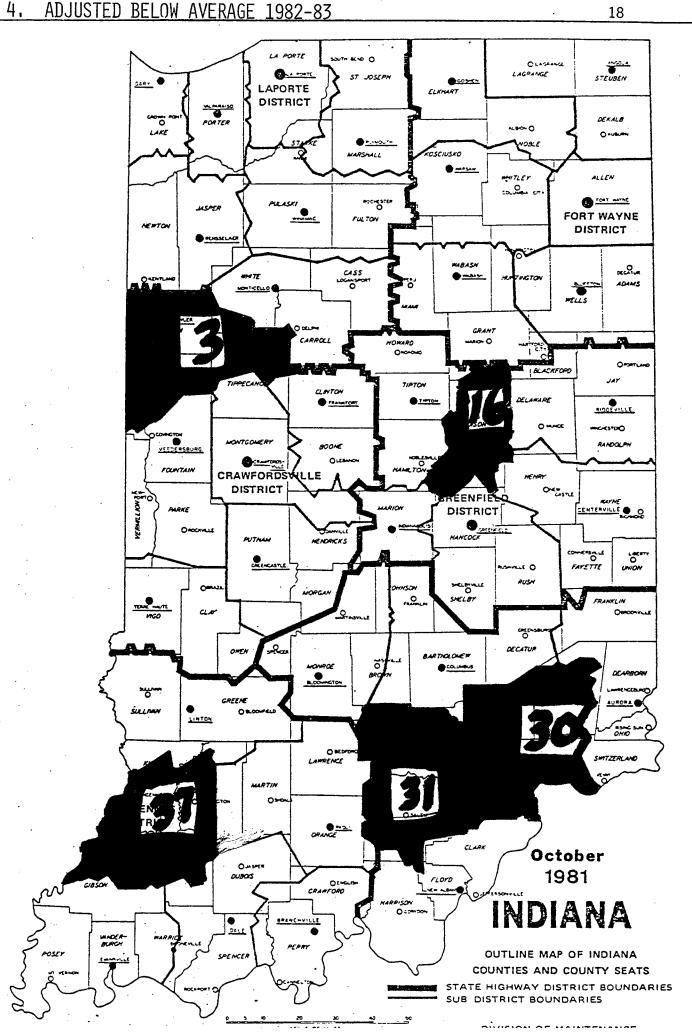
The lines drawn below "most cost" and above "least cost" indicate the  $\overline{Y} \pm S_{\overline{Y}}$ . If the distributions were truly normal one would expect about 6 or 7 subdistricts on each end of the distribution (about 17% of 37). For the unadjusted means (the underlying distribution was accepted as normal) the 6 or 7 was exact, but for the adjusted mean distribution only 4 and 5 subdistricts appear outside the limits. Remember the underlying distribution was very narrow, except for the two points from subdistricts 15 and 37, and most symmetric. In addition, notice that subdistrict 15 has the highest mean and 37 has the lowest (being influenced by the two odd points) however these means are not terribly out of line.

Next, notice how the districts outside the limits on unadjusted do not appear outside the limits on adjusted except for subdistrict 5.

All of this is summarized on the following maps of Indiana with subdistricts indicated.







The four Indiana maps indicate the location of the subdistricts above and below the one standard deviation limits of the mean cost of shallow patching using unadjusted and adjusted values.

- Notice that only subdistrict #5 remains in the <u>above</u> category going from unadjusted to adjusted (1 vs. 2 maps).
- Similarly only subdistrict #31 remains in the below category going from unadjusted to adjusted (3 vs. 4 maps).

This rather violent change of the particular subdistricts in the above or below category when using unadjusted to adjusted is due only to the labor, X4, variable.

It should be understood that this method of separating subdistricts into low and high cost groups is not absolute. If you turn back to page 13 and look at the Unadjusted Means there is no way that subdistrict 24 should be considered less than 23. In the Adjusted Means column, 12 is no better than 36 and 10 is no worse than 16 from these results.

This method of separation into  $\overline{Y}\pm S_{\overline{Y}}$  is merely an indication of what is going on in the field. The engineers certainly may use the results as guidelines but nothing more.

#### III. CONCLUSIONS

The following two general types of conclusions are drawn from this paper:

- 1. Statistical Analysis
- 2. Engineering Meaning of the results

## 1. Statistical Analysis Conclusions

To use the classical parametric analysis methods; analysis of variance, covariance analysis and regression analysis, certain assumptions are necessary. When these assumptions cannot be tested directly, the methods used in this paper are suggested for engineers' consideration. In this case these methods worked fairly well.

The test for the interaction of subdistricts by months using a covariate with one degree of freedom worked exceptionally well for this case in which there was only one observation per cell. It was most informative later to find out that this interaction, even though it was significant when used with subdistricts and months, did not show any effect on Y (cost) when the other 16 covariates were introduced in the model. The statistical concept is that the contribution to explaining Y using X was not large when the effects of other X's, namely X4, X5, X7, X12, and X13, were used in the covariate model.

It was shown that the regression coefficients across months for the subdistricts, when X5, X7, and X12 were used individually versus Y, were not equal. In addition, losing the information from X5, X7, and X12 did not reduce the  $R^2$  substantially, even though it was reduced significantly in the statistical sense.

The strange result of testing for the equality of the regression coefficients using X13 over subdistricts and finding the coefficients equal, but when plotted showed many subdistricts with all X13's = 0 (which we knew) and the remaining being very heterogeneous was a surprise. Anyway, one should not use a variable such as X13 in the final model under these circumstances.

The final adjusted model

Y = S X4

made statistical sense for the comparison of the original rank of the costs across subdistricts versus the ranks after adjustment using only X4 out of all those data.

One may say, using the correlation matrix, that this final model should have been obvious immediately, but if one looks carefully the other X's, X5, X7, X12, and X13 were contributing quite a bit of information on Y but from assumptions on the analysis should not be used if subdistrict means are to be compared. It should be understood that if subdistricts by months are to be compared then the adjustment using X5, X7, X12 and X13 should definitely be considered.

#### 2. Engineering Meaning of the Results

From an engineering standpoint, the results of the statistical analysis coincides with the expectation of the engineer.

Shallow patching costs consists of labor costs and material costs. Labor costs are approximately 60-75% greater than material cost per ton of mix placed. Labor cost is a variable cost depending on work practices, weather, road conditions, and material workability, yet material costs are fixed and depend only on the amount purchased.

In short, then, an increase in production would be expected to be achieved by focusing attention to labor practices used in shallow patching.

#### IV. REFERENCES

- Anderson, V. L., and McLean, R. A., "Design of Experiments: A Realistic Approach", Marcel Dekker, 1974.
- 2. Draper, Norman and Smith, Harry, "Applied Regression Analysis", Second Edition, Wiley, 1981.
- 3. Ostle, Bernard and Mensing, R. W., "Statistics in Research", Third Edition, Iowa State University, 1975.

Sharaf, E. A., Sinha, K. C., Whitmire, C. and Yoder, E. J.,
"Field Investigation of Resource Requirements for State Highway
Routine Maintenance Activities". Paper presented at Annual
Meeting of the Transportation Research Board in January 1983.

#### Abstract

This paper discribes the first phase of a comprehensive study to identify potential areas of cost and energy savings in the operations of routine maintenance activities on the state highway system of Indiana.

In this phase, the current highway routine maintenance standards of the Indiana Department of Highways (IDOH) were reviewed and updated based on data collected directly from the field. Furthermore, guidelines for estimating fuel consumption of equipment were established.

The needs for different resource elements (materials, labor, and equipment) for various routine maintenance activities in terms of types, rates of consumption, and frequencies of use were identified. Energy consumed in each activity was determined as the number of fuel gallons required to produce one production unit of an activity.

The preliminary data analysis indicated that there is a potential for considerable cost and energy savings could be achieved through better assignment of equipment in different activities.

Sanderson, V. A. and Sinha, K. C., "A Management Information System to Monitor Routine Maintenance Productivity," September 1983.

#### Abstract

This paper presents an analysis of pavement routine maintenance costs for the state highway system in Indiana. The data base includes pavement maintenance records for four years, 1980-83. The results include a trend analysis as well as a correlation analysis. First, the total cost trends as well as individual activity costs are examined. Then the resource consumption trends are analyzed in terms of labor and materials use. Finally, a statistical correlation analysis is presented in order to examine the relationship of maintenance expenditures in the earlier years on the level of maintenance expenditures of subsequent years. The analysis showed high correlation between maintenance expenditures in one year and the expenditures of the previous two or three years. The results on this study should be of direct use in the planning and management of annual pavement maintenance programs.

Sharaf, E. A. and Sinha, K. C., "An Analysis of Pavement Routine Maintenance Activities in Indiana," Presented at Annual Meeting of TRB, Washington, D.C., January, 1984.

#### Abstract

This paper discussed some of the results of an ongoing research project sponsored jointly by the Federal Highway Administration and Indiana Department of Highways in the area of highway routine maintenance productivity improvement.

An effective management approach is essential in planning and controlling highway activities, particularly in an era of limited resources. The experience gained in both pavement management and maintenance management systems indicate that efficient maintenance operations depend to a great degree on management factors, particularly the ability of the first- and second-line supervisors responsible for the planning and the application of proper mix of personnel, equipment, materials and procedures to routine task and emergency requirements in the maintenance program.

The wide use of computers has greatly contributed to the development of maintenance management systems. A large amount of information can now be processed to distill the necessary information for the highway managers. However, most of the available procedures for pursuing maintenance

management reports are generally too detailed and complex for easy and ready use of the managers. A need exists to identify the essential level of information required by routine maintenance managers and to develop procedures that can produce the necessary information in forms useful in improved scheduling and controlling routine maintenance activities.

The paper discusses the types of measures most suitable to reflect maintenance productivity and presents a procedure to produce straight-forward reports of maintenance unit productivity levels. The generated information is then examined to identify maintenance units with low productivity, and compare units on a statewide basis. Higher levels of management will be able to relay this information to the individual unit, indicating that unit's production level and how it compares with other units and the statewide average. Providing maintenance unit personnel with a guideline to evaluate their operations, in the form of a check list of factors found to contribute to low productivity, will help them to identify areas for improvement, resulting in cost savings as well as improving the quality of maintenance operations.

Sharaf E. A., Sinha, K. C., and Anderson, V. L., "Pavement Routine Maintenance Cost Prediction Models," August 1984, to be presented at 1985 TRB Annual Meeting in Washington, D. C.

#### Abstract

In this paper a methodology is presented for using the available data on pavement routine maintenance from the Indiana Department of Highways (IDOH) to develop models relating the cost of pavement routine maintenance to pavement system characteristics. The results showed that total pavement routine maintenance costs are affected by traffic level and by climatic zone (weather effect). Furthermore, the analysis of costs of individual activities showed that the extent of patching work (amount of potholes repair taking place after winter) is negatively correlated to the amount of sealing activities taking place before winter. The implication of this result is that by increasing sealing activity, not only a higher level of service may be achieved (less potholes), but also a considerable savings in overall pavement routine maintenance expenditure could be accomplished.

O'Brien, S., Sinha, K. C., and Anderson, V. L., "A Procedure to Identify Factors Influencing Pavement Routine Maintenance Productivity," October, 1984, to be presented at the International Pavement Management Conference, Ontario, Canada, March 1985.

#### Abstract

This paper will present a procedure that is being developed to identify the factor which may influence the routine maintenance productivity at the subdistrict level of the Indiana Department of Highways. The procedure follows the four steps:

- An analysis of crew day cards for three consecutive fiscal years to determine if the grouping of subdistricts of equal productivity is random or there seems to be a pattern that occurs over the three years.
- 2. Field investigations of subdistricts within each of the production groups considered. This will allow observation of work patterns, work habits, equipment usage, and personal interviews with the people who actually perform the routine maintenance tasks. Some judgement can be made as to the quality of work performed although no physical measurement of quality can be ascertained.

- 3. Statistical analysis of the data used to develop the production groups. This includes testing for homogeneity of the variances and normality of the raw data, investigating the correlation of the input factors, running one-way and two-way ANOVA', using covariance in the determination of variables that significantly affect the production unit, and showing how the dependent variables change when the influence of the significant independent variables are removed. The analysis will be done both by year and for each bimonthly grouping.
- 4. A survey of subdistrict personnel. A questionnaire sent to each subdistrict includes information that is not found on the crew day cards. The questionnaire is to be filled out by the superintendent, general foreman, and a unit foreman collectively. Answers are being compared to see if any trends in the subdistricts are present.

The combination of the results from the four steps is expected to provide the reasons for the differences in subdistrict routine maintenance productivity levels currently present within the State of Indiana. Once the reasons are established, measures can then be taken to improve productivity levels thereby reducing routine maintenance costs.

```
SAS Program for IDF for nonadditivity (Tukey)
     DATA A1;
     INPUT A 1 B 2 Y 3-5;
     K = 1;
                                        В
     CARDS;
                                             1 observ./cell
          Data from a 2 way class A
     PROC SORT DATA = A1; BY A;
     PROC MEANS DATA = A1 NOPRINT; BY A;
     VAR Y;
     OUTPUT OUT = A2 MEAN = MYA;
     DATA A3; MERGE A1 A2; BY A;
     PROC SORT DATA = A3; BY K B;
     PROC MEANS DATA = A3 NOPRINT; BY K B;
     VAR Y;
     OUTPUT OUT = A4 MEAN = MYB;
     DATA A5; MERGE A3 A4; BY K B;
     PROC MEANS DATA = A5 NOPRINT: By K;
     VAR Y;
     OUTPUT OUT = A6 MEAN = MY;
     DATA A7; MERGE A5 A6; BY K;
     X = (MYA-MY)*(MYB-MY);
     PROC PRINT DATA = A7;
     PROC GLM DATA = A7;
     CLASSES A B;
     MODEL Y = A B X;
```