ON MOMENTS OF ORDER STATISTICS FROM INDEPENDENT BINOMIAL POPULATIONS*

by

Shanti S. Gupta and S. Panchapakesan

Purdue University and

Southern Illinois University

Department of Statistics
Division of Mathematical Science
Mimeograph Series #297

July 12, 1972

---

*This research was supported by the Office of Naval Research Contract N00014-67-A-0226-00014 and by the Aerospace Research Laboratories Contract AF33(615)67C1244 at Purdue University. Reproduction in whole or in part is permitted for any purpose of The United States Government.

$$B(p,q) = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}$$

$$I_p(a,b) = \frac{1}{B(a,b)} \int_0^p u^{a-1}\,(1-u)^{b-1}\,du, \quad a,b > 0.$$

Let $p_i(x)$ be the probability that the ith order statistic $X_{(i)}$ is equal to x and let $P_i(x) = P\{X_{(i)} \leq x\}$ be the c.d.f. of $X_{(i)}$. Khatri (1962) has obtained the following results.

$$(2.1) \qquad p_i(x) = \sum_{k=0}^{i-1} \sum_{m=0}^{M-i} \frac{M!}{(i-1-k)!\,(k+m+1)!\,(M-i-m)!} \{B(x-1)\}^{i-1-k} \{b(x)\}^{k+m+1} \{1-B(x)\}^{M-i-m}$$

where $B(x-1) = 0$ for $x = 0$. This can be rewritten as

$$(2.2) \qquad p_i(x) = i \binom{M}{i} \int_{B(x-1)}^{B(x)} \omega^{i-1}(1-\omega)^{M-i}\,d\omega = I_{B(x)}(i,M-i+1) - I_{B(x-1)}(i,M-i+1).$$

Further

$$(2.3) \qquad P_i(x) = i \binom{M}{i} \int_0^{B(x)} \omega^{i-1}(1-\omega)^{M-i}\,d\omega = I_{B(x)}(i,M-i+1),$$

$$(2.4) \qquad E(X_{(i)}) = \sum_{x=0}^{N-1} [1-P_i(x)] \qquad \text{and}$$

$$(2.5) \qquad E(X^2_{(i)}) = 2 \sum_{x=0}^{N-1} x[1-P_i(x)] + \sum_{x=0}^{N-1} [1-P_i(x)].$$

It should be pointed out that the expressions (2.1) through (2.5) hold for any discrete distribution with proper choice of the probability mass $b(\alpha)$. Also, it can be easily seen that for both discrete and continuous random variables

$$(2.6) \qquad P_i(x) = \sum_{t=i}^{M} \binom{M}{t} [B(x)]^t [1-B(x)]^{M-t}$$

where $B(x)$ is to be interpreted as the usual c.d.f. in the continuous case. Now (2.3) follows from (2.6), since

$$(2.7) \qquad \sum_{t=i}^{M} \binom{M}{t} p^t (1-p)^{M-t} = \frac{1}{B(i,M-i+1)} \int_0^p \omega^{i-1} (1-\omega)^{M-i} d\omega$$

where $0 \leq p < 1$. (When $B(x) = 1$, (2.3) is obviously true). Now (2.2) follows at once from (2.3) by noting that

$$p_i(x) = P_i(x) - P_i(x-1).$$

For the special cases i=1 and i=M, we obtain the following results from (2.3), (2.4) and (2.5).

$$(2.8) \qquad p_1(x) = [1-B(x-1)]^M - [1-B(x)]^M$$

$$(2.9) \qquad P_1(x) = 1 - [1-B(x)]^M$$

$$(2.10) \qquad p_M(x) = [B(x)]^M - [B(x-1)]^M$$

$$(2.11) \qquad P_M(x) = [B(x)]^M$$

$$(2.12) \qquad E(X_{(1)}) = \sum_{x=0}^{N-1} [1-B(x)]^M$$

$$(2.13) \qquad E(X^2_{(1)}) = 2 \sum_{x=0}^{N-1} x[1-B(x)]^M + E(X_{(1)})$$

$$(2.14) \qquad E(X_{(M)}) = \sum_{x=0}^{N-1} [1-\{B(x)\}^M]$$

$$(2.15)\qquad E(X^2_{(M)}) = 2\sum_{x=0}^{N-1} x[1-\{B(x)\}^M] + E(X_{(M)})$$

3. Joint distribution of $X_{(i)}$ and $X_{(j)}$, $i < j$.

Let $p_{i,j}(x,y)(i < j)$ be the probability that $X_{(i)}$ is equal to x and $X_{(j)}$ is equal to y and let $P_{i,j}(x,y) = P(X_{(i)} \leq x,\ X_{(j)} \leq y)$. If $x \geq y$,

$$(3.1)\qquad P_{i,j}(x,y) = P\{X_{(j)} \leq y\}$$
$$= j\binom{M}{j}\int_0^{B(y)} u^{i-1}(1-u)^{M-j}du.$$

If $x < y$, then a combinatorial argument leads to

$$P_{i,j}(x,y) = \sum_{\substack{s=i \\ s+t\leq M}}^{M}\sum_{t=0}^{M-j} \frac{M!}{s!(M-s-t)!t!}\{B(x)\}^s\{B(y)-B(x)\}^{M-s-t}\{1-B(y)\}^t$$

$$= \sum_{s=i}^{j}\sum_{t=0}^{M-j} \frac{M!}{s!(M-s-t)!t!}\{B(x)\}^s\{B(y)-B(x)\}^{M-s-t}\{1-B(y)\}^t$$

$$+ (1-\delta_{jM})\sum_{s=j+1}^{M}\sum_{t=0}^{M-s} \frac{M!}{s!(M-s-t)!t!}\{B(x)\}^s\{B(y)-B(x)\}^{M-s-t}\{1-B(y)\}^t$$

where δ_{jM} is the Kronecker delta.

By repeated application of the results

$$\sum_{t=a}^{n}\binom{n}{t}p^t(k-p)^{n-t} = \frac{1}{B(a,n-a+1)}\int_0^p u^{a-1}(k-u)^{n-a}du \qquad \text{and}$$

$$\sum_{t=0}^{b}\binom{n}{t}p^t(k-p)^{n-t} = k^n - \frac{1}{B(b+1,n-b)}\int_0^p u^b(k-u)^{n-b-1}du$$

where $0 \leq p < 1$ and $p < k$, we obtain

$$(3.2)\qquad P_{i,j}(x,y) = i\binom{M}{i}\int_0^{B(x)} u^{i-1}(1-u)^{M-i}du - \frac{M!}{(i-1)!(j-i-1)!(M-j)!}\int_{B(y)}^{1} dv \int_0^{B(x)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}d\omega.$$

Now we can write

$$(3.3)\qquad p_{i,j}(x,y) = \begin{cases} 0 & , \text{ if } x > y \\ P_{i,j}(x,x) - P_{i,j}(x-1;x) & , \text{ if } x = y \\ P_{i,j}(x,y) - P_{i,j}(x-1,y) & \\ \qquad - P_{i,j}(x,y-1) + P_{i,j}(x-1,y-1) & , \text{ if } x < y. \end{cases}$$

Khatri (1962) has obtained the joint distribution directly in the form

$$(3.4)\qquad P_{i,j}(x,y) = \frac{M!}{(i-1)!(j-i-1)!(M-j)!}\int\int \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}d\omega dv$$

where the double integration is performed over the region given by

$$v \geq \omega,$$
$$P(x) \geq \omega \geq P(x-1),$$
$$P(y) \geq v \geq P(y-1).$$

But Khatri has obtained the c.d.f. $P_{i,j}(x,y)\,(x \leq y)$ in the form

$$(3.5)\qquad P_{i,j}(x,y) = \frac{M!}{(i-1)!(j-i-1)!(M-j)!}\int_{B(x)}^{B(y)} dv \int_0^{B(x)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}d\omega.$$
$$+ j\binom{M}{j}\int_0^{B(x)} v^{j-1}(1-v)^{M-j}dv \text{ is valid only for } x \leq y.$$

Incidentally, from (3.2) and (3.5) we get the relation

$$(3.6)\qquad \frac{M!}{(i-1)!(j-i-1)!(M-j)!}\int_{B(x)}^{1} dv \int_{0}^{B(x)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}\,d\omega$$

$$= i\binom{M}{i}\int_{0}^{B(x)} u^{i-1}(1-u)^{M-i}\,du - j\binom{M}{j}\int_{0}^{B(x)} u^{j-1}(1-u)^{M-j}\,du$$

$$= I_{B(x)}(i,M-i+1) - I_{B(x)}(j,M-j+1).$$

Now $E(X_{(i)}X_{(j)}) = \sum_{x=0}^{N}\sum_{y=0}^{N} xy\, p_{i,j}(x,y)$

$$= \sum_{x=0}^{N} x^2\, p_{i,j}(x,x) + \sum_{x=0}^{N-1}\sum_{y=x+1}^{N} xy p_{i,j}(x,y).$$

So

$$(3.7)\qquad \mathrm{Cov}(X_{(i)},X_{(j)}) = \sum_{x=0}^{N} x^2\, p_{i,j}(x,x) + \sum_{x=0}^{N-1}\sum_{y=x+1}^{N} xy p_{i,j}(x,y)$$

$$- \{\sum_{x=0}^{N-1}[1-P_i(x)]\}\{[\sum_{x=0}^{N-1}[1-P_j(x)]\}.$$

An explicit expression for $\mathrm{Cov}(x_{(i)},X_{(j)})$ is very complicated. However, for the special case where i=1 and j=M, we obtain by usual algebraic simplifications,

$$(3.8)\qquad \mathrm{Cov}(X_{(1)},X_{(M)}) = NE(X_{(1)}) - (1-\delta_{N1})\sum_{y=1}^{N-1}\sum_{x=0}^{y-1}[B(y)-B(x)]^M$$

$$- E(X_{(1)})\,E(X_{(M)}),$$

where $E(X_{(1)})$ and $E(X_{(M)})$ are given by (2.12) and (2.14) and δ_{N1} is the Kronecker delta.

4. Distribution of $Y_{i,j} = X_{(j)} - X_{(i)} (j > i)$.

$Y_{i,j}$ represents a generalized range and can take values $0,1,\ldots,N$. For $r \geq 0$,

$$(4.1) \qquad P(Y_{i,j}=r) = \sum_{k=0}^{N-r} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_A \int \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j} dv d\omega$$

where A is the region given by

$$v \geq \omega,$$

$$B(k) \geq \omega \geq B(k-1),$$

$$B(k+r) \geq v \geq B(k+r-1).$$

This can be rewritten as

$$(4.2) \qquad P\{Y_{i,j}=r\} = \begin{cases} \sum_{k=0}^{N} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{\omega}^{B(k)} \omega^{i-1}(v-\omega)^{j-i-1} \\ \qquad (1-v)^{M-j} dv, r=0, \\ \sum_{k=0}^{N-r} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{B(k+r-1)}^{B(k+r)} \omega^{i-1}(v-\omega)^{j-i-1} \\ \qquad (1-v)^{M-j} dv, r > 0. \end{cases}$$

So $E(Y_{i,j})$

$$(4.3) \qquad = \sum_{r=1}^{N} r \sum_{k=0}^{N-r} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{B(k+r-1)}^{B(k+r)} \omega^{i-1}(v-\omega)^{j-i-1} (1-v)^{M-j} dv$$

But we also know that

$$(4.4) \qquad E(Y_{i,j}) = E(X_{(j)}) - E(X_{(i)})$$

$$= \sum_{x=0}^{N-1} i \binom{M}{i} \int_0^{B(x)} \omega^{i-1}(1-\omega)^{M-i} d\omega - \sum_{x=0}^{N-1} j \binom{M}{j} \int_0^{B(x)} \omega^{j-1} (1-\omega)^{M-j} d\omega$$

In particular if i=M-1, we obtain

$$(4.8)\qquad P(Y_{M-1,M}=r)=\begin{cases}\sum_{k=0}^{N}[\{B(k)\}^{M}-\{B(k-1)\}^{M-1}\{MB(k)-(M-1)B(k-1)\}], & r=0\\ \sum_{k=0}^{N-r}M[\{B(k)\}^{M-1}-\{B(k-1\}^{M-1}]b(k+r), & r>0\end{cases}$$

5. Asymptotic Results.

Let $Z_i = \dfrac{X_i - Np}{\sqrt{Np(1-p)}}$. Then, for large N, $P\{X_1 \le x\} \approx P\{Z_i \le z\}$, where Z_i is a normal variate with mean zero and variance unity and $z = \dfrac{x - Np}{\sqrt{Np(1-p)}}$.

As the transformation from X to Z preserves order, we have

$$(5.1)\qquad P\{X_{(i)} \le x\} \approx P\{Z_{(i)} \le z\} = i\binom{M}{i}\int_0^{\Phi(z)} u^{i-1}(1-u)^{M-i}du,$$

where

$$\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{t^2}{2}}dt.$$

Further we can write

$$(5.2)\qquad E(X_{(i)}^r) \approx E([Z_{(i)}\sqrt{Np(1-p)} + Np]^r) = \sum_{\alpha=0}^{r}\binom{r}{\alpha}(Np)^{r-\frac{\alpha}{2}}(1-p)^{\frac{\alpha}{2}}E(z_{(i)}^{\alpha}).$$

In particular

$$(5.3)\qquad EX_{(i)} \approx Np + \sqrt{Np(1-p)}\ EZ_{(i)}$$

and

$$(5.4)\qquad V(X_{(i)}) \approx Np(1-p)V(Z_{(i)}).$$

6. Applications and Description of the Tables.

The binomial model is of interest in some statistical inference problems. For example, in a life test experiment, truncated at a fixed time, the number of failures is a binomial random variable. Hence if we assume the c.d.f. of the life distribution to be $p_i=F(t,\theta_i)$ and we are interested in testing hypotheses about p_i or, alternatively, about θ_i, then the distribution of the ordered number of failures becomes relevant especially if one is interested in a ranking or selection problems. More specifically, we give the following examples.

Sobel and Huyett (1957), Gupta and Sobel (1960) and Gupta (1966) have discussed the problems of selection and ranking for the parameters of several binomial populations. The problems discussed deal with selecting a subset or selecting a single population using the indifference zone approach. For both formulations, the probability of a correct selection depends on the distribution of X_{max} or $X_{max}-X$ where X_{max} is the largest of a set of independent and identical binomial random variables and X is another binomial random variable distributed independently of X_{max}. For subset selection problems we are interested in evaluating the probability

$$(6.1)\qquad P\{X_{max}-X \leq d\} = \sum_{x=0}^{N} \binom{N}{x} p^x (1-p)^{N-x} \Big\{ \sum_{\alpha=0}^{x+d} \binom{N}{\alpha} p^{\alpha} (1-p)^{N-\alpha} \Big\}^{M-1}$$

$$= \sum_{x=0}^{N} b(x)\{B(x+d)\}^{M-1}.$$

Thus, the tables similar to Table II. of the present paper are useful in evaluating the above probability. The expected size of the selected subset is given by a similar expression (see (6.3) of Gupta and Sobel

(1960)) which can be computed with the help of such tables.

Siotani (1956) has considered tests of hypotheses of the type $p_1=p_2=\ldots=p_M=p$ and suggested the use of the range for this case. He has constructed tables for the distribution of the range. If we are interested in testing $H:p_{(M)} \leq p_0$, then a quick test for this is as follows:

Reject H if $x_{(1)} > x_0(\alpha,N,M)$ where α is the level of significance. To construct this test, we wish to obtain $x_0(\alpha,N,M)$ satisfying

$$(6.2) \qquad \sup_{0<p_{(M)}\leq p_0<1} P[x_{(1)} > x_0(\alpha,N,M)] \leq \alpha.$$

It can be shown that the probability on the left hand side of (6.2) is maximized when $p_{(M)}=p_0$. Hence the necessary constants $x_0(\alpha,N,M)$ can be obtained from the results of this paper. A similar test for the hypothesis $H:p_{(1)} \geq p_0$, can be constructed using the rejection region $x_{(M)} < x_0'(\alpha,N,M)$.

The usual tests for outliers suggest the use of statistics $X_{(M)}-X_{(M-i)}$, $1 \leq i \leq M-1$. The results in Section 4, deal with the distribution of statistics of this type. For the particular case $X_{(M)}-X_{(M-1)}$, a simpler form of the distribution is given by (4.8).

Description of the Tables.

The tables at the end give the first two moments and the c.d.f. of the largest and smallest of M independent and identical binomial random variables, each denoting the number of successes in N independent trials with p as the associated parameter. For the table of moments the range of values of common p is: p=0.1(0.1)0.5; the values of N are: N=1(1)15;

the values of M are: M=1(1)10. These tables were computed in 1960 by Miss Ann Elmer of Bell Telephone Laboratories while one of the authors was a member of the technical staff of Bell Laboratories.

More extensive set of tables for the moments and the c.d.f. of the largest and the smallest order statistic is available and can be obtained by request from the authors.

References

Gupta, S.S. (1966). Selection and ranking procedures and order statistics for the binomial distribution. Proceedings of the International Symposium on classical and contagious discrete distributions held in Montreal (1963), Pergamon Press, New York, 219-230.

Gupta, S.S. and Sobel, M. (1960). Selecting a subset containing the best of several binomial populations. Chapter 20 in Contributions to Probability and Statistics (Hotelling Volume), Stanford University Press, Stanford.

Khatri, C.G. (1962). Distributions of order statistics for discrete case. Ann. Inst. Stat. Math., 14, 167-171.

Siotani, M. (1956). Order statistics for discrete case with a numerical application to the binomial distribution. Ann. Inst. Stat. Math., 8, 95-104.

Siotani, M. and Ozawa, M. (1958). Tables for testing the homogeneity of k independent binomial experiments on a certain event based on the range. Ann. Inst. Stat. Math., 10, 47-63.

Sobel, M. and Huyett, M.J. (1957). Selecting the best one of several binomial populations. Bell System Tech. J., 36, 537-576.

Table 1.

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.1

N/M	1	2	3	4	5	6	7	8	9	10
1	0.1000	0.1900	0.2710	0.3439	0.4095	0.4686	0.5217	0.5695	0.6126	0.6513
	0.0900	0.1539	0.1976	0.2256	0.2418	0.2490	0.2495	0.2452	0.2373	0.2271
	0.1000	0.0100	0.0010	0.0001	0.0000					
	0.0900	0.0099	0.0010	0.0001	0.0000					
2	0.2000	0.3638	0.4983	0.6089	0.7003	0.7761	0.8392	0.8920	0.9364	0.9740
	0.1800	0.2712	0.3094	0.3169	0.3079	0.2908	0.2708	0.2509	0.2325	0.2165
	0.2000	0.0362	0.0069	0.0013	0.0002	0.0000				
	0.1800	0.0351	0.0068	0.0013	0.0002	0.0000				
3	0.3000	0.5258	0.6972	0.8289	0.9315	1.0126	1.0778	1.1314	1.1764	1.2148
	0.2700	0.3678	0.3864	0.3725	0.3485	0.3246	0.3046	0.2896	0.2795	0.2734
	0.3000	0.0742	0.0199	0.0054	0.0015	0.0004	0.0001	0.0000		
	0.2700	0.0703	0.0196	0.0054	0.0015	0.0004	0.0001	0.0000		
4	0.4000	0.6790	0.8778	1.0232	1.1328	1.2184	1.2874	1.3450	1.3945	1.4382
	0.3600	0.4524	0.4510	0.4243	0.3971	0.3766	0.3635	0.3563	0.3532	0.3525
	0.4000	0.1210	0.0408	0.0140	0.0048	0.0017	0.0006	0.0002	0.0001	0.0000
	0.3600	0.1119	0.0394	0.0138	0.0048	0.0017	0.0006	0.0002	0.0001	0.0000
5	0.5000	0.8256	1.0460	1.2022	1.3188	1.4101	1.4850	1.5487	1.6045	1.6544
	0.4500	0.5304	0.5123	0.4797	0.4544	0.4386	0.4299	0.4253	0.4225	0.4202
	0.5000	0.1744	0.0692	0.0282	0.0115	0.0047	0.0019	0.0008	0.0003	0.0001
	0.4500	0.1576	0.0655	0.0275	0.0114	0.0047	0.0019	0.0008	0.0003	0.0001
6	0.6000	0.9671	1.2058	1.3720	1.4958	1.5940	1.6754	1.7454	1.8069	1.8619
	0.5400	0.6046	0.5735	0.5386	0.5153	0.5015	0.4926	0.4857	0.4790	0.4719
	0.6000	0.2329	0.1044	0.0484	0.0226	0.0106	0.0050	0.0023	0.0011	0.0005
	0.5400	0.2058	0.0965	0.0464	0.0221	0.0105	0.0049	0.0023	0.0011	0.0005
7	0.7000	1.1047	1.3596	1.5354	1.6670	1.7720	1.8597	1.9350	2.0011	2.0597
	0.6300	0.6768	0.6353	0.5991	0.5761	0.5608	0.5487	0.5375	0.5263	0.5152
	0.7000	0.2953	0.1454	0.0746	0.0387	0.0202	0.0105	0.0055	0.0029	0.0015
	0.6300	0.2556	0.1310	0.0700	0.0374	0.0198	0.0104	0.0055	0.0029	0.0015
8	0.8000	1.2392	1.5090	1.6942	1.8334	1.9450	2.0381	2.1180	2.1875	2.2490
	0.7200	0.7477	0.6977	0.6596	0.6348	0.6162	0.6000	0.5848	0.5706	0.5574
	0.8000	0.3608	0.1913	0.1064	0.0602	0.0342	0.0194	0.0111	0.0063	0.0036
	0.7200	0.3064	0.1680	0.0976	0.0570	0.0331	0.0191	0.0110	0.0063	0.0036
9	0.9000	1.3712	1.6550	1.8494	1.9960	2.1135	2.2114	2.2949	2.3675	2.4315
	0.8100	0.8180	0.7604	0.7194	0.6912	0.6687	0.6488	0.6310	0.6153	0.6017
	0.9000	0.4288	0.2414	0.1434	0.0868	0.0530	0.0324	0.0198	0.0121	0.0074
	0.8100	0.3580	0.2066	0.1280	0.0805	0.0504	0.0314	0.0195	0.0120	0.0074
10	1.0000	1.5010	1.7982	2.0015	2.1550	2.2780	2.3801	2.4671	2.5425	2.6089
	0.9000	0.8879	0.8229	0.7782	0.7460	0.7196	0.6970	0.6777	0.6614	0.6478
	1.0000	0.4990	0.2950	0.1848	0.1185	0.0767	0.0498	0.0324	0.0211	0.0137
	0.9000	0.4100	0.2461	0.1605	0.1070	0.0715	0.0475	0.0314	0.0207	0.0136
11	1.1000	1.6292	1.9391	2.1511	2.3111	2.4390	2.5451	2.6353	2.7135	2.7825
	0.9900	0.9576	0.8852	0.8360	0.7995	0.7699	0.7452	0.7249	0.7081	0.6943
	1.1000	0.5708	0.3515	0.2302	0.1547	0.1052	0.0719	0.0492	0.0337	0.0232
	0.9900	0.4624	0.2863	0.1942	0.1359	0.0956	0.0672	0.0469	0.0327	0.0226
12	1.2000	1.7558	2.0780	2.2983	2.4644	2.5971	2.7069	2.8003	2.8813	2.9529
	1.0800	1.0270	0.9471	0.8929	0.8523	0.8199	0.7936	0.7722	0.7547	0.7399
	1.2000	0.6442	0.4105	0.2788	0.1949	0.1381	0.0985	0.0705	0.0505	0.0362
	1.0800	0.5150	0.3268	0.2287	0.1662	0.1222	0.0899	0.0659	0.0481	0.0349
13	1.3000	1.8812	2.2153	2.4435	2.6155	2.7526	2.8660	2.9626	3.0464	3.1205
	1.1700	1.0963	1.0087	0.9491	0.9047	0.8697	0.8418	0.8192	0.8004	0.7842
	1.3000	0.7188	0.4716	0.3303	0.2386	0.1751	0.1295	0.0962	0.0716	0.0533
	1.1700	0.5679	0.3676	0.2636	0.1974	0.1503	0.1149	0.0878	0.0668	0.0506
14	1.4000	2.0056	2.3511	2.5869	2.7644	2.9059	3.0229	3.1225	3.2090	3.2854
	1.2600	1.1655	1.0698	1.0049	0.9567	0.9194	0.8897	0.8655	0.8451	0.8274
	1.4000	0.7944	0.5345	0.3842	0.2853	0.2156	0.1644	0.1261	0.0969	0.0746
	1.2600	0.6211	0.4086	0.2987	0.2290	0.1794	0.1417	0.1119	0.0882	0.0693
15	1.5000	2.1289	2.4856	2.7288	2.9116	3.0572	3.1777	3.2802	3.3693	3.4480
	1.3500	1.2346	1.1306	1.0603	1.0085	0.9687	0.9371	0.9111	0.8890	0.8699
	1.5000	0.8711	0.5989	0.4402	0.3347	0.2592	0.2029	0.1599	0.1264	0.1001
	1.3500	0.6744	0.4496	0.3338	0.2608	0.2090	0.1694	0.1377	0.1119	0.0908

A missing entry in the above table denotes a zero correct to four decimal places.

Table I. (con't)

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.2

N/M	1	2	3	4	5	6	7	8	9	10
1	0.2000	0.3600	0.4880	0.5904	0.6723	0.7379	0.7903	0.8322	0.8658	0.8926
	0.1600	0.2304	0.2499	0.2418	0.2203	0.1934	0.1657	0.1396	0.1162	0.0958
	0.2000	0.0400	0.0080	0.0016	0.0003	0.0001	0.0000			
	0.1600	0.0384	0.0079	0.0016	0.0003	0.0001	0.0000			
2	0.4000	0.6688	0.8531	0.9829	1.0773	1.1485	1.2046	1.2505	1.2895	1.3236
	0.3200	0.3783	0.3558	0.3181	0.2860	0.2639	0.2507	0.2440	0.2417	0.2420
	0.4000	0.1312	0.0467	0.0168	0.0060	0.0022	0.0008	0.0003	0.0001	0.0000
	0.3200	0.1172	0.0447	0.0165	0.0060	0.0022	0.0008	0.0003	0.0001	0.0000
3	0.6000	0.9510	1.1703	1.3184	1.4267	1.5116	1.5818	1.6421	1.6951	1.7425
	0.4800	0.5047	0.4573	0.4177	0.3937	0.3800	0.3711	0.3637	0.3562	0.3481
	0.6000	0.2490	0.1173	0.0568	0.0277	0.0135	0.0066	0.0032	0.0016	0.0008
	0.4800	0.2089	0.1058	0.0538	0.0269	0.0133	0.0065	0.0032	0.0016	0.0008
4	0.8000	1.2180	1.4657	1.6323	1.7563	1.8551	1.9372	2.0071	2.0676	2.1206
	0.6400	0.6261	0.5642	0.5232	0.4969	0.4766	0.4584	0.4413	0.4253	0.4106
	0.8000	0.3820	0.2117	0.1226	0.0719	0.0424	0.0250	0.0148	0.0087	0.0051
	0.6400	0.3044	0.1788	0.1097	0.0671	0.0407	0.0244	0.0145	0.0086	0.0051
5	1.0000	1.4756	1.7489	1.9332	2.0711	2.1806	2.2709	2.3472	2.4128	2.4703
	0.8000	0.7465	0.6720	0.6242	0.5895	0.5612	0.5371	0.5171	0.5006	0.4873
	1.0000	0.5244	0.3222	0.2091	0.1386	0.0927	0.0622	0.0418	0.0281	0.0189
	0.8000	0.4011	0.2554	0.1749	0.1219	0.0848	0.0585	0.0401	0.0273	0.0185
6	1.2000	1.7267	2.0238	2.2242	2.3740	2.4925	2.5900	2.6723	2.7435	2.8061
	0.9600	0.8667	0.7786	0.7211	0.6786	0.6453	0.6191	0.5982	0.5814	0.5674
	1.2000	0.6733	0.4436	0.3106	0.2236	0.1630	0.1196	0.0881	0.0649	0.0479
	0.9600	0.4984	0.3326	0.2427	0.1834	0.1398	0.1065	0.0807	0.0608	0.0456
7	1.4000	1.9732	2.2924	2.5075	2.6680	2.7948	2.8992	2.9876	3.0642	3.1316
	1.1200	0.9867	0.8837	0.8162	0.7672	0.7300	0.7011	0.6776	0.6579	0.6407
	1.4000	0.8268	0.5727	0.4226	0.3219	0.2494	0.1950	0.1532	0.1207	0.0952
	1.1200	0.5961	0.4096	0.3101	0.2457	0.1987	0.1618	0.1318	0.1070	0.0865
8	1.6000	2.2162	2.5560	2.7848	2.9552	3.0899	3.2008	3.2949	3.3762	3.4478
	1.2800	1.1065	0.9876	0.9103	0.8551	0.8136	0.7808	0.7538	0.7308	0.7110
	1.6000	0.9838	0.7075	0.5423	0.4298	0.3473	0.2840	0.2338	0.1933	0.1603
	1.2800	0.6942	0.4866	0.3768	0.3069	0.2570	0.2183	0.1866	0.1596	0.1364
9	1.8000	2.4563	2.8156	3.0571	3.2369	3.3790	3.4961	3.5952	3.6809	3.7562
	1.4400	1.2259	1.0907	1.0036	0.9421	0.8956	0.8588	0.8286	0.8032	0.7817
	1.8000	1.1437	0.8468	0.6677	0.5447	0.4536	0.3828	0.3259	0.2791	0.2399
	1.4400	0.7927	0.5638	0.4431	0.3669	0.3134	0.2728	0.2402	0.2127	0.1888
10	2.0000	2.6941	3.0719	3.3254	3.5140	3.6632	3.7859	3.8898	3.9796	4.0586
	1.6000	1.3449	1.1931	1.0963	1.0281	0.9767	0.9361	0.9031	0.8755	0.8521
	2.0000	1.3059	0.9897	0.7977	0.6649	0.5661	0.4888	0.4262	0.3742	0.3301
	1.6000	0.8916	0.6413	0.5094	0.4263	0.3684	0.3252	0.2911	0.2631	0.2391
11	2.2000	2.9299	3.3252	3.5903	3.7873	3.9430	4.0711	4.1796	4.2734	4.3559
	1.7600	1.4636	1.2949	1.1882	1.1134	1.0572	1.0129	0.9770	0.9470	0.9215
	2.2000	1.4701	1.1356	0.9315	0.7897	0.6836	0.6003	0.5325	0.4760	0.4279
	1.7600	0.9909	0.7191	0.5760	0.4858	0.4230	0.3763	0.3400	0.3106	0.2860
12	2.4000	3.1640	3.5762	2.8521	4.0572	4.2192	4.3525	4.4653	4.5629	4.6487
	1.9200	1.5820	1.3962	1.2796	1.1981	1.1370	1.0891	1.0502	1.0176	0.9899
	2.4000	1.6360	1.2842	1.0685	0.9181	0.8052	0.7162	0.6437	0.5830	0.5312
	1.9200	1.0906	0.7974	0.6428	0.5455	0.4777	0.4273	0.3880	0.3565	0.3305
13	2.6000	3.3967	3.8249	4.1114	4.3242	4.4922	4.6304	4.7474	4.8486	4.9375
	2.0800	1.7001	1.4971	1.3705	1.2823	1.2164	1.1647	1.1227	1.0876	1.0578
	2.6000	1.8033	1.4350	1.2083	1.0498	0.9304	0.8361	0.7589	0.6942	0.6389
	2.0800	1.1906	0.8759	0.7100	0.6055	0.5327	0.4785	0.4362	0.4022	0.3742
14	2.8000	3.6280	4.0718	4.3683	4.5885	4.7624	4.9053	5.0263	5.1309	5.2229
	2.2400	1.8179	1.5975	1.4609	1.3660	1.2952	1.2398	1.1947	1.1572	1.1254
	2.8000	1.9720	1.5877	1.3506	1.1844	1.0588	0.9593	0.8778	0.8093	0.7506
	2.2400	1.2909	0.9548	0.7775	0.6658	0.5880	0.5300	0.4848	0.4483	0.4181
15	3.0000	3.8582	4.3169	4.6232	4.8506	5.0300	5.1775	5.3024	5.4103	5.5052
	2.4000	1.9354	1.6976	1.5510	1.4494	1.3737	1.3144	1.2664	1.2264	1.1925
	3.0000	2.1418	1.7422	1.4951	1.3214	1.1899	1.0856	0.9999	0.9277	0.8658
	2.4000	1.3915	1.0341	0.8453	0.7264	0.6435	0.5817	0.5336	0.4947	0.4626

A missing entry in the above table denotes a zero correct to four decimal places.

Table I. (con't)

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.3

N/M	1	2	3	4	5	6	7	8	9	10
1	0.3000	0.5100	0.6570	0.7599	0.8319	0.8824	0.9176	0.9424	0.9596	0.9718
	0.2100	0.2499	0.2254	0.1825	0.1398	0.1038	0.0756	0.0543	0.0387	0.0274
	0.3000	0.0900	0.0270	0.0081	0.0024	0.0007	0.0002	0.0001	0.0000	0.0000
	0.2100	0.0819	0.0263	0.0080	0.0024	0.0007	0.0002	0.0001	0.0000	0.0000
2	0.6000	0.9318	1.1288	1.2566	1.3477	1.4183	1.4765	1.5264	1.5704	1.6098
	0.4200	0.4073	0.3475	0.3061	0.2833	0.2710	0.2630	0.2559	0.2483	0.2395
	0.6000	0.2682	0.1334	0.0677	0.0345	0.0176	0.0090	0.0046	0.0023	0.0012
	0.4200	0.2125	0.1170	0.0633	0.0333	0.0173	0.0089	0.0046	0.0023	0.0012
3	0.9000	1.3210	1.5566	1.7121	1.8270	1.9176	1.9917	2.0537	2.1064	2.1517
	0.6300	0.5598	0.4852	0.4401	0.4084	0.3818	0.3580	0.3371	0.3191	0.3042
	0.9000	0.4790	0.2937	0.1885	0.1229	0.0805	0.0529	0.0347	0.0228	0.0150
	0.6300	0.3458	0.2277	0.1573	0.1087	0.0742	0.0501	0.0335	0.0223	0.0148
4	1.2000	1.6942	1.9641	2.1434	2.2756	2.3790	2.4630	2.5335	2.5940	2.6471
	0.8400	0.7129	0.6200	0.5609	0.5176	0.4846	0.4594	0.4400	0.4247	0.4122
	1.2000	0.7058	0.4816	0.3482	0.2585	0.1943	0.1469	0.1114	0.0846	0.0642
	0.8400	0.4787	0.3365	0.2566	0.2020	0.1601	0.1266	0.0994	0.0776	0.0602
5	1.5000	2.0578	2.3585	2.5580	2.7049	2.8201	2.9144	2.9938	3.0623	3.1222
	1.0500	0.8661	0.7519	0.6794	0.6286	0.5911	0.5619	0.5380	0.5174	0.4993
	1.5000	0.9422	0.6852	0.5293	0.4220	0.3426	0.2810	0.2319	0.1920	0.1594
	1.0500	0.6117	0.4433	0.3511	0.2911	0.2473	0.2125	0.1830	0.1575	0.1351
6	1.8000	2.4148	2.7433	2.9611	3.1216	3.2476	3.3507	3.4375	3.5122	3.5775
	1.2600	1.0189	0.8823	0.7970	0.7378	0.6936	0.6586	0.6301	0.6063	0.5864
	1.8000	1.1852	0.8990	0.7235	0.6015	0.5102	0.4385	0.3802	0.3316	0.2903
	1.2600	0.7452	0.5497	0.4434	0.3752	0.3270	0.2906	0.2613	0.2365	0.2146
7	2.1000	2.7670	3.1211	3.3557	3.5286	3.6643	3.7754	3.8689	3.9494	4.0200
	1.4700	1.1712	1.0118	0.9133	0.8452	0.7945	0.7549	0.7230	0.6967	0.6744
	2.1000	1.4330	1.1203	0.9271	0.7918	0.6900	0.6096	0.5439	0.4889	0.4418
	1.4700	0.8791	0.6564	0.5356	0.4581	0.4035	0.3627	0.3308	0.3051	0.2835
8	2.4000	3.1153	3.4934	3.7435	3.9279	4.0727	4.1911	4.2909	4.3769	4.4523
	1.6800	1.3231	1.1404	1.0287	0.9518	0.8948	0.8504	0.8146	0.7874	0.7593
	2.4000	1.6847	1.3474	1.1379	0.9905	0.8790	0.7906	0.7181	0.6572	0.6050
	1.6800	1.0136	0.7636	0.6282	0.5415	0.4802	0.4342	0.3981	0.3691	0.3452
9	2.7000	3.4606	3.8610	4.1259	4.3210	4.4742	4.5996	4.7053	4.7963	4.8761
	1.8900	1.4745	1.2685	1.1434	1.0576	0.9942	0.9448	0.9049	0.8718	0.8438
	2.7000	1.9394	1.5792	1.3546	1.1960	1.0755	0.9797	0.9003	0.8343	0.7773
	1.8900	1.1484	0.8712	0.7211	0.6251	0.5574	0.5065	0.4664	0.4339	0.4069
10	3.0000	3.8034	4.2250	4.5036	4.7089	4.8701	5.0020	5.1132	5.2090	5.2929
	2.1000	1.6256	1.3959	1.2575	1.1628	1.0929	1.0386	0.9949	0.9585	0.9279
	3.0000	2.1966	1.8149	1.5761	1.4070	1.2782	1.1754	1.0907	1.0191	0.9574
	2.1000	1.2837	0.9791	0.8144	0.7090	0.6348	0.5790	0.5352	0.4997	0.4700
11	3.3000	4.1439	4.5857	4.8775	5.0924	5.2611	5.3992	5.5156	5.6160	5.7039
	2.3100	1.7763	1.5230	1.3710	1.2674	1.1910	1.1318	1.0840	1.0445	1.0110
	3.3000	2.4561	2.0539	1.8017	1.6226	1.4859	1.3767	1.2865	1.2101	1.1442
	2.3100	1.4192	1.0875	0.9081	0.7933	0.7124	0.6516	0.6040	0.5654	0.5334
12	3.6000	4.4826	4.9436	5.2480	5.4721	5.6480	5.7920	5.9134	6.0180	6.1098
	2.5200	1.9268	1.6496	1.4841	1.3715	1.2887	1.2244	1.1727	1.1300	1.0938
	3.6000	2.7174	2.2957	2.0307	1.8421	1.6981	1.5827	1.4873	1.4064	1.3365
	2.5200	1.5551	1.1962	1.0021	0.8778	0.7903	0.7245	0.6730	0.6312	0.5965
13	3.9000	4.8197	5.2992	5.6155	5.8484	6.0313	6.1809	6.3071	6.4158	6.5112
	2.7300	2.0769	1.7758	1.5967	1.4752	1.3859	1.3167	1.2610	1.2150	1.1761
	3.9000	2.9803	2.5399	2.2627	2.0652	1.9140	1.7928	1.6924	1.6072	1.5336
	2.7300	1.6912	1.3052	1.0963	0.9627	0.8684	0.7977	0.7422	0.6972	0.6598
14	4.2000	5.1554	5.6525	5.9804	6.2218	6.4113	6.5664	6.6971	6.8098	6.9086
	2.9400	2.2269	1.9017	1.7091	1.5785	1.4827	1.4085	1.3489	1.2996	1.2580
	4.2000	3.2446	2.7864	2.4974	2.2912	2.1333	2.0065	1.9014	1.8121	1.7348
	2.9400	1.8276	1.4144	1.1909	1.0478	0.9469	0.8711	0.8117	0.7635	0.7234
15	4.5000	5.4897	6.0040	6.3430	6.5926	6.7885	6.9488	7.0839	7.2004	7.3026
	3.1500	2.3766	2.0274	1.8211	1.6815	1.5792	1.5001	1.4365	1.3839	1.3396
	4.5000	3.5102	3.0348	2.7345	2.5200	2.3555	2.2234	2.1137	2.0205	1.9398
	3.1500	1.9642	1.5239	1.2857	1.1332	1.0256	0.9448	0.8814	0.8300	0.7873

Table 1. (con't)

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.4

N/M	1	2	3	4	5	6	7	8	9	10
1	0.4000	0.6400	0.7840	0.8704	0.9222	0.9533	0.9720	0.9832	0.9899	0.9940
	0.2400	0.2304	0.1693	0.1128	0.0717	0.0445	0.0272	0.0165	0.0100	0.0060
	0.4000	0.1600	0.0640	0.0256	0.0102	0.0041	0.0016	0.0007	0.0003	0.0001
	0.2400	0.1344	0.0599	0.0249	0.0101	0.0041	0.0016	0.0007	0.0003	0.0001
2	0.8000	1.1648	1.3606	1.4853	1.5757	1.6465	1.7041	1.7518	1.7917	1.8251
	0.4800	0.3968	0.3239	0.2834	0.2564	0.2329	0.2099	0.1871	0.1651	0.1444
	0.8000	0.4352	0.2662	0.1684	0.1075	0.0687	0.0440	0.0281	0.0180	0.0115
	0.4800	0.2970	0.2035	0.1414	0.0961	0.0640	0.0421	0.0274	0.0177	0.0114
3	1.2000	1.6573	1.8978	2.0540	2.1669	2.2534	2.3226	2.3798	2.4284	2.4708
	0.7200	0.5664	0.4719	0.4124	0.3694	0.3377	0.3146	0.2977	0.2852	0.2753
	1.2000	0.7427	0.5253	0.3932	0.3016	0.2341	0.1827	0.1430	0.1120	0.0878
	0.7200	0.4553	0.3376	0.2694	0.2215	0.1831	0.1507	0.1230	0.0996	0.0801
4	1.6000	2.1342	2.4124	2.5934	2.7248	2.8268	2.9095	2.9787	3.0378	3.0891
	0.9600	0.7361	0.6153	0.5415	0.4913	0.4545	0.4252	0.4007	0.3795	0.3611
	1.6000	1.0658	0.8097	0.6508	0.5396	0.4557	0.3894	0.3352	0.2897	0.2512
	0.9600	0.6131	0.4663	0.3831	0.3288	0.2900	0.2597	0.2343	0.2118	0.1912
5	2.0000	2.6014	2.9127	3.1156	3.2634	3.3783	3.4714	3.5491	3.6156	3.6736
	1.2000	0.9055	0.7578	0.6688	0.6080	0.5631	0.5282	0.5006	0.4781	0.4595
	2.0000	1.3986	1.1085	0.9269	0.7985	0.7013	0.6241	0.5608	0.5074	0.4615
	1.2000	0.7711	0.5946	0.4953	0.4301	0.3835	0.3486	0.3214	0.2996	0.2814
6	2.4000	3.0618	3.4031	3.6257	3.7880	3.9143	4.0169	4.1029	4.1766	4.2409
	1.4400	1.0745	0.8993	0.7949	0.7240	0.6722	0.6321	0.5999	0.5730	0.5500
	2.4000	1.7382	1.4176	1.2158	1.0723	0.9629	0.8756	0.8036	0.7429	0.6907
	1.4400	0.9295	0.7230	0.6075	0.5321	0.4780	0.4368	0.4041	0.3775	0.3556
7	2.8000	3.5172	3.8860	4.1266	4.3022	4.4390	4.5502	4.6435	4.7234	4.7932
	1.6800	1.2432	1.0402	0.9202	0.8392	0.7798	0.7340	0.6972	0.6669	0.6415
	2.8000	2.0828	1.7344	1.5141	1.3570	1.2368	1.1405	1.0608	0.9932	0.9349
	1.6800	1.0881	0.8515	0.7197	0.6339	0.5726	0.5262	0.4893	0.4591	0.4337
8	3.2000	3.9686	4.3629	4.6203	4.8082	4.9547	5.0739	5.1739	5.2597	5.3347
	1.9200	1.4115	1.1807	1.0450	0.9536	0.8869	0.8355	0.7943	0.7604	0.7317
	3.2000	2.4314	2.0572	1.8199	1.6502	1.5201	1.4156	1.3289	1.2553	1.1917
	1.9200	1.2471	0.9803	0.8321	0.7356	0.6668	0.6148	0.5737	0.5402	0.5123
9	3.6000	4.4167	4.8351	5.1081	5.3075	5.4631	5.5897	5.6960	5.7873	5.8670
	2.1600	1.5796	1.3207	1.1693	1.0676	0.9935	0.9364	0.8906	0.8530	0.8212
	3.6000	2.7832	2.3843	2.1317	1.9503	1.8108	1.6987	1.6056	1.5264	1.4578
	2.1600	1.4062	1.1093	0.9447	0.8376	0.7613	0.7035	0.6578	0.6206	0.5896
10	4.0000	4.8622	5.3032	5.5910	5.8014	5.9655	6.0991	6.2113	6.3077	6.3919
	2.4000	1.7475	1.4605	1.2932	1.1812	1.0996	1.0368	0.9866	0.9452	0.9104
	4.0000	3.1378	2.7165	2.4484	2.2560	2.1078	1.9886	1.8894	1.8049	1.7317
	2.4000	1.5656	1.2386	1.0574	2.9397	0.8558	0.7924	0.7422	0.7014	0.6672
11	4.4000	5.3054	5.7680	6.0698	6.2905	6.4627	6.6030	6.7208	6.8220	6.9105
	2.6400	1.9152	1.5999	1.4169	1.2944	1.2053	1.1369	1.0821	1.0371	0.9991
	4.4000	3.4946	3.0516	2.7693	2.5664	2.4101	2.2841	2.1792	2.0898	2.0122
	2.6400	1.7251	1.3680	1.1703	1.0420	0.9505	0.8813	0.8267	0.7822	0.7451
12	4.8000	5.7467	6.2298	6.5451	6.7755	6.9555	7.1021	7.2252	7.3311	7.4236
	2.8800	2.0828	1.7391	1.5402	1.4074	1.3108	1.2367	1.1774	1.1286	1.0876
	4.8000	3.8533	3.3897	3.0939	2.8811	2.7169	2.5845	2.4742	2.3801	2.2983
	2.8800	1.8848	1.4976	1.2834	1.1444	1.0454	0.9704	0.9113	0.8631	0.8229
13	5.2000	6.1862	6.6890	7.0172	7.2571	7.4444	7.5970	7.7253	7.8355	7.9319
	3.1200	2.2502	1.8782	1.6634	1.5201	1.4161	1.3362	1.2724	1.2199	1.1758
	5.2000	4.2138	3.7304	3.4216	3.1993	3.0277	2.8891	2.7736	2.6750	2.5893
	3.1200	2.0447	1.6273	1.3967	1.2470	1.1404	1.0597	0.9960	0.9441	0.9008
14	5.6000	6.6242	7.1460	7.4865	7.7354	7.9298	8.0883	8.2215	8.3359	8.4360
	3.3600	2.4174	2.0170	1.7863	1.6326	1.5211	1.4355	1.3671	1.3110	1.2637
	5.6000	4.5758	4.0734	3.7522	3.5207	3.3419	3.1975	3.0770	2.9741	2.8847
	3.3600	2.2046	1.7572	1.5101	1.3497	1.2355	1.1491	1.0808	1.0253	0.9789
15	6.0000	7.0608	7.6009	7.9533	8.2110	8.4122	8.5763	8.7142	8.8327	8.9364
	3.6000	2.5846	2.1557	1.9091	1.7449	1.6259	1.5346	1.4617	1.4018	1.3514
	6.0000	4.9392	4.4184	4.0852	3.8449	3.6592	3.5091	3.3839	3.2769	3.1838
	3.6000	2.3647	1.8873	1.6236	1.4526	1.3307	1.2386	1.1658	1.1065	1.0570

Table I. (con't)

Mean and Variance of the Largest (Top two entries) and the Smallest (Bottom two entries) of M Order Statistics from a Binomial Population with Parameter p and N trials.

p=0.5

N/M	1	2	3	4	5	6	7	8	9	10
1	0.5000	0.7500	0.8750	0.9375	0.9687	0.9844	0.9922	0.9961	0.9980	0.9990
	0.2500	0.1875	0.1094	0.0586	0.0303	0.0154	0.0078	0.0039	0.0019	0.0010
	0.5000	0.2500	0.1250	0.0625	0.0312	0.0156	0.0078	0.0039	0.0020	0.0010
	0.2500	0.1875	0.1094	0.0586	0.0303	0.0154	0.0078	0.0039	0.0019	0.0010
2	1.0000	1.3750	1.5625	1.6797	1.7617	1.8218	1.8665	1.8999	1.9249	1.9437
	0.5000	0.3594	0.2773	0.2255	0.1835	0.1469	0.1158	0.0901	0.0695	0.0531
	1.0000	0.6250	0.4375	0.3203	0.2383	0.1782	0.1335	0.1001	0.0751	0.0563
	0.5000	0.3594	0.2773	0.2255	0.1835	0.1469	0.1158	0.0901	0.0695	0.0531
3	1.5000	1.9687	2.2031	2.3511	2.4558	2.5356	2.5995	2.6525	2.6974	2.7359
	0.7500	0.5303	0.4197	0.3538	0.3107	0.2800	0.2557	0.2346	0.2149	0.1963
	1.5000	1.0312	0.7969	0.6489	0.5442	0.4644	0.4005	0.3475	0.3026	0.2641
	0.7500	0.5303	0.4197	0.3538	0.3107	0.2800	0.2557	0.2346	0.2149	0.1963
4	2.0000	2.5469	2.8203	2.9946	3.1192	3.2145	3.2906	3.3533	3.4062	3.4519
	1.0000	0.7009	0.5615	0.4796	0.4241	0.3834	0.3525	0.3287	0.3100	0.2949
	2.0000	1.4531	1.1797	1.0054	0.8808	0.7855	0.7094	0.6467	0.5938	0.5481
	1.0000	0.7009	0.5615	0.4796	0.4241	0.3834	0.3525	0.3287	0.3099	0.2949
5	2.5000	3.1152	3.4229	3.6197	3.7612	3.8701	3.9577	4.0305	4.0923	4.1457
	1.2500	0.8715	0.7023	0.6041	0.5388	0.4914	0.4547	0.4250	0.4002	0.3791
	2.5000	1.8848	1.5771	1.3803	1.2388	1.1299	1.0423	0.9695	0.9077	0.8543
	1.2500	0.8715	0.7023	0.6041	0.5388	0.4914	0.4547	0.4250	0.4002	0.3791
6	3.0000	3.6768	4.0151	4.2323	4.3888	4.5096	4.6069	4.6879	4.7569	4.8169
	1.5000	1.0420	0.8428	0.7281	0.6518	0.5966	0.5544	0.5210	0.4937	0.4707
	3.0000	2.3232	1.9849	1.7677	1.6112	1.4904	1.3931	1.3121	1.2431	1.1831
	1.5000	1.0420	0.8428	0.7281	0.6518	0.5966	0.5544	0.5210	0.4937	0.4707
7	3.5000	4.2332	4.5997	4.8354	5.0056	5.1372	5.2435	5.3321	5.4077	5.4734
	1.7500	1.2125	0.9831	0.8517	0.7647	0.7018	0.6536	0.6152	0.5837	0.5573
	3.5000	2.7668	2.4003	2.1646	1.9944	1.8628	1.7565	1.6679	1.5923	1.5266
	1.7500	1.2125	0.9831	0.8517	0.7647	0.7018	0.6536	0.6152	0.5837	0.5573
8	4.0000	4.7855	5.1783	5.4311	5.6140	5.7556	5.8701	5.9636	6.0473	6.1183
	2.0000	1.3830	1.1233	0.9752	0.8772	0.8065	0.7526	0.7097	0.6745	0.6450
	4.0000	3.2145	2.8217	2.5689	2.3860	2.2444	2.1299	2.0344	1.9527	1.8817
	2.0000	1.3830	1.1233	0.9752	0.8772	0.8065	0.7526	0.7097	0.6745	0.6450
9	4.5000	5.3346	5.7519	6.0208	6.2156	6.3664	6.4886	6.5907	6.6779	6.7539
	2.2500	1.5534	1.2634	1.0985	0.9896	0.9111	0.8512	0.8036	0.7646	0.7318
	4.5000	3.6654	3.2481	2.9792	2.7844	2.6336	2.5114	2.4093	2.3221	2.2461
	2.2500	1.5534	1.2634	1.0985	0.9896	0.9111	0.8512	0.8036	0.7646	0.7318
10	5.0000	5.8810	6.3215	6.6056	6.8114	6.9711	7.1004	7.2086	7.3011	7.3817
	2.5000	1.7239	1.4035	1.2217	1.1019	1.0156	0.9498	0.8974	0.8546	0.8186
	5.0000	4.1190	3.6785	3.3944	3.1886	3.0289	2.8996	2.7914	2.6989	2.6183
	2.5000	1.7239	1.4035	1.2217	1.1019	1.0156	0.9498	0.8974	0.8546	0.8186
11	5.5000	6.4250	6.8875	7.1860	7.4025	7.5704	7.7066	7.8205	7.9180	8.0030
	2.7500	1.8943	1.5435	1.3449	1.2141	1.1200	1.0482	0.9912	0.9444	0.9053
	5.5000	4.5750	4.1124	3.8140	3.5975	3.4296	3.2934	3.1795	3.0820	2.9970
	2.7500	1.8943	1.5435	1.3449	1.2141	1.1200	1.0482	0.9912	0.9444	0.9053
12	6.0000	6.9671	7.4506	7.7628	7.9894	8.1652	8.3079	8.4272	8.5295	8.6186
	3.0000	2.0648	1.6835	1.4681	1.3263	1.2242	1.1465	1.0848	1.0342	0.9919
	6.0000	5.0329	4.5494	4.2372	4.0106	3.8348	3.6921	3.5727	3.4705	3.3814
	3.0000	2.0648	1.6835	1.4681	1.3263	1.2242	1.1465	1.0848	1.0342	0.9919
13	6.5000	7.5074	8.0111	8.3364	8.5726	8.7560	8.9049	9.0295	9.1363	9.2294
	3.2500	2.2352	1.8235	1.5912	1.4384	1.3285	1.2448	1.1784	1.1239	1.0784
	6.5000	5.4926	4.9889	4.6636	4.4274	4.2440	4.0951	3.9705	3.8637	3.7706
	3.2500	2.2352	1.8235	1.5912	1.4384	1.3285	1.2448	1.1784	1.1239	1.0784
14	7.0000	8.0461	8.5692	8.9072	9.1526	9.3433	9.4981	9.6278	9.7389	9.8358
	3.5000	2.4056	1.9634	1.7142	1.5504	1.4328	1.3430	1.2718	1.2136	1.1648
	7.0000	5.9539	5.4308	5.0928	4.8474	4.6567	4.5019	4.3722	4.2611	4.1642
	3.5000	2.4056	1.9634	1.7142	1.5504	1.4328	1.3430	1.2718	1.2136	1.1648
15	7.5000	8.5835	9.1252	9.4754	9.7298	9.9275	10.0880	10.2225	10.3378	10.4383
	3.7500	2.5761	2.1034	1.8373	1.6625	1.5369	1.4412	1.3654	1.3032	1.2512
	7.5000	6.4165	5.8748	5.5246	5.2702	5.0725	4.9120	4.7775	4.6622	4.5617
	3.7500	2.5761	2.1034	1.8373	1.6625	1.5369	1.4412	1.3654	1.3032	1.2512

Table II.

The cumulative distribution of the largest of M order statistics from a binomial population with parameter p and N trials.

p=.25

	M 1	2	3	4	5	6	7	8	9	10
N=1 X=0	0.75000	0.56250	0.42187	0.31641	0.23730	0.17798	0.13348	0.10011	0.07508	0.05631
N=2 X=0	0.56250	0.31641	0.17798	0.10011	0.05631	0.03168	0.01782	0.01002	0.00564	0.00317
1	0.93750	0.87891	0.82397	0.77248	0.72420	0.67893	0.63650	0.59672	0.55942	0.52446
N=3 X=0	0.42187	0.17798	0.07508	0.03168	0.01336	0.00564	0.00238	0.00100	0.00042	0.00018
1	0.84375	0.71191	0.60068	0.50682	0.42763	0.36081	0.30444	0.25687	0.21673	0.18287
2	0.98437	0.96899	0.95385	0.93895	0.92428	0.90984	0.89562	0.88163	0.86785	0.85429
N=4 X=0	0.31641	0.10011	0.03168	0.01002	0.00317	0.00100	0.00032	0.00010	0.00003	0.00001
1	0.73828	0.54506	0.40241	0.29709	0.21934	0.16193	0.11955	0.08826	0.06516	0.04811
2	0.94922	0.90102	0.85526	0.81183	0.77060	0.73147	0.69433	0.65907	0.62560	0.59383
3	0.99609	0.99220	0.98833	0.98447	0.98062	0.97679	0.97297	0.96917	0.96539	0.96162
N=5 X=0	0.23730	0.05631	0.01336	0.00317	0.00075	0.00018	0.00004	0.00001		
1	0.63281	0.40045	0.25341	0.16036	0.10148	0.06422	0.04064	0.02572	0.01627	0.01030
2	0.89648	0.80368	0.72049	0.64591	0.57905	0.51911	0.46537	0.41720	0.37401	0.33530
3	0.98437	0.96899	0.95385	0.93895	0.92428	0.90984	0.89562	0.88163	0.86785	0.85429
4	0.99902	0.99805	0.99707	0.99610	0.99513	0.99415	0.99318	0.99221	0.99124	0.99028
N=6 X=0	0.17798	0.03168	0.00564	0.00100	0.00018	0.00003	0.00001			
1	0.53394	0.28509	0.15222	0.08127	0.04340	0.02317	0.01237	0.00661	0.00353	0.00188
2	0.83057	0.68984	0.57296	0.47588	0.39525	0.32828	0.27266	0.22646	0.18809	0.15622
3	0.96240	0.92622	0.89139	0.85788	0.82563	0.79458	0.76471	0.73596	0.70829	0.68166
4	0.99536	0.99074	0.98615	0.98157	0.97702	0.97249	0.96798	0.96349	0.95902	0.95457
5	0.99976	0.99951	0.99927	0.99902	0.99878	0.99854	0.99829	0.99805	0.99780	0.99756
N=7 X=0	0.13348	0.01782	0.00238	0.00032	0.00004	0.00001				
1	0.44495	0.19798	0.08809	0.03919	0.01744	0.00776	0.00345	0.00154	0.00068	0.00030
2	0.75641	0.57215	0.43278	0.32736	0.24762	0.18730	0.14168	0.10716	0.08106	0.06131
3	0.92944	0.86386	0.80291	0.74626	0.69361	0.64467	0.59918	0.55691	0.51761	0.48109
4	0.98712	0.97441	0.96186	0.94947	0.93725	0.92517	0.91326	0.90150	0.88989	0.87843
5	0.99866	0.99732	0.99598	0.99464	0.99330	0.99197	0.99064	0.98931	0.98798	0.98665
6	0.99994	0.99988	0.99982	0.99976	0.99969	0.99963	0.99957	0.99951	0.99945	0.99939
N=8 X=0	0.10011	0.01002	0.00100	0.00010	0.00001					
1	0.36708	0.13475	0.04946	0.01816	0.00667	0.00245	0.00090	0.00033	0.00012	0.00004
2	0.67854	0.46042	0.31242	0.21199	0.14384	0.09760	0.06623	0.04494	0.03049	0.02069
3	0.88618	0.78532	0.69594	0.61673	0.54654	0.48433	0.42921	0.38036	0.33707	0.29870
4	0.97270	0.94615	0.92032	0.89520	0.87076	0.84699	0.82387	0.80138	0.77950	0.75822
5	0.99577	0.99156	0.98737	0.98320	0.97904	0.97491	0.97079	0.96668	0.96260	0.95853
6	0.99962	0.99924	0.99886	0.99847	0.99809	0.99771	0.99733	0.99695	0.99657	0.99619
7	0.99998	0.99997	0.99995	0.99994	0.99992	0.99991	0.99989	0.99988	0.99986	0.99985
N=9 X=0	0.07508	0.00564	0.00042	0.00003						
1	0.30034	0.09020	0.02709	0.00814	0.00244	0.00073	0.00022	0.00007	0.00002	0.00001
2	0.60068	0.36081	0.21673	0.13019	0.07820	0.04697	0.02822	0.01695	0.01018	0.00612
3	0.83427	0.69601	0.58067	0.48443	0.40415	0.33717	0.28129	0.23468	0.19579	0.16334
4	0.95107	0.90454	0.86028	0.81819	0.77816	0.74009	0.70388	0.66944	0.63558	0.60553
5	0.99001	0.98011	0.97032	0.96062	0.95102	0.94151	0.93210	0.92279	0.91356	0.90443
6	0.99866	0.99732	0.99598	0.99464	0.99330	0.99197	0.99064	0.98931	0.98798	0.98665
7	0.99989	0.99979	0.99968	0.99957	0.99947	0.99936	0.99925	0.99915	0.99904	0.99893
8		0.99999	0.99999	0.99998	0.99998	0.99998	0.99997	0.99997	0.99997	0.99996
N=10 X=0	0.05631	0.00317	0.00018	0.00001						
1	0.24403	0.05955	0.01453	0.00355	0.00087	0.00021	0.00005	0.00001		
2	0.52559	0.27625	0.14519	0.07631	0.04011	0.02108	0.01108	0.00582	0.00306	0.00161
3	0.77588	0.60198	0.46706	0.36238	0.28116	0.21815	0.16926	0.13132	0.10189	0.07905
4	0.92187	0.84985	0.78345	0.72224	0.66582	0.61380	0.56585	0.52164	0.48088	0.44331
5	0.98027	0.96093	0.94198	0.92339	0.90518	0.88732	0.86982	0.85266	0.83583	0.81935
6	0.99649	0.99300	0.98952	0.98605	0.98259	0.97915	0.97572	0.97230	0.96889	0.96549
7	0.99958	0.99917	0.99875	0.99834	0.99792	0.99751	0.99709	0.99668	0.99626	0.99585
8	0.99997	0.99994	0.99991	0.99988	0.99985	0.99982	0.99979	0.99976	0.99973	0.99970
9					0.99999	0.99999	0.99999	0.99999	0.99999	0.99999

A missing entry in the above table denotes a 0 or a 1 correct to five decimal places.

Table II. (con't)

The cumulative distribution of the largest of M order statistics from a binomial population with parameter p and N trials.

p=.50

	M	1	2	3	4	5	6	7	8	9	10
N=1	X=0	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00098
N=2	X=0	0.25000	0.06250	0.01562	0.00391	0.00098	0.00024	0.00006	0.00002		
	1	0.75000	0.56250	0.42187	0.31641	0.23730	0.17798	0.13348	0.10011	0.07508	0.05631
N=3	X=0	0.12500	0.01562	0.00195	0.00024	0.00003					
	1	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00092
	2	0.87500	0.76562	0.66992	0.58618	0.51291	0.44880	0.39270	0.34361	0.30066	0.26308
N=4	X=0	0.06250	0.00391	0.00024	0.00002						
	1	0.31250	0.09766	0.03052	0.00954	0.00298	0.00093	0.00029	0.00009	0.00003	0.00001
	2	0.68750	0.47266	0.32495	0.22340	0.15359	0.10559	0.07260	0.04991	0.03431	0.02359
	3	0.93750	0.87891	0.82397	0.77248	0.72420	0.67893	0.63650	0.59672	0.55942	0.52446
N=5	X=0	0.03125	0.00098	0.00003							
	1	0.18750	0.03516	0.00659	0.00124	0.00023	0.00004	0.00001			
	2	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00098
	3	0.81250	0.66016	0.53638	0.43581	0.35409	0.28770	0.23376	0.18993	0.15432	0.12538
	4	0.96875	0.93848	0.90915	0.88074	0.85322	0.82655	0.80072	0.77570	0.75146	0.72798
N=6	X=0	0.01562	0.00024								
	1	0.10937	0.01196	0.00131	0.00014	0.00002					
	2	0.34375	0.11816	0.04062	0.01396	0.00480	0.00165	0.00057	0.00019	0.00007	0.00002
	3	0.65625	0.43066	0.28262	0.18547	0.12172	0.07988	0.05242	0.03440	0.02257	0.01481
	4	0.89062	0.79321	0.70646	0.62919	0.56037	0.49908	0.44449	0.39588	0.35258	0.31401
	5	0.98437	0.96899	0.95385	0.93895	0.92428	0.90984	0.89562	0.88163	0.86785	0.85429
N=7	X=0	0.00781	0.00006								
	1	0.06250	0.00391	0.00024	0.00002						
	2	0.22656	0.05133	0.01163	0.00263	0.00060	0.00014	0.00003	0.00001		
	3	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00098
	4	0.77344	0.59821	0.46267	0.35785	0.27677	0.21407	0.16557	0.12806	0.09904	0.07660
	5	0.93750	0.87891	0.82397	0.77248	0.72420	0.67893	0.63650	0.59672	0.55942	0.52446
	6	0.99219	0.98444	0.97674	0.96911	0.96154	0.95403	0.94658	0.93918	0.93184	0.92456
N=8	X=0	0.00391	0.00002								
	1	0.03516	0.00124	0.00004							
	2	0.14453	0.02089	0.00302	0.00044	0.00006	0.00001				
	3	0.36328	0.13197	0.04794	0.01742	0.00633	0.00230	0.00084	0.00030	0.00011	0.00004
	4	0.63672	0.40541	0.25813	0.16436	0.10465	0.06663	0.04243	0.02701	0.01720	0.01095
	5	0.85547	0.73183	0.62605	0.53557	0.45816	0.39194	0.33530	0.28684	0.24538	0.20991
	6	0.96484	0.93092	0.89820	0.86662	0.83615	0.80676	0.77839	0.75103	0.72462	0.69915
	7	0.99609	0.99220	0.98833	0.98447	0.98062	0.97679	0.97297	0.96917	0.96539	0.96162
N=9	X=0	0.00195									
	1	0.01953	0.00038	0.00001							
	2	0.08984	0.00807	0.00073	0.00007	0.00001					
	3	0.25391	0.06447	0.01637	0.00416	0.00106	0.00027	0.00007	0.00002		
	4	0.50000	0.25000	0.12500	0.06250	0.03125	0.01562	0.00781	0.00391	0.00195	0.00098
	5	0.74609	0.55666	0.41532	0.30987	0.23119	0.17249	0.12869	0.09602	0.07164	0.05345
	6	0.91016	0.82838	0.75396	0.68622	0.62457	0.56845	0.51738	0.47090	0.42859	0.39009
	7	0.98047	0.96132	0.94254	0.92413	0.90608	0.88839	0.87104	0.85402	0.83734	0.82099
	8	0.99805	0.99610	0.99415	0.99221	0.99027	0.98834	0.98641	0.98448	0.98256	0.98064
N=10	X=0	0.00098									
	1	0.01074	0.00012								
	2	0.05469	0.00299	0.00016	0.00001						
	3	0.17187	0.02954	0.00508	0.00087	0.00015	0.00003				
	4	0.37695	0.14209	0.05356	0.02019	0.00761	0.00287	0.00108	0.00041	0.00015	0.00006
	5	0.62305	0.38819	0.24186	0.15069	0.09389	0.05850	0.03645	0.02271	0.01415	0.00881
	6	0.82812	0.68579	0.56792	0.47031	0.38947	0.32253	0.26710	0.22119	0.18317	0.15169
	7	0.94531	0.89362	0.84475	0.79855	0.75488	0.71360	0.67457	0.63768	0.60281	0.56984
	8	0.98926	0.97863	0.96812	0.95772	0.94743	0.93725	0.92718	0.91722	0.90737	0.89762
	9	0.99902	0.99805	0.99707	0.99610	0.99513	0.99415	0.99318	0.99221	0.99124	0.99028

A missing entry in the above table denotes a 0 or a 1 correct to five decimal places.

Table II.

The cumulative distribution of the smallest of M order statistics from a binomial population with parameter p and N trials.

p=0.25

	M	1	2	3	4	5	6	7	8	9	10
N=1	X=0	0.75000	0.93750	0.98437	0.99609	0.99902	0.99976	0.99994	0.99998		
N=2	X=0	0.56250	0.80859	0.91626	0.96336	0.98397	0.99299	0.99693	0.99866	0.99941	0.99974
	1	0.93750	0.99609	0.99976	0.99998						
N=3	X=0	0.42187	0.66577	0.80677	0.88829	0.93542	0.96266	0.97841	0.98752	0.99279	0.99583
	1	0.84375	0.97559	0.99619	0.99940	0.99991	0.99999				
	2	0.98437	0.99976								
N=4	X=0	0.31641	0.53270	0.68056	0.78163	0.85072	0.89796	0.93024	0.95231	0.96740	0.97772
	1	0.73828	0.93150	0.98207	0.99531	0.99877	0.99968	0.99992	0.99998	0.99999	
	2	0.94922	0.99742	0.99987	0.99999						
	3	0.99609	0.99998								
N=5	X=0	0.23730	0.41830	0.55634	0.66162	0.74192	0.80316	0.84987	0.88550	0.91267	0.93339
	1	0.63281	0.86517	0.95049	0.98182	0.99333	0.99755	0.99910	0.99967	0.99988	0.99996
	2	0.89648	0.98928	0.99889	0.99989	0.99999					
	3	0.98437	0,99976								
	4	0.99902									
N=6	X=0	0.17798	0.32428	0.44454	0.54340	0.62467	0.69147	0.74638	0.79152	0.82862	0.85913
	1	0.53394	0.78278	0.89876	0.95282	0.97801	0.98975	0.99522	0.99777	0.99896	0.99952
	2	0.83057	0.97129	0.99514	0.99918	0.99986	0.99998				
	3	0.96240	0.99859	0.99995							
	4	0.99536	0.99998								
	5	0.99976									
N=7	X=0	0.13348	0.24915	0.34938	0.43622	0.51148	0.57669	0.63319	0.68216	0.72458	0.76135
	1	0.44495	0.69192	0.82900	0.90508	0.94732	0.97076	0.98377	0.99099	0.99500	0.99722
	2	0.75641	0.94066	0.98555	0.99648	0.99914	0.99979	0.99995	0.99999	1.00000	
	3	0.92944	0.99502	0.99965	0.99998						
	4	0.98712	0.99983								
	5	0.99866									
	6	0.99994									
N=8	X=0	0.10011	0.19020	0.27127	0.34423	0.40988	0.46896	0.52212	0.56996	0.61302	0.65176
	1	0.36708	0.59941	0.74646	0.83953	0.89844	0.93572	0.95931	0.97425	0.98370	0.98968
	2	0.67854	0.89667	0.96678	0.98932	0.99657	0.99890	0.99965	0.99989	0.99996	0.99999
	3	0.88618	0.98705	0.99853	0.99983	0.99998					
	4	0.97270	0.99925	0.99998							
	5	0.99577	0.99998								
	6	0.99962									
	7	0.99998									
N=9	X=0	0.07508	0.14453	0.20876	0.26817	0.32312	0.37395	0.42095	0.46443	0.50464	0.54184
	1	0.30034	0.51047	0.65750	0.76036	0.83234	0.88269	0.91792	0.94257	0.95982	0.97189
	2	0.60068	0.84054	0.93632	0.97457	0.98985	0.99595	0.99838	0.99935	0.99974	0.99990
	3	0.83427	0.97253	0.99545	0.99925	0.99987	0.99998				
	4	0.95107	0.99761	0.99988	0.99999						
	5	0.99001	0.99990								
	6	0.99866									
	7	0.99989									
N=10	X=0	0.05631	0.10946	0.15961	0.20693	0.25159	0.29374	0.33351	0.37104	0.40646	0.43988
	1	0.24403	0.42850	0.56796	0.67339	0.75309	0.81334	0.85889	0.89333	0.91936	0.93904
	2	0.52559	0.77494	0.89323	0.94935	0.97597	0.98860	0.99459	0.99743	0.99878	0.99942
	3	0.77588	0.94977	0.98874	0.99748	0.99943	0.99987	0.99997	0.99999		
	4	0.92187	0.99390	0.99952	0.99996						
	5	0.98027	0.99961	0.99999							
	6	0.99649	0.99999								
	7	0.99958									
	8	0.99997									

A missing entry in this table denotes 1 correct to five decimal places.

Table II.

The cumulative distribution of the smallest of M order statistics from a binomial population with parameter p and N trials.

p=0.50

	M	1	2	3	4	5	6	7	8	9	10
N=1	X=0	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
N=2	X=0	0.25000	0.43750	0.57812	0.68359	0.76270	0.82202	0.86652	0.89989	0.92492	0.94369
	1	0.75000	0.93750	0.98437	0.99609	0.99902	0.99976	0.99994	0.99998		
N=3	X=0	0.12500	0.23437	0.33008	0.41382	0.48709	0.55120	0.60730	0.65639	0.69934	0.73692
	1	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
	2	0.87500	0.98437	0.99805	0.99976	0.99997					
N=4	X=0	0.06250	0.12109	0.17603	0.22752	0.27580	0.32107	0.36350	0.40328	0.44058	0.47554
	1	0.31250	0.52734	0.67505	0.77660	0.84641	0.89441	0.92740	0.95009	0.96569	0.97641
	2	0.68750	0.90234	0.96948	0.99046	0.99702	0.99907	0.99971	0.99991	0.99997	0.99999
	3	0.93750	0.99609	0.99976	0.99998						
N=5	X=0	0.03125	0.06152	0.09085	0.11926	0.14678	0.17345	0.19928	0.22430	0.24854	0.27202
	1	0.18750	0.33984	0.46362	0.56419	0.64591	0.71230	0.76624	0.81007	0.84568	0.87462
	2	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
	3	0.81250	0.96484	0.99341	0.99876	0.99977	0.99996	0.99999			
	4	0.96875	0.99902	0.99997							
N=6	X=0	0.01562	0.03101	0.04615	0.06105	0.07572	0.09016	0.10438	0.11837	0.13215	0.14571
	1	0.10937	0.20679	0.29354	0.37081	0.43963	0.50092	0.55551	0.60412	0.64742	0.68599
	2	0.34375	0.56934	0.71738	0.81453	0.87828	0.92012	0.94758	0.96560	0.97743	0.98519
	3	0.65625	0.88184	0.95938	0.98604	0.99520	0.99835	0.99943	0.99981	0.99993	0.99998
	4	0.89062	0.98804	0.99869	0.99986	0.99998					
	5	0.98437	0.99976								
N=7	X=0	0.00781	0.01556	0.02325	0.03089	0.03846	0.04597	0.05342	0.06082	0.06815	0.07543
	1	0.06250	0.12109	0.17603	0.22752	0.27580	0.32107	0.36350	0.40328	0.44058	0.47554
	2	0.22656	0.40179	0.53733	0.64215	0.72323	0.78593	0.83443	0.97194	0.90096	0.92340
	3	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
	4	0.77344	0.94867	0.98837	0.99737	0.99940	0.99986	0.99997	0.99999		
	5	0.93750	0.99609	0.99976	0.99998						
	6	0.99219	0.99994								
N=8	X=0	0.00391	0.00780	0.01167	0.01553	0.01938	0.02321	0.02703	0.03083	0.03461	0.03838
	1	0.03516	0.06908	0.10180	0.13338	0.16385	0.19324	0.22161	0.24897	0.27538	0.30085
	2	0.14453	0.26817	0.37395	0.46443	0.54184	0.60806	0.66470	0.71316	0.75462	0.79009
	3	0.36328	0.59459	0.74187	0.83564	0.89535	0.93337	0.95757	0.97299	0.98280	0.98905
	4	0.63672	0.86803	0.95206	0.98258	0.99367	0.99770	0.99916	0.99970	0.99989	0.99996
	5	0.85547	0.97911	0.99698	0.99956	0.99994	0.99999				
	6	0.96484	0.99876	0.99996							
	7	0.99609	0.99998								
N=9	X=0	0.00195	0.00390	0.00585	0.00779	0.00973	0.01166	0.01359	0.01552	0.01744	0.01936
	1	0.01953	0.03868	0.05746	0.07587	0.09392	0.11161	0.12896	0.14598	0.16266	0.17901
	2	0.08984	0.17162	0.24604	0.31378	0.37543	0.43155	0.48262	0.52910	0.57141	0.60991
	3	0.25391	0.44334	0.58468	0.69013	0.76881	0.82751	0.87131	0.90398	0.92836	0.94655
	4	0.50000	0.75000	0.87500	0.93750	0.96875	0.98437	0.99219	0.99609	0.99805	0.99902
	5	0.74609	0.93553	0.98363	0.99584	0.99894	0.99973	0.99993	0.99998		
	6	0.91016	0.99193	0.99927	0.99993	0.99999					
	7	0.98047	0.99962	0.99999							
	8	0.99805									
N=10	X=0	0.00098	0.00195	0.00293	0.00390	0.00487	0.00585	0.00682	0.00779	0.00875	0.00972
	1	0.01074	0.02137	0.03188	0.04228	0.05257	0.06275	0.07281	0.08277	0.09263	0.10238
	2	0.05469	0.10638	0.15525	0.20145	0.24512	0.28640	0.32543	0.36232	0.39719	0.43016
	3	0.17187	0.31421	0.43208	0.52969	0.61053	0.67747	0.73290	0.77881	0.81683	0.84831
	4	0.37695	0.61181	0.75814	0.84931	0.90611	0.94150	0.96355	0.97729	0.98585	0.99119
	5	0.62305	0.85791	0.94644	0.97981	0.99239	0.99713	0.99892	0.99959	0.99985	0.99994
	6	0.82812	0.97046	0.99492	0.99913	0.99985	0.99997				
	7	0.94531	0.99701	0.99984	0.99999						
	8	0.98926	0.99988								
	9	0.99902									

A missing entry in the above table denotes a 1 correct to five decimal places.

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Purdue University

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

On Moments of Order Statistics from Independent Binomial Populations*

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report, July, 1972

5. AUTHOR(S) (Last name, first name, initial)

Gupta, Shanti S. and Panchapakesan, S.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1967		6

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
N00014-67-A-0226-00014 & AF33(615)67C	
b. PROJECT NO. 1244	Mimeograph Series #297
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Aerospace Research Laboratories Wright Patterson Air Force Base Dayton, Ohio

13. ABSTRACT

This paper deals with order statistics from the binomial distribution. Let $X_i (i=1,2,\ldots,M)$ be the number of successes in N independent trials from a binomial distribution with p as the probability of success, in each trial. Let X_i be arranged in order to give $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(M)}$. The discrete variable X_i or $X_{(i)}$ take values $0,1,\ldots,N (i=1),\ldots,M)$. Tables are provided giving the cumulative distribution and the expected value and variance of $X_{(1)}$ and $X_{(M)}$. Joint distribution of $X_{(i)}$ and $X_{(j)}$ is obtained. The distribution of $X_{(j)}-X_{(i)} (j > i)$ is also derived and special cases are studied.

DD FORM 1 JAN 64 1473

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Order Statistics Binomial Populations Expected Values, Variance Largest and Smallest Applications and Tables						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through

_____."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through

_____."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through

_____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS), (S), (C), or (U)*

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.