ON A CONJECTURE CONCERNING LEAST FAVORABLE CONFIGURATIONS IN CERTAIN TWO-STAGE SELECTION PROCEDURES

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ABSTRACT

Given k normal populations with unknown means and a common known (or unknown) variance a two-stage procedure $\mathcal{P}_1$ with screening in the first stage to find the population with the largest mean is under concern. It was proposed and studied previously by Cohen(1959),Alam(1970), Tamhane and Bechhofer(1977,1979) and Gupta and Miescke(1979). But up to now a conjecture concerning least favorable parameter configurations in an indifference zone approach remained unproved for $k \geq 3$. In this paper we give a non-standard proof of the conjecture in case of $k = 3$ for $\mathcal{P}_1$ which (under minor changes) works also for a simplified version $\mathcal{P}_2$. Besides, the point is exposed where another (more intuitive) method of proof fails to work .

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1. INTRODUCTION

Suppose we are given k normal populations $\pi_1, \ldots, \pi_k$ with unknown means $\mu_1, \ldots, \mu_k$ and a common known (or unknown) variance $\sigma^2 > 0$. The following two-stage procedure $\mathcal{P}_1$ to find the population with the largest mean was studied by Alam(1970), Cohen(1959), Tamhane and Bechhofer(1977,1979) and Gupta and Miescke(1979) :

Procedure $\mathcal{P}_1$:

Stage 1 : Take k independent samples $(X_{i1}, \ldots, X_{in_1})$ of size n_1, $i = 1, \ldots, k$, from $\pi_1, \ldots, \pi_k$ and compute $X_i = (X_{i1} + \cdots + X_{in_1}) / n_1$, $i = 1, \ldots, k$. Select all populations π_i with $X_i \geq \max\{X_j \mid j = 1, \ldots, k\} - c$, where $c > 0$ is fixed. If only one population is selected, stop and assert that this one has the largest mean. Otherwise proceed to Stage 2.

Stage 2 : Take additional independent samples $(Y_{i1}, \ldots, Y_{in_2})$ of size n_2 from those populations being selected in Stage 1 and compute $Y_i = (Y_{i1} + \cdots + Y_{in_2}) / n_2$ for them. Among the selected populations decide finally in favor of that population yielding the largest $n_1 X_i + n_2 Y_i$.

Thus procedure $\mathcal{P}_1$ is a combination of two classical one-stage procedures where the first one (in Stage 1) is due to Gupta(1956) and the second one (in Stage 2) is due to Bechhofer(1954).

Now in all papers dealing with $\mathcal{P}_1$ the following conjecture concerning the least favorable parameter configurations w.r.t. the probability of a correct selection , $P_{\underline{\mu}}\{C.S.\mathcal{P}_1\}$, $\underline{\mu} = (\mu_1,\ldots,\mu_k)$, in an indifference zone approach was stated but remained unproved for $k \geq 3$:

Conjecture : Let $\delta^* > 0$ be fixed and consider $\Omega_{\delta^*} = \{\underline{\mu} \in \mathbb{R}^k \mid \mu_{[k-1]} \leq \mu_{[k]} - \delta^*\}$,where for $\underline{\mu} \in \mathbb{R}^k$ $\mu_{[1]} \leq \ldots \leq \mu_{[k]}$ denote the ordered coordinates. Then for every $t \in \mathbb{R}$

$$\inf_{\underline{\mu} \in \Omega_{\delta^*}} P_{\underline{\mu}}\{C.S.\mathcal{P}_1\} = P_{(t,t,\ldots,t,t+\delta^*)}\{C.S.\mathcal{P}_1\} .$$

In Section 3 we shall prove the conjecture for $k = 3$. But we do not see any way to adapt this proof properly to cases where $k > 3$.The point where another (more intuitive) method of proof fails to work will be exposed in Section 2 , where also some general auxiliary results are given .

As a by-product (with minor changes) our proof works also for procedure $\mathcal{P}_2$, say , which differs from $\mathcal{P}_1$ only in Stage 2 where final decisions are made in terms of the Y_i's instead of the $n_1 X_i + n_2 Y_i$'s.

2. SOME GENERAL PROPERTIES OF $\mathcal{P}_1$ AND $\mathcal{P}_2$

In this section we study the behavior of $\mathcal{P}_1$ and $\mathcal{P}_2$ in the general situation ($k \geq 2$) and derive some preliminary results which will be useful in Section 3 when we shall prove the conjecture for $k = 3$.

We start with

$$P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m\} = \int_{\mathbb{R}^k} P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m \mid \underline{X} = \underline{x}\}\, dP_{\underline{\mu}}\{\underline{X} = \underline{x}\}, \quad (2.1)$$

where $\underline{X} = (X_1,\ldots,X_k)$, $\underline{x} = (x_1,\ldots,x_k)$, $\underline{\mu} \in \mathbb{R}^k$ and $m = 1,2$, and state without proof some properties of the terms appearing in (2.1). They hold for both, $\mathcal{P}_1$ and $\mathcal{P}_2$ and are well known or easy to prove .

$$P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m\} = P_{\underline{\tilde{\mu}}}\{\text{C.S.}\mathcal{P}_m\} \quad (2.2)$$

for every $\underline{\mu}, \underline{\tilde{\mu}} \in \mathbb{R}^k$ with $\mu_{[i]} = \tilde{\mu}_{[i]}$, $i = 1,\ldots,k$.

Thus from now on we restrict our considerations to parameter configurations $\underline{\mu} \in \mathbb{R}^k$ with $\mu_1 \leq \mu_2 \leq \ldots \leq \mu_k$.

$$\begin{aligned} P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m \mid \underline{X} = \underline{x}\} &= P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m \mid \underline{X} = \underline{x} + a\,\underline{1}\} \\ &= P_{\underline{\mu} + a\underline{1}}\{\text{C.S.}\mathcal{P}_m \mid \underline{X} = \underline{x}\} \end{aligned} \quad (2.3)$$

for every $\underline{\mu}, \underline{x} \in \mathbb{R}^k$ and $a \in \mathbb{R}$, where $\underline{1} = (1,1,\ldots,1) \in \mathbb{R}^k$.

$$P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m\} = P_{\underline{\mu} + a\underline{1}}\{\text{C.S.}\mathcal{P}_m\}, \underline{\mu} \in \mathbb{R}^k, a \in \mathbb{R}. \quad (2.4)$$

For $\underline{x} \in \mathbb{R}^k$ fixed, $P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m \mid \underline{X} = \underline{x}\}$ is non-decreasing in μ_k and non-increasing in $\mu_1, \ldots, \mu_{k-1}$. (2.5)

For $\underline{\mu} \in \mathbb{R}^k$ fixed, $P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m \mid \underline{X} = \underline{x}\}$ in non-decreasing in x_k . (2.6)

$P_{\underline{\mu}}\{\text{C.S.}\mathcal{P}_m\}$ is non-decreasing in μ_k , $\underline{\mu} \in \mathbb{R}^k$. (2.7)

Obviously, (2.1) and (2.3) imply (2.4) , whereas (2.7) (which was proved already by Tamhane and Bechhofer (1977)) follows from (2.1) , (2.5) (the "μ_k -part") and (2.6) . Analogously it could be demonstrated easily that $P_{\underline{\mu}}\{C.S.\mathcal{P}_m\}$ is non-increasing in $\mu_1, \ldots, \mu_{k-1}$ if it were true that for every fixed $\underline{\mu} \in \mathbb{R}^k$ $P_{\underline{\mu}}\{C.S.\mathcal{P}_m \mid \underline{X} = \underline{x}\}$ were non-increasing in $x_1, \ldots, x_{k-1}$. But this does not hold true for $k \geq 3$!

<u>Counterexample :</u> For $k \geq 3$ let $\mu_1 \leq \mu_2 \leq \ldots \leq \mu_{k-1} \leq \mu_k - \delta^*$ and $0 < \varepsilon < c/2$ be fixed. Then for $\underline{x} = (x_1, x_2, \ldots, x_k)$ with $x_k - c < x_2, x_3, \ldots, x_{k-1} < x_k - c + \varepsilon$ and $x_k - \varepsilon \leq x_1 \leq x_k$ and for $\underline{x}' = (x_1', x_2, \ldots, x_k)$ with $x_k + \varepsilon \leq x_1' < x_k + 2\varepsilon$, we have $P_{\underline{\mu}}\{C.S.\mathcal{P}_2 \mid \underline{X} = \underline{x}\} < P_{\underline{\mu}}\{C.S.\mathcal{P}_2 \mid \underline{X} = \underline{x}'\}$, since in Stage 1 , under $\underline{X} = \underline{x}$, all populations are selected whereas under $\underline{X} = \underline{x}'$, π_1 and π_k only are selected . And it is not difficult to see that for sufficiently small $\varepsilon > 0$, $P_{\underline{\mu}}\{C.S.\mathcal{P}_1 \mid \underline{X} = \underline{x}\} < P_{\underline{\mu}}\{C.S.\mathcal{P}_1 \mid \underline{X} = \underline{x}'\}$ holds, too.

It should be pointed out clearly that though we are able to prove the conjecture for $k = 3$, the interesting question whether for $k \geq 3$ and $m \in \{1,2\}$ $P_{\underline{\mu}}\{C.S.\mathcal{P}_m\}$ really is non-increasing in $\mu_1, \ldots, \mu_{k-1}$ or not still remains open .

3. PROOF OF THE CONJECTURE FOR $k = 3$

Now we shall study the case of $k = 3$ in more detail . Let $h(x) = (2\pi\sigma^2/n_1)^{-1/2} \exp(-n_1 x^2 / 2\sigma^2)$, $x \in \mathbb{R}$, such that X_i has the density $h(x - \mu_i)$, $x \in \mathbb{R}$, $i = 1,2,3$. Before we present our main result we state the following key lemma . Its proof is of very technical nature and may be skipped at the first reading .

Lemma : For every $v \geq 0$, $w \geq 0$ and $m \in \{1,2\}$

$$P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid X_1 = -v + w\} \tag{3.1}$$
$$\leq \quad P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid X_1 = -v - w\} .$$

Proof : Let $v,w \geq 0$ and $m \in \{1,2\}$ be fixed and let us denote the difference of the r.h.s. minus the l.h.s. of (3.1) by A ,say.

Then

$$A = \int_{\mathbb{R}}\int_{\mathbb{R}} \Big[P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid \underline{X} = (-v-w,x_2,x_3)\}\, h(x_2)\, h(x_3-\delta^*)$$
$$- P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid \underline{X} = (-v+w,x_2,x_3)\}\, h(x_2)\, h(x_3-\delta^*)\Big]\, dx_2 dx_3$$
$$= \int_{\mathbb{R}}\int_{\mathbb{R}} \Big[P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid \underline{X}=(-v-w,x_2-w,x_3-w)\}\, h(x_2-w)\, h(x_3-w-\delta^*)$$
$$- P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid \underline{X}=(-v+w,x_2+w,x_3+w)\}\, h(x_2+w)\, h(x_3+w-\delta^*)\Big]\, dx_2 dx_3 .$$

Thus by (2.3) we get

$$A = \int_{\mathbb{R}} \int_{\mathbb{R}} P_{(0,0,\delta^*)} \left\{ C.S.\mathcal{P}_m \mid \underline{X} = (-v,x_2,x_3) \right\} H(x_2,x_3)\, dx_2\, dx_3 \quad ,$$

where $H(x_2,x_3) = h(x_2-w)\, h(x_3-w-\delta^*) - h(x_2+w)\, h(x_3+w-\delta^*), (x_2,x_3) \in \mathbb{R}^2$.

Now let $C = \{ (\xi,\eta) \in \mathbb{R}^2 \mid H(\xi,\eta) > 0 \}$ and $\tilde{C} = \{ (\xi,\eta) \in \mathbb{R}^2 \mid H(\xi,\eta) < 0 \}$. Then the monotone likelihood ratio property of normal distributions w.r.t. location parameters implies $C = \{ (\xi,\eta) \in \mathbb{R}^2 \mid \xi + \eta > \delta^* \}$ and $\tilde{C} = \{ (\xi,\eta) \in \mathbb{R}^2 \mid \xi + \eta < \delta^* \}$.

Moreover, let $\alpha : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be defined by $\alpha(\xi,\eta) = (\delta^*-\eta, \delta^*-\xi)$ and let $(\xi^\alpha, \eta^\alpha) = \alpha(\xi,\eta)$, $(\xi,\eta) \in \mathbb{R}^2$ in the following. Then in view of $\alpha(C) = \tilde{C}$ we get

$$A = \left[\int_C + \int_{\tilde{C}} \right] P_{(0,0,\delta^*)} \left\{ C.S.\mathcal{P}_m \mid \underline{X} = (-v,x_2,x_3) \right\} H(x_2,x_3) \quad d(x_2,x_3)$$

$$= \int_{\tilde{C}} \Big[P_{(0,0,\delta^*)} \left\{ C.S.\mathcal{P}_m \mid \underline{X} = (-v,x_2^\alpha,x_3^\alpha) \right\} H(x_2^\alpha,x_3^\alpha)$$

$$+ P_{(0,0,\delta^*)} \left\{ C.S.\mathcal{P}_m \mid \underline{X} = (-v,x_2,x_3) \right\} H(x_2,x_3) \Big] \, d(x_2,x_3) \quad .$$

Finally, since $H(\xi^\alpha, \eta^\alpha) = -H(\xi,\eta)$, $(\xi,\eta) \in \mathbb{R}^2$, we arrive at

$$A = \int_{\tilde{C}} \Big[P_{(0,0,\delta^*)} \left\{ C.S.\mathcal{P}_m \mid \underline{X} = (-v,x_2,x_3) \right\}$$

$$- P_{(0,0,\delta^*)} \left\{ C.S.\mathcal{P}_m \mid \underline{X} = (-v,x_2^\alpha,x_3^\alpha) \right\} \Big] \, H(x_2,x_3) \quad d(x_2,x_3) \quad .$$

Thus to complete the proof in view of $H(\xi,\eta) < 0$ for $(\xi,\eta) \in \tilde{C}$ we only have to show that for every $(x_2,x_3) \in \tilde{C}$

$$P_{(0,0,\delta^*)} \left\{ \text{C.S. } \mathcal{P}_m \mid \underline{X} = (-v, x_2, x_3) \right\} \tag{3.2}$$

$$\leq \quad P_{(0,0,\delta^*)} \left\{ \text{C.S. } \mathcal{P}_m \mid \underline{X} = (-v, x_2^{\alpha}, x_3^{\alpha}) \right\} \quad .$$

Now let $(x_2, x_3) \in \tilde{C}$ be fixed. For notational convenience, let for $\underline{x} \in \mathbb{R}^3$, $S(\underline{x}) = \{ i \in \{1,2,3\} \mid x_i \geq x_{[3]} - c \}$ denote the set of indices of those populations being selected at Stage 1 in case of $\underline{X} = \underline{x}$. We are stepping now through different cases for $S(-v, x_2, x_3)$, showing that always $S(-v, x_2^{\alpha}, x_3^{\alpha})$ is as favorable to π_3 (i.e. a correct selection) as $S(-v, x_2, x_3)$ (Note that this already will suffice to complete the proof for $\mathcal{P}_2$) and moreover, that thereby the relevant x-values corresponding to π_2 and π_3 do not change to the disadvantage of π_3 .

Obviously, $x_3 > x_2 + c$ implies $3 \notin S(-v, x_2, x_3)$ (cf. next case) or $3 \in S(-v, x_2^{\alpha}, x_3^{\alpha}) \subseteq S(-v, x_2, x_3) \subseteq \{1,3\}$. Thus (3.2) holds for $\mathcal{P}_2$. And since we have $x_3^{\alpha} > x_3$, (3.2) is proved for $\mathcal{P}_1$, too .

Moreover, $x_3 < x_2 - c$ implies $3 \notin S(-v, x_2, x_3)$ and thus for $\mathcal{P}_1$ as well as for $\mathcal{P}_2$ the l.h.s. of (3.2) equals zero .

Finally, let $x_2 - c \leq x_3 \leq x_2 + c$. If $3 \notin S(-v, x_2, x_3)$ the same argument as before applies. Otherwise, we have to distinguish between three possibilities for $S(-v, x_2, x_3)$:

The first one is $S(-v, x_2, x_3) = \{1,2,3\}$. This implies $\{2,3\} \subseteq S(-v, x_2^{\alpha}, x_3^{\alpha}) \subseteq S(-v, x_2, x_3)$ which proves (3.2) for $\mathcal{P}_2$ and in view of $x_3^{\alpha} - x_2^{\alpha} = x_3 - x_2$ and $x_3^{\alpha} > x_3$ for $\mathcal{P}_1$, too .

The second one is $S(-v,x_2,x_3) = \{2,3\}$ which implies $S(-v,x_2^{\alpha},x_3^{\alpha}) = \{2,3\}$ and can be handled analogously .

The third one is $S(-v,x_2,x_3) = \{1,3\}$ implying $S(-v,x_2^{\alpha},x_3^{\alpha}) = \{2,3\}$ in view of $x_3^{\alpha} - c > v + \delta^* > 0 > -v$. This point requires a bit more care since , at the same time , one population (π_1) leaves the subset of populations being selected whereas another one (π_2) enters it . But this does not really cause difficulties since our parameter configuration is $\underline{\mu} = (0,0,\delta^*)$ and therefore π_1 and π_2 are "interchangeable" . Thus (3.2) follows immediately for $\mathcal{P}_2$ and the additional argument $x_2 \leq -v$, i.e. $x_3^{\alpha} - x_2^{\alpha} \geq x_3 + v$, implies (3.2) for $\mathcal{P}_1$. This completes the proof of our Lemma .

The following representation of the probability of a correct selection under $\mathcal{P}_1$ or $\mathcal{P}_2$, respectively , will be useful in the sequel :

$$P_{\underline{\mu}}\{C.S.\mathcal{P}_m\} = \left[\int_{-\infty}^{\gamma} + \int_{\gamma}^{\infty}\right] P_{\underline{\mu}}\{C.S.\mathcal{P}_m \mid X_1 = x_1\}\ h(x_1 - \mu_1)\, dx_1 \quad (3.3)$$

$$= \int_0^{\infty} \Big[P_{\underline{\mu}}\{C.S.\mathcal{P}_m \mid X_1 = \gamma - w\}\ h(\gamma - w - \mu_1)$$

$$+ P_{\underline{\mu}}\{C.S.\mathcal{P}_m \mid X_1 = \gamma + w\}\ h(\gamma + w - \mu_1)\Big]\, dw \ ,$$

where $\underline{\mu} \in \mathbb{R}^k$, $\gamma \in \mathbb{R}$ and $m = 1,2$. It is derived by substituting $x_1 = \gamma - w$ in the first integral and $x_1 = \gamma + w$ in the second one .

Theorem : For $k = 3$ the conjecture holds true for $\mathcal{P}_1$ as well as for $\mathcal{P}_2$.

Proof : In view of (2.4) and (2.7) it suffices to prove that for every $v \geq 0$ and $m \in \{1,2\}$

$$P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m\} \leq P_{(-2v,0,\delta^*)}\{C.S.\mathcal{P}_m\} .$$

Now let $v \geq 0$ and $m \in \{1,2\}$ be fixed . Then by (3.3) for $\gamma = -v$ and by the symmetry of h we get

$$P_{(-2v,0,\delta^*)}\{C.S.\mathcal{P}_m\} - P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m\}$$

$$= \int_0^\infty \Big[P_{(-2v,0,\delta^*)}\{C.S.\mathcal{P}_m \mid X_1 = -v-w\}\, h(v-w)$$

$$+ P_{(-2v,0,\delta^*)}\{C.S.\mathcal{P}_m \mid X_1 = -v+w\}\, h(v+w)$$

$$- P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid X_1 = -v-w\}\, h(v+w)$$

$$- P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid X_1 = -v+w\}\, h(v-w) \Big]\, dw .$$

By (2.5) this is bounded from below by

$$\int_0^\infty \Big[P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid X_1=-v-w\} - P_{(0,0,\delta^*)}\{C.S.\mathcal{P}_m \mid X_1=-v+w\} \Big] \Big[h(v-w) - h(v+w) \Big]\, dw \geq 0 ,$$

where the last inequality follows from the fact that for $v,w \geq 0$ we have $h(v-w) \geq h(v+w)$ and (3.1) . Thus the proof is completed.

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BIBLIOGRAPHY

Alam,K.(1970). A two-sample procedure for selecting the population with the largest mean from k normal populations. Ann.Inst. Statist.Math. 22 , 127 - 136 .

Bechhofer,R.E.(1954). A single-sample multiple decision procedure for ranking means of normal populations with known variances. Ann. Math.Statist. 25 , 16 - 39 .

Cohen,D.S.(1959). A two-sample decision procedure for ranking means of normal populations with a common known variance. Unpublished M.S. thesis, Dept. of Ind. Eng., Cornell Univ., Ithaca, New York .

Gupta,S.S.(1956). On a decision rule for a problem in ranking means . Mimeo.Series No. 150 , Inst. of Statist. , Univ. of Noth Carolina, Chapel Hill , North Carolina .

Gupta,S.S. and Miescke,K.J.(1979). On the least favorable configurations in certain two-stage selection procedures. Mimeo.Series No. 79 - 6 , Dept. of Statist., Purdue Univ.,West-Lafayette,Indiana.

Tamhane,A.C. and Bechhofer,R.E.(1977). A two-stage minimax procedure with screening for selecting the largest normal mean. Commun. Statist. A6 , 1003 - 1033 .

Tamhane,A.C. and Bechhofer,R.E.(1979). A two-stage minimax procedure with screening for selecting the largest normal mean (II) : An improved lower bound and associated tables. Commun.Statist. A8 , 337-358.

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