Γ-OPTIMAL DECISION PROCEDURES FOR SELECTING THE BEST POPULATION IN RANDOMIZED COMPLETE BLOCK DESIGN

by

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ABSTRACT

In randomized complete block design, we face the problem of selecting the best population. If some partial information about the unknown parameters is available, then we wish to determine the optimal decision rule to select the best population.

In this paper, in the class of natural selection rules, we employ the Γ-optimal criterion to determine optimal decision rules that will minimize the maximum expected risk over the class of some partial information. Furthermore, the traditional hypothesis testing is briefly discussed from the view point of ranking and selection.

1. INTRODUCTION

In randomized complete block design (R.C.B.D) with one observation per cell, we can express the observable random variable $X_{i\ell}$ ($i = 1,\ldots,k$, $\ell = 1,\ldots,n$) as

$$X_{i\ell} = \mu + \tau_i + \beta_\ell + \epsilon_{i\ell}, \quad \sum_{i=1}^{k} \tau_i = 0. \tag{1.1}$$

where μ is the overall mean, τ_i is the i-th treatment effect, β_ℓ is the ℓ-th block effect, and $\epsilon_{i\ell}$ is the error component of (i,ℓ) cell. We assume that the errors within each block are jointly normally distributed. We also assume that the quality of a treatment is judged by the largeness of τ_i's values. The i-th population is called the best if $\tau_i = \max_{1 \le \ell \le k} \tau_\ell$. In many practical situations, the goal of the experimenter is to select the best population.

In this paper, we shall use Γ-optimal criterion to determine the sample size of a natural selection procedure so that it will minimize the maximum expected risk over the class of some partial information [cf. Gupta and Huang (1976)].

In Section 2 some basic definitions and notations are introduced and basic formulation of the problem is also given. In Section 3, some useful expressions for the probability of correct selection (PCS) is derived, and Γ-optimal sample size is determined. Section 4 deals with a numerical example for illustrative purpose. In Section 5, we discuss the relationship between Δ and n^*. Section 6 includes some conclusions and a discussion of the traditional hypothesis testing from the view point of ranking and selection. For general reference of multiple decision procedures, see Gupta and Panchapakesan (1979) and Gupta and Huang (1981).

2. BASIC FORMULATION OF THE SELECTION PROBLEM

In R.C.B.D., as (1.1), we assume that $\underset{\sim}{\epsilon}_\ell = (\epsilon_{1\ell},\ldots,\epsilon_{k\ell})'$: error components within ℓ-th block have jointly a multivariate normal distribution with mean vector $\underset{\sim}{0} = (0,\ldots,0)'$ and covariance matrix $\Sigma = \sigma^2 \begin{pmatrix} 1 & \cdots & \lambda \\ \vdots & & \vdots \\ \lambda & \cdots & 1 \end{pmatrix}_k$, where σ^2 is unknown and λ is a known constant. Thus, $(\underset{\sim}{X}_\ell = X_{1\ell},\ldots,X_{k\ell})'$ have joint multivariate normal distribution with mean vector $\underset{\sim}{\theta}_\ell = (\theta_{1\ell},\ldots,\theta_{k\ell})'$ and covariance matrix Σ, where $\theta_{i\ell} = (\mu + \tau_i + \beta_\ell)$ for all i, $1 \le i \le k$, and any ℓ, $1 \le \ell \le n$. For all i, $1 \le i \le k$, define $\bar{X}_i = (\sum_{\ell=1}^{n} X_{i\ell}/n)$. Then $\underset{\sim}{Y}_i = (\bar{X}_i-\bar{X}_1,\ldots,\bar{X}_i-\bar{X}_k)'$ are jointly sufficient for $(\tau_i - \tau_1,\ldots,\tau_i - \tau_k)'$. Now, if $\tau_i = \max_{\ell\neq i} \tau_\ell$ then $\tau_i - \tau_\ell \ge 0$ for all $1 \le \ell \le k$. We consider a class of natural selection rules for i-th population, $1 \le i \le k$, as:

$$\delta^{(i)}(\underset{\sim}{x}_n) = \begin{cases} 1 \\ 0 \end{cases} \text{if } \bar{X}_i \begin{matrix} \ge \\ < \end{matrix} \max_{\ell\neq i} \bar{X}_\ell \tag{2.1}$$

where $\underset{\sim}{x}_n = (\bar{X}_1,\bar{X}_2,\ldots,\bar{X}_k)'$. Some optimal properties have been studied by several authors (see Gupta and Panchapakesan (1979)). So the class of natural selection rules can be denoted by:

$$D = \{\underset{\sim}{\delta}(\underset{\sim}{x}_n) \mid \underset{\sim}{\delta}(\underset{\sim}{x}_n) = (\delta^{(1)}(\underset{\sim}{x}_n),\ldots,\delta^{(k)}(\underset{\sim}{x}_n))'\}. \tag{2.2}$$

The parameter space Ω is as follows:

$$\Omega = \{\underset{\sim}{\tau} = (\tau_1,\ldots,\tau_k)' \mid \tau_i \in R \text{ for all } i = 1,\ldots,k\}. \tag{2.3}$$

Let Δ be a given positive constant, and for all i, $1 \le i \le k$,

$$\Omega_i = \{\underset{\sim}{\tau} = (\tau_1,\ldots,\tau_k)' \mid \tau_i \ge \tau_\ell + \Delta\sigma \text{ for all } \ell \neq i\}, \tag{2.4}$$

$$\Omega_0 = \{\underset{\sim}{\tau} \mid \tau_1 = \ldots = \tau_k\}, \tag{2.5}$$

and $\Omega_{k+1} = \Omega - (\bigcup_{i=0}^{k} \Omega_i)$.

Let $L^{(i)}(\underset{\sim}{\tau};\ \delta^{(j)}(\underset{\sim}{x}_n))$ represent the loss function for $\underset{\sim}{\tau} \in \Omega_i$, $0 \le i \le k+1$, when the j-th population, $1 \le j \le k$, is selected. Let for $1 \le j \le k$,

$$L^{(i)}(\underset{\sim}{\tau};\ \delta^{(j)}(\underset{\sim}{x}_n)) = \begin{cases} (c_0 n)\delta^{(j)}(\underset{\sim}{x}_n) \\ \ell(\frac{\tau_i-\tau_j}{\sigma})\delta^{(j)}(\underset{\sim}{x}_n) \\ 0 \end{cases} \text{for } \underset{\sim}{\tau} \in \begin{cases} \Omega_0 \ (i = 0) \\ \Omega_i (1 \le i \le k) \\ \Omega_{k+1} (i=k+1) \end{cases} \tag{2.6}$$

where ℓ is some positive increasing function such that $\ell(0) = 0$ and $\ell(x) = o(e^{cx^2})$, $c > 0$, and c_0 represents the sampling cost from each population ($c_0 > 0$). So, for all $\underset{\sim}{\tau} \in \Omega_i$, $0 \leq i \leq k+1$, the loss function of $\underset{\sim}{\delta}(\underset{\sim}{x}_n)$ is defined as:

$$L^{(i)}(\underset{\sim}{\tau};\ \underset{\sim}{\delta}(\underset{\sim}{x}_n)) = \sum_{j=1}^{k} L^{(i)}(\underset{\sim}{\tau};\ \delta^{(j)}(\underset{\sim}{x}_n)). \tag{2.7}$$

Similarly, we have

$$R^{(i)}(\underset{\sim}{\tau};\ \underset{\sim}{\delta}_n) = E\{L^{(i)}(\underset{\sim}{\tau};\ \underset{\sim}{\delta}(\underset{\sim}{x}_n))\} \tag{2.8}$$

and for some ρ (prior distribution) over Ω, $\gamma^{(i)}(\rho;\ \underset{\sim}{\delta}_n)$ is defined as:

$$\gamma^{(i)}(\rho;\ \underset{\sim}{\delta}_n) = E\{R^{(i)}(\underset{\sim}{\tau};\ \underset{\sim}{\delta}_n)\}. \tag{2.9}$$

Thus, the Bayes risk of $\underset{\sim}{\delta}_n$ w.r.t. ρ is defined as

$$\gamma(\rho;\ \underset{\sim}{\delta}_n) = \sum_{i=0}^{k+1} \gamma^{(i)}(\rho;\ \underset{\sim}{\delta}_n). \tag{2.10}$$

In this selection problem, it is assumed that some partial information is available. So that we can specify $\pi_i = P_r(\underset{\sim}{\tau} \in \Omega_i)$, for all i, $0 \leq i \leq k+1$ and define

$$\Gamma = \{\rho \mid \int_{\Omega_i} d\rho(\underset{\sim}{\tau}) = \pi_i,\ \sum_{i=0}^{k+1} \pi_i = 1, 0 \leq i \leq k+1\}. \tag{2.11}$$

If there is no prior information, we can assume that $\pi_0 = \ldots = \pi_k = (1-\pi_{k+1})/(k+1)$.

Now if there exists n^* such that

$$\sup_{\rho \in \Gamma} \gamma(\rho;\ \underset{\sim}{\delta}_{n^*}) = \inf_{\underset{\sim}{\delta}_n \in D}\{\sup_{\rho \in \Gamma} \gamma(\rho;\ \underset{\sim}{\delta}_n)\}. \tag{2.12}$$

Then $\underset{\sim}{\delta}_{n^*}$ is called a Γ-optimal decision rule and n^* is the Γ-optimal decision. In the following discussion, we will determine $\underset{\sim}{\delta}_{n^*}$ for this selection problem.

3. MAIN RESULTS

We can easily show the following lemma

Lemma 3.1. Suppose for any ℓ, $1 \le \ell \le n$, $\underset{\sim}{X}_\ell = (X_{1\ell},\ldots,X_{k\ell})'$ follows a multivariate normal distribution with mean vector $\underset{\sim}{\theta}_\ell = (\mu + \tau_1 + \beta_\ell,\ldots, \mu + \tau_k + \beta_\ell)'$ and covariance matrix

$$\Sigma = \sigma^2\begin{pmatrix} 1 & \cdots & \lambda \\ \vdots & & \vdots \\ \lambda & \cdots & 1 \end{pmatrix}_k .$$

Let $\underset{\sim}{Y}_i = (\bar{X}_i-\bar{X}_1,\ldots,\bar{X}_i-\bar{X}_k)'$ and $p_i(\underset{\sim}{\tau}) = P_r(\bar{X}_i \ge \max_{\ell\ne i} \bar{X}_\ell)$ where

$\bar{X}_i = (\sum_{\ell=1}^{n} X_{i\ell}/n)$ for any i, (i = 1,...,k).

Then

a) $\underset{\sim}{Y}_i$ follows multivariate normal distribution with mean vector $(\tau_i-\tau_1,\ldots,\tau_i-\tau_k)'$ and covariance

$$\text{matrix } \frac{2\sigma^2(1-\lambda)}{n}\begin{pmatrix} 1 & \cdots & \frac{1}{2} \\ \vdots & & \vdots \\ \frac{1}{2} & \cdots & 1 \end{pmatrix}_{(k-1)} \qquad (3.1)$$

b) $$p_i(\underset{\sim}{\tau}) = \Phi_{k-1}\left(\frac{(\tau_i-\tau_1)/\sigma}{\sqrt{2(1-\lambda)/n}},\ldots,\frac{(\tau_i-\tau_k)/\sigma}{\sqrt{2(1-\lambda)/n}}\right), \qquad (3.2)$$

and

c) $$p_i(\underset{\sim}{\tau}) = \int_{-\infty}^{\infty} \prod_{\ell\ne i}^{k} \Phi\left(z + \frac{(\tau_i-\tau_\ell)/\sigma}{\sqrt{(1-\lambda)/n}}\right) d\Phi(z), \qquad (3.3)$$

where $\Phi_{k-1}(\cdot)$ denotes the c.d.f. of (k-1)-variate normal distribution with mean vector $\underset{\sim}{0} = (0,\ldots,0)'$ and covariance matrix

$\Lambda = \begin{pmatrix} 1 & \cdots & \frac{1}{2} \\ \vdots & & \vdots \\ \frac{1}{2} & \cdots & 1 \end{pmatrix}_{(k-1)}$ and $\Phi(\cdot)$ denotes the c.d.f. of standard normal distribution.

Theorem 3.1. In R.C.B.D., for fixed Δ and λ, let

$$Q_M(n) = \sup_{g\geq\Delta}\{\ell(g)\cdot(1-\Phi_{k-1}(\frac{g}{\sqrt{2(1-\lambda)/n}},\dots,\frac{g}{2(1-\lambda)/n}))\} \quad (3.4)$$

and $H(n) = \sup_{\rho\in\Gamma}\gamma(\rho;\underset{\sim}{\delta}_n)$. Then there exists n^* such that

$$H(n^*) = \inf_{n\geq 1} H(n).$$

where $n^* = \begin{cases} \langle n_0^*\rangle & \text{if } H(\langle n_0^*\rangle) \leq H([n_0^*]) \\ [n_0^*] & \text{if } H(\langle n_0^*\rangle) > H([n_0^*]) \end{cases}$ and n_0^* is a positive real number assumed to exist satisfying the following equations.

$$\begin{cases} \text{a)} & Q_M'(n_0^*) = -(c_0\pi_0)/(1-\pi_0-\pi_{k+1}) \\ \text{b)} & Q_M''(n_0^*) > 0 \end{cases} \quad (3.5)$$

$\langle x\rangle$ ($[x]$) denotes the smallest (largest) integer which is larger (less) than or equal to x.

Proof. For any $\underset{\sim}{\delta}_n \in D$, we have

$$\gamma(\rho;\underset{\sim}{\delta}_n) = \sum_{i=0}^{k+1}\int_{\Omega_i} R^{(i)}(\underset{\sim}{\tau};\underset{\sim}{\delta}_n)d\rho(\underset{\sim}{\tau})$$

$$= (c_0 n)\sum_{j=1}^{k}\int_{\Omega_0} p_j(\underset{\sim}{\tau})d\rho(\underset{\sim}{\tau}) + \sum_{i=1}^{k}\int_{\Omega_i}\sum_{j=1}^{k}\ell(\frac{\tau_i-\tau_j}{\sigma}) p_j(\underset{\sim}{\tau})d\rho(\underset{\sim}{\tau}).$$

Since

1) $\sup_{\underset{\sim}{\tau}\in\Omega_0} p_j(\underset{\sim}{\tau}) = \int_{-\infty}^{\infty}\Phi^{k-1}(z)d\Phi(z) = \frac{1}{k}.$

2) By Somerville's paper (1954) for $1 \leq i \leq k$, we have

$$\sup_{\underset{\sim}{\tau}\in\Omega_i}\{\sum_{j=1}^{k}\ell(\frac{\tau_i-\tau_j}{\sigma})p_j(\underset{\sim}{\tau})\} = \sup_{g_i\geq\Delta}\{\sum_{j\neq i}^{k}\ell(g_i)p_j\}$$

$$= \sup_{g_i\geq\Delta}\{\ell(g_i)(1-\Phi_{k-1}(\frac{g_i}{\sqrt{2(1-\lambda)/n}},\dots,\frac{g_i}{\sqrt{2(1-\lambda)/n}}))\} = Q_M(n).$$

Thus, $H(n) = \sup_{\rho \in \Gamma} \gamma(\rho; \underset{\sim}{\delta}_n) = n \cdot (c_0 \pi_0) + Q_M(n)(1-\pi_0-\pi_{k+1})$.

Since there exists n_0^* such that

$$\begin{cases} \text{a)} & Q_M'(n_0^*) = -(c_0\pi_0)/(1-\pi_0-\pi_{k+1}) \\ \text{b)} & Q_M''(n_0^*) > 0, \end{cases}$$

thus, we have

$$H'(n_0^*) = 0 \text{ and } H''(n_0^*) > 0.$$

$$\text{So} \quad n^* = \begin{cases} \langle n_0^* \rangle \text{ if } H(\langle n_0^* \rangle) \le H([n_0^*]) \\ [n_0^*] \text{ if } H(\langle n_0^* \rangle) > H([n_0^*]). \end{cases}$$

<u>Lemma 3.2</u>. (Slepian (1962)) Let $(X_1,\ldots,X_n)$ be multivariate normal with zero mean and positive definite covariance matrix $\Sigma_1 = \{\rho_{ij}\}$ and $(Y_1,\ldots,Y_n)$ be multivariate normal with zero mean and positive definite covariance matrix $\Sigma_2 = \{\kappa_{ij}\}$. Let $\rho_{ij} \ge \kappa_{ij}$ for $i, j = 1,\ldots,n$ and $\rho_{ii} = \kappa_{ii}$, $i = 1,\ldots,n$. Then

$$P_r(X_1 \ge a_1,\ldots,X_n \ge a_n) \ge P_r(Y_1 \ge a_1,\ldots,Y_n \ge a_n).$$

We need a numerical solution for n to satisfy the infimum of H(n).

<u>Theorem 3.2</u>. Let $\ell(\cdot)$ be a positive increasing function such that $\ell(x) = o(e^{cx^2})$, $(c > 0)$,

$$Q_M(n) = \sup_{g \ge \Delta}\{\ell(g)(1-\Phi_{k-1}(\frac{g}{\sqrt{\frac{2(1-\lambda)}{n}}},\ldots,\frac{g}{\sqrt{\frac{2(1-\lambda)}{n}}}))\}$$

and $H(n) = nc_0\pi_0 + (1-\pi_0-\pi_{k+1})Q_M(n)$. Let

$$n_0 = \langle (k-1)^2(\frac{1-\lambda}{\pi})(\frac{1-\pi_0-\pi_{k+1}}{c_0\pi_0})^2 \frac{(\ell(g_*))^2}{g_*^2\, e^{g_*^2/2(1-\lambda)}} \rangle. \tag{3.6}$$

Then

$$\inf_{n \ge 1} H(n) = \inf_{n \le n_0} H(n)$$

where g_* is such that $\frac{\ell(g_*)}{g_*} e^{-\frac{g_*^2}{4(1-\lambda)}} = \sup_{g \geq \Delta} \left\{ \frac{\ell(g)}{g} e^{-\frac{g^2}{4(1-\lambda)}} \right\}$.

Proof. By Lemma 3.2, we have

$$1-\Phi_{k-1}(t,\ldots,t) \leq 1-\Phi^{k-1}(t) \leq (k-1)(1-\Phi(t))$$

and $1-\Phi(t) \leq \frac{1}{t} \frac{e^{-t^2/2}}{\sqrt{2\pi}}$, for $t > 0$. Thus

$$Q_M(n) \leq \sup_{g \geq \Delta} \left\{ \ell(g)(k-1)\left(1-\Phi\left(\frac{g}{\sqrt{\frac{2(1-\lambda)}{n}}}\right)\right) \right\} \tag{3.7}$$

$$\leq (k-1) \frac{\sqrt{(1-\lambda)/\pi}}{\sqrt{n}} \sup_{g \geq \Delta} \left\{ \frac{\ell(g)}{g} e^{-\frac{g^2}{4(1-\lambda)}} \right\}.$$

Since $\ell(g)$ is a positive increasing function such that $\ell(g) = o(e^{cg^2})$ $(c > 0)$, then there exists g_* such that

$$\sup_{g \geq \Delta} \left\{ \frac{\ell(g)}{g} e^{-\frac{g^2}{4(1-\lambda)}} \right\} = \frac{\ell(g_*)}{g_*} e^{-g_*^2/4(1-\lambda)}. \tag{3.8}$$

By (3.7) and (3.8), as $n \to \infty$, $Q_M(n)$ decreases to 0. We can find

$$n_0 = \left< \left\{ (k-1)^2 \left(\frac{1-\lambda}{\pi}\right) \left(\frac{1-\pi_0-\pi_{k+1}}{c_0\pi_0}\right)^2 \frac{(\ell(g_*))^2}{g_*^2 e^{g_*^2/2(1-\lambda)}} \right\} \right>$$

to satisfy

$$Q_M(n_0) \leq \frac{c_0\pi_0}{1-\pi_0-\pi_{k+1}}.$$

Now for any $n \geq n_0$, $Q_M(n)-Q_M(n+1) \leq Q_M(n_0) \leq \frac{c_0\pi_0}{(1-\pi_0-\pi_{k+1})}$ and $H(n+1)-H(n) = c_0\pi_0-(1-\pi_0-\pi_{k+1})(Q_M(n)-Q_M(n+1)) \geq 0$. In other words, $H(n)$ is an increasing function of n. Thus,

$$\inf_{n \geq 1} H(n) = \inf_{n \leq n_0} H(n).$$

Under a finite domain of n, we can solve for the infimum of n^* numerically by the following algorithm.

1. Determine n_0 such that (3.6) holds.

2. Determine a non-empty set $\mathbb{C}$, where

$$\mathbb{C} = \left\{ n' \middle| \begin{array}{l} Q_M(n') \leq Q_M(n'-1) - c_0\pi_0/(1-\pi_0-\pi_{k+1}) \\ \\ Q_M(n') \leq Q_M(n'+1) + c_0\pi_0/(1-\pi_0-\pi_{k+1}) \end{array}, \; n' \leq n_0 \right\}.$$

3. If $\mathbb{C}$ is a singleton consisting of n', then $n^* = n'$; if $\mathbb{C}$ has a cardinality ≥ 2, then choose n^* such that

$$H(n^*) = \inf_{n \in \mathbb{C}} H(n).$$

An example is considered in Section 4.

4. Numerical example for the existence of n^*

We consider a special case of the loss function, namely, $\ell(g) = c'g^{\alpha}$, $c' > 0$, $g \geq \Delta$, $\alpha \geq 1$, then $\frac{\ell(g)}{g} e^{-g^2/4(1-\lambda)} = c'g^{\alpha-1}e^{-g^2/4(1-\lambda)}$ has the maximum point at $g_{\star} = \max(\sqrt{2(1-\lambda)(\alpha-1)};\Delta)$. Thus n_0 can be expressed as

$$n_0 = \langle (k-1)^2 \left(\frac{1-\lambda}{\pi}\right)\left(\frac{c'}{c_0} \cdot \frac{1-\pi_0-\pi_{k+1}}{\pi_0}\right)^2 g_{\star}^{2(\alpha-1)} e^{-g_{\star}^2/2(1-\lambda)} \rangle.$$

Let

$$M_{k-1}^{\alpha}(x) = \sup_{t \geq x}\{t^{\alpha}(1-\Phi_{k-1}(t,\ldots,t))\}, \text{ then}$$

$$Q_M(n) = c'\left(\sqrt{\frac{2(1-\lambda)}{n}}\right)^{\alpha} M_{k-1}^{\alpha}\left(\frac{\Delta}{\sqrt{\frac{2(1-\lambda)}{n}}}\right).$$

Now, $\mathbb{C}$ is a set of all $n' \leq n_0$ such that

$$\begin{cases} (1)\ M_{k-1}^{\alpha}\left(\Delta\sqrt{\frac{n'}{2(1-\lambda)}}\right)\Big/(n')^{\alpha/2} - M_{k-1}^{\alpha}\left(\Delta\sqrt{\frac{n'+1}{2(1-\lambda)}}\right)\Big/(n'+1)^{\alpha/2} \leq \dfrac{c_0\pi_0/1-\pi_0-\pi_{k+1}}{c'(2(1-\lambda))^{\alpha/2}} \\ (2)\ M_{k-1}^{\alpha}\left(\Delta\sqrt{\frac{n'}{2(1-\lambda)}}\right)\Big/(n')^{\alpha/2} - M_{k-1}^{\alpha}\left(\Delta\sqrt{\frac{n'-1}{2(1-\lambda)}}\right)\Big/(n'-1)^{\alpha/2} \leq -\dfrac{c_0\pi_0/1-\pi_0-\pi_{k+1}}{c'(2(1-\lambda))^{\alpha/2}}. \end{cases}$$

By using the table 3.1 of Somerville's paper (1954), we can compute $H(n^*) = \inf_{n\in\mathbb{C}} H(n)$ directly. Some Γ-optimal sample sizes are given in Tables I and II for $\lambda = 0.0, 0.5$, $\pi_0 = 0.05, 0.10, 0.15$, $\alpha = 1.0, 2.0$, $\Delta \leq 0.05$, $\pi_{k+1} \doteqdot 0.0$ and $c'/c_0 = 15, 30, 45, 60$.

5. Sensitivity analysis between Δ and n^*

In this section, we discuss some relationships between Δ and n^*. Since Δ and n^* depend on λ, α, k, c'/c_0, π_0, π_{k+1}, we fix $\lambda = 0.5$, $\alpha = 1.0$, $k = 4$ and $\pi_{k+1} = 0$. Let c'/c_0 change from 15 to 30 and π_0 change from 0.10 to 0.15. With different values of c'/c_0 and π_0, we get a clear idea of the relationship between Δ and n^*. The results are shown in Table III and Fig. 1. We observe that the relation in Fig. 1.b is more stable than in Fig. 1.a and the relation in Fig. 1.d is more stable than in Fig. 1.c. Thus for fixed c'/c_0, the larger π_0 corresponds to more stable relationship between Δ and n^*. Similarly, Fig. 1.a is more stable than Fig. 1.c and Fig. 1.b is more stable than Fig. 1.d; this means that for fixed π_0, the smaller c'/c_0 corresponds to more stable relationship between Δ and n^*.

6. Discussion

In the special case of k = 2, we have

$\Omega = \{\underset{\sim}{\tau} = (\tau_1; \tau_2)' | \tau_i \in R, i = 1,2\}$,

$\Omega_0 = \{\underset{\sim}{\tau} | \tau_1 = \tau_2\}$,

$\Omega_1 = \{\underset{\sim}{\tau} | \tau_1 \geq \tau_2 + \Delta\sigma\}$,

$\Omega_2 = \{\underset{\sim}{\tau} | \tau_2 \geq \tau_1 + \Delta\sigma\}$,

and $\Omega_3 = \Omega - (\bigcup_{i=0}^{2} \Omega_i)$.

If we do not know any prior information about the parameters we can take $P_r(\underset{\sim}{\tau} \in \Omega_0) = P_r(\underset{\sim}{\tau} \in \Omega_1) = \frac{1}{2}$. Then this is reduced to the traditional problem of testing

(*) $H_0: \tau_1 = \tau_2$ vs $H_1: \tau_1 \geq \tau_2 + \Delta\sigma$.

It should be pointed out that both the type I and type II errors are controlled simultaneously.

TABLE III.

Relationship between Δ and n*

c'/c_0	π_0	n* \ Δ	≤ 0.25	0.275	0.30	0.35	0.40	0.45	0.50
15	0.10	n_1^*	8	8	9	11	12	12	11
15	0.15	n_2^*	6	6	8	8	9	8	8
30	0.10	n_3^*	13	15	17	19	18	17	16
30	0.15	n_4^*	9	9	12	13	13	12	12

TABLE I ($\alpha = 1.0$)

Γ-optimal Sample Size for R.C.B.D. Problem

k	λ / π_0 / c'/c_0	($\lambda = 0.0$)			($\lambda = 0.5$)		
		0.05	0.10	0.15	0.05	0.10	0.15
2	15	11	7	5	9	6	4
	30	17	11	8	14	9	6
	45	22	14	10	18	11	8
	60	27	17	12	22	13	10
3	15	15	9	7	12	7	6
	30	23	14	11	18	11	8
	45	30	18	14	24	15	11
	60	36	22	16	29	18	13
4	15	17	10	8	14	8	6
	30	26	16	12	21	13	9
	45	34	21	16	27	17	13
	60	42	26	19	33	20	15
5	15	18	11	8	15	9	7
	30	29	18	13	23	14	11
	45	38	23	17	30	18	14
	60	46	28	21	36	22	16
6	15	20	12	9	16	10	7
	30	31	19	14	25	15	11
	45	40	25	18	32	20	15
	60	49	30	22	39	24	18

Table II (α = 2.0)

Γ-optimal Sample Size for R.C.B.D. Problem

k	λ	(λ = 0.0)			(λ = 0.5)		
	π_0 / c'/c_0	0.05	0.10	0.15	0.05	0.10	0.15
2	15	10	7	6	7	5	4
	30	14	10	8	10	7	6
	45	17	12	10	12	9	7
	60	20	14	11	14	10	8
3	15	13	9	7	9	7	5
	30	18	13	10	13	9	7
	45	22	15	12	16	11	9
	60	26	18	14	18	13	10
4	15	15	10	8	11	7	6
	30	21	14	12	15	10	8
	45	25	18	14	18	13	10
	60	29	20	16	21	14	12
5	15	16	11	9	12	8	7
	30	22	16	13	16	11	9
	45	27	19	15	20	14	11
	60	32	22	17	23	16	13
6	15	17	12	10	12	9	7
	30	24	17	13	17	12	10
	45	29	20	16	21	14	12
	60	34	23	19	24	17	13

Figure 1

Graphical relationship between Δ and n*

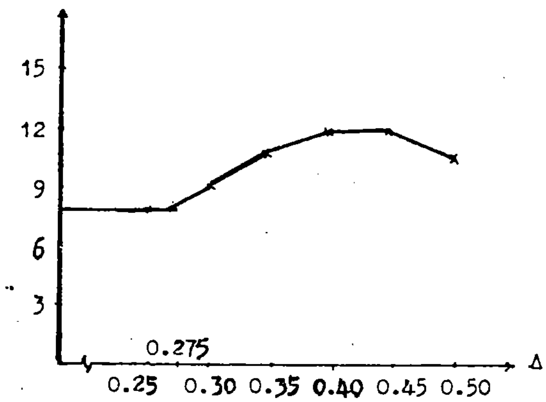

Fig 1.a under $c'/c_0 = 15$, $\pi_0 = 0.10$.

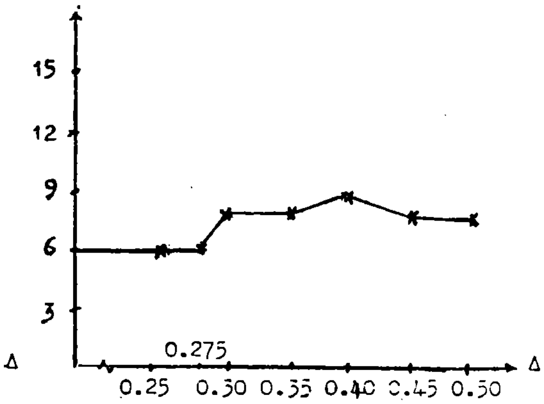

Fig 1.b under $c'/c_0 = 15$, $\pi_0 = 0.15$.

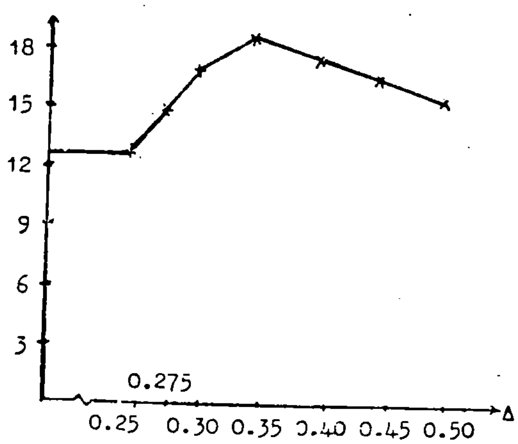

Fig 1.c under $c'/c_0 = 30$, $\pi_0 = 0.10$.

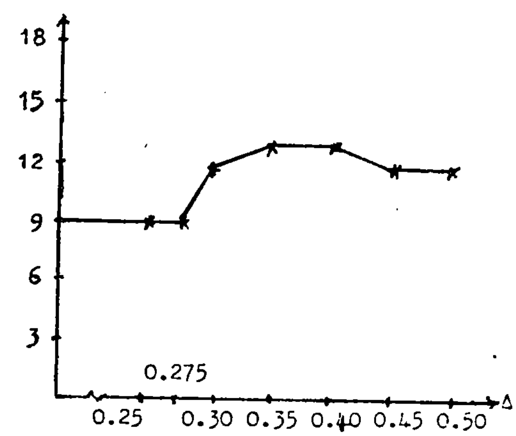

Fig 1.d under $c'/c_0 = 30$, $\pi_0 = 0.15$.

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In randomized complete block design, we face the problem of selecting the best population. If some partial information about the unknown parameters is available, then we wish to determine the optimal decision rule to select the best population.

In this paper, in the class of natural selection rules, we employ the Γ-optimal criterion to determine optimal decision rules that will minimize the maximum expected risk over the class of some partial information. Furthermore, the traditional hypothesis testing is briefly discussed from the view point of ranking and selection.

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