Buffon in the Round

by

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1. Introduction. Suppose that a needle of length 2d is thrown at random onto a circle of radius R so that its mid-point M falls within or on the circumference. We can define the term "at random" in a number of ways. In this paper we consider the following alternative models.

A. The needle is tossed so that the position of M is uniformly distributed along the radius vector passing through it. Let U be the distance from the center O of the circle to M. Then U is a random variable with density function f given by

$$f(u) = \begin{cases} 1/R & 0 \leq u \leq R \\ 0 & \text{otherwise.} \end{cases}$$

B. The needle is tossed so that the position of M is uniformly distributed over the whole circle. In this case U has density function g, where

$$g(u) = \begin{cases} \frac{2u}{R^2} & 0 \leq u \leq R \\ 0 & \text{otherwise} \end{cases}$$

In both cases we assume that the angle the needle makes with a fixed vector in the plane of the circle is uniformly distributed on $[0,2\pi]$

We shall find the probability distribution of the number of intersections of the needle with the circumference of the circle. In case A these probabil-

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ities are expressed in terms of elliptic integrals of the 1st and 2nd kinds. On the other hand the results for case B are expressed in terms of elementary functions only. In appendix 1 a table of these probabilities is given for certain $d|R$ ratios.

2. Derivation of the Probabilities.

We define the random variable Z as the number of intersections of the needle with the circumference of the circle. It is clear, that for $d > 2R$, $P[Z=2]=1$, so we need only consider the case $0 < d \leq 2R$. There are 2 main sub-cases to consider (a) $R \leq d \leq 2R$ and (b) $0 < d \leq R$. We shall deal with the case (a) first.

(a) $R \leq d \leq 2R$. We shall break this case down into two further sub-cases depending on the distance from 0 to M. Let $p_i(u)$ be the conditional probability $P[Z=i|U = u]$, $i = 0,1,2$ then:

(i) $0 < u \leq d-R$. Under these assumptions we always have exactly two intersections, so that:

$$p_0(u) = p_1(u) = 0$$
$$p_2(u) = 1$$

(ii) $d-R < u \leq R$. Referring to fig. 1 for notation we have:

$$\begin{aligned} p_0(u) &= 0 \\ p_1(u) &= 2 - 2\Psi/\pi \\ p_2(u) &= 2\Psi/\pi - 1 \\ R^2 &= u^2 + d^2 - 2ud\cos(\pi-\Psi) \end{aligned}$$

On expressing Ψ in terms of u,d and R and substituting we get

$$\begin{aligned} p_0(u) &= 0 \\ p_1(u) &= 2 - 2/\pi \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) \\ p_2(u) &= 2/\pi \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) - 1 \end{aligned}$$

$P_A[Z = i]$ denotes the probability of i intersections in case A. Similarly for $P_B[Z = i]$. It follows that for $R \leq d \leq 2R$ we have:

$$P_A[Z=0] = 0$$

$$P_A[Z=1] = \int_{d-R}^{R} \left[2 - \frac{2}{\pi} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right)\right] \frac{du}{R}$$

$$P_A[Z=2] = \int_0^{d-R} \frac{du}{R} + \int_{d-R}^{R} \left[\frac{2}{\pi} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) - 1\right] \frac{du}{R}$$

In case B we have

$$P_B[Z=0] = 0$$

$$P_B[Z=1] = \int_{d-R}^{R} \left[2 - \frac{2}{\pi} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right)\right] \frac{2udu}{R^2}$$

$$P_B[Z=2] = \int_0^{d-R} \frac{2udu}{R^2} + \int_{d-R}^{R} \left[\frac{2}{\pi} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) - 1\right] \frac{2udu}{R^2}$$

Simplifying these expressions we get

$$P_A[Z=0] = 0,$$

$$P_A[Z=1] = 4 - \frac{2d}{R} - \frac{2}{\pi R} \int_{d-R}^{R} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du,$$

$$P_A[Z=2] = \frac{2d}{R} - 3 + \frac{2}{\pi R} \int_{d-R}^{R} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du,$$

$$P_B[Z=0] = 0,$$

$$P_B[Z=1] = \frac{2d}{R^2}(2R-d) - \frac{4}{\pi R^2} \int_{d-R}^{R} u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du$$

$$P_B[Z=2] = 1 - \frac{2d}{R^2}(2R-d) + \frac{4}{\pi R^2} \int_{d-R}^{R} u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du$$

We will leave the evaluation of the above integrals to Section 3.

(b) $0 < d \leq R$. We break this case down into three sub-cases depending on the distance from 0 to M.

(i) $0 \leq u \leq R-d$. Under these conditions no intersections are possible so we have

$$p_0(u) = 1$$

$$p_1(u) = p_2(u) = 0.$$

(ii) $R-d \leq u < \sqrt{R^2-d^2}$. Now only zero or one intersections are possible. Referring to fig. 2 we see that:

$$p_0(u) = 1 - 2\Psi/\pi$$

$$p_1(u) = 2\Psi/\pi$$

$$p_2(u) = 0$$

$$\Psi = \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right)$$

(iii) $\sqrt{R^2-d^2} \leq u \leq R$. Only one or two intersections are possible and, referring to fig. 3, we see that:

$$p_0(u) = 0$$

$$p_1(u) = 2 - 2\Psi/\pi$$

$$p_2(u) = 2\Psi/\pi - 1$$

As for case (a) we obtain

$$P_A[Z=0] = \frac{\sqrt{R^2-d^2}}{R} - \frac{2}{\pi R} \int_{R-d}^{\sqrt{R^2-d^2}} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du,$$

$$P_A[Z=1] = 2 - \frac{2\sqrt{R^2-d^2}}{R} + \frac{2}{\pi R} \int_{R-d}^{\sqrt{R^2-d^2}} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du$$

$$- \frac{2}{\pi R} \int_{\sqrt{R^2-d^2}}^{R} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du,$$

$$P_A[Z=2] = \frac{\sqrt{R^2-d^2}}{R} - 1 + \frac{2}{\pi R} \int_{\sqrt{R^2-d^2}}^{R} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du.$$

$$P_B[Z=0] = \frac{R^2-d^2}{R^2} - \frac{4}{\pi R^2} \int_{R-d}^{\sqrt{R^2-d^2}} u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du,$$

$$P_B[Z=1] = 2 - \frac{2}{R^2}(R^2-d^2) + \frac{4}{\pi R^2} \int_{R-d}^{\sqrt{R^2-d^2}} u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du$$

$$- \frac{4}{\pi R^2} \int_{\sqrt{R^2-d^2}}^{R} u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du$$

$$P_B[Z=2] = \frac{1}{R^2}(R^2-d^2) - 1 + \frac{4}{\pi R^2} \int_{\sqrt{R^2-d^2}}^{R} u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du.$$

3. <u>Evaluation of the integrals.</u>

Let $I = \int_a^b \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du.$

On integrating by parts we get

$$I = u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right)\Big|_a^b - \int_a^b \frac{(R^2-d^2-u^2)\,du}{\sqrt{[u^2-(R-d)^2][(R+d)^2-u^2]}}$$

Let I_1 denote the remaining integral. By means of the substitution $u = (R+d)\cos\phi$, we obtain

$$I_1 = -\int_{a_1}^{b_1} \frac{[(R^2-d^2) + (R+d)^2\cos^2\phi]d\phi}{\sqrt{4Rd - (R+d)^2\sin^2\phi}}$$

For the moment we do not bother to express a_1 and b_1 in terms of a and b. Now, letting $(R+d)\sin\phi = 2\sqrt{Rd}\sin\theta$, we obtain

$$I_1 = -\frac{2R}{R+d}\int_{a_2}^{b_2} \frac{[(R+d) - 2d\sin^2\theta]\,d\theta}{\sqrt{1 - k^2\sin^2\theta}}$$

where $$k^2 = 4Rd/(R+d)^2$$

Upon rewriting this expression we get

$$I_1 = -2R\int_{a_2}^{b_2} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} + \frac{4dR}{R+d}\int_{a_2}^{b_2} \frac{\sin^2\theta d\theta}{\sqrt{1-k^2\sin^2\theta}}.$$

Setting: $$F(\eta|k) = \int_0^\eta \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$$

$$E(\eta|k) = \int_0^\eta \sqrt{1-k^2\sin^2\theta}\,d\theta$$

where $F(\eta|k)$ is the elliptic integral of the 1st kind and $E(\eta|k)$ is the elliptic integral of the 2nd kind and observing that

$$\int_{a_2}^{b_2} \frac{\sin^2\theta d\theta}{\sqrt{1-k^2\sin^2\theta}} = \frac{1}{k^2} \int_{a_2}^{b_2} \frac{[1-(1-k^2\sin^2\theta)]\, d\theta}{\sqrt{1-k^2\sin^2\theta}}$$

$$= \frac{1}{k^2} [F(b_2|k) - F(a_2|k) - E(b_2|k) + E(a_2|k)]$$

we obtain,

$$I = u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) \Big|_a^b - \Big\{ (d-R)[F(b_2|k) - F(a_2|k)] - (d+R)[E(b_2|k) - E(a_2|k)] \Big\}$$

To express a_2 and b_2 in terms of a and b we must express θ in terms of u. Following through the substitutions we get

$$\theta = \arcsin \sqrt{\frac{(R+d)^2-u^2}{4Rd.}}$$

So we can easily find a_2 and b_2 in terms of a and b. We leave off doing this until later.

$$\text{Let} \quad I' = \int_a^b u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du.$$

On integrating by parts we obtain

$$I' = \frac{u^2}{2} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) \Big|_a^b - 1/2\ I_1'$$

where $$I_1' = \int_a^b \frac{u(R^2-d^2-u^2)\, du}{\sqrt{[u^2-(R-d)^2][(R+d)^2-u^2]}}$$

By the substitutions, $u = (R+d)\cos\phi$, followed by $(R+d)\sin\phi = 2\sqrt{Rd}\sin\theta$,

we obtain

$$I_1' = -\,2R^2(b_2-a_2) - 2Rd\sin\theta\cos\theta\,\Big|_{\theta=a_2}^{b_2}$$

Hence we have

$$I' = \frac{u^2}{2}\arccos\left(\frac{R^2-d^2-u^2}{2ud}\right)\Big|_{u=a}^{b} + R^2(b_2-a_2)$$

$$+\ Rd\sin\theta\cos\theta\,\Big|_{\theta=a_2}^{b_2}$$

We gather together in table 1 the values of (a,b) and (a_2,b_2) we need to evaluate the integrals involved.

Table 1

u	:	$d-R$	$R-d$	$\sqrt{R^2-d^2}$	R
θ	:	$\pi/2$	$\pi/2$	$\arcsin\sqrt{\frac{1+\gamma}{2}}$	$\arcsin\sqrt{\frac{2+\gamma}{4}}$

where $\gamma = d/R$.

Let $\theta_1 = \pi/2$, $\theta_2 = \arcsin\sqrt{\frac{1+\gamma}{2}}$ and $\theta_3 = \arcsin\sqrt{\frac{2+\gamma}{4}}$

<u>Case A</u>

(i)

$$\int_{d-R}^{R} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du$$

$$= 2\pi R - \pi d - R \arccos\left(\frac{\gamma}{2}\right) - \Big\{(d-R)[F(\theta_3|k) - F(\theta_1|k)]$$

$$- (d+R)[E(\theta_3|k) - E(\theta_1|k)]\Big\}$$

(ii)

$$\int_{R-d}^{\sqrt{R^2-d^2}} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du.$$

$$= \frac{\pi}{2}\sqrt{R^2-d^2} - \Big\{(d-R)[F(\theta_2|k) - F(\theta_1|k)]$$

$$- (d+R)[E(\theta_2|k) - E(\theta_1|k)]\Big\}$$

(iii)

$$\int_{\sqrt{R^2-d^2}}^{R} \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du$$

$$= \pi R - R \arccos\left(\frac{\gamma}{2}\right) - \frac{\pi}{2}\sqrt{R^2-d^2} - \Big\{(d-R)[F(\theta_3|k) - F(\theta_2|k)]$$

$$- (d+R)[E(\theta_3|k) - E(\theta_2|k)]\Big\}$$

<u>Case B</u>

(i)

$$\int_{d-R}^{R} u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du$$

$$= \frac{\pi}{2}[2dR-d^2-R^2] - \frac{R^2}{2}\arccos\left(\frac{\gamma}{2}\right) + R^2 \arcsin\sqrt{\frac{2+\gamma}{4}} + \frac{Rd}{4}\sqrt{4-\gamma^2}$$

(ii) $$\int_{R-d}^{\sqrt{R^2-d^2}} u \arccos\left(\frac{R^2-d^2-u^2}{2ud}\right) du.$$

$$= R^2 \arcsin\sqrt{\frac{1+\gamma}{2}} + \frac{Rd}{2}\sqrt{1-\gamma^2} - \frac{\pi}{4}(R^2+d^2)$$

(iii) $$\int_{\sqrt{R^2-d^2}}^{R} u \arccos \frac{R^2-d^2-u^2}{2ud} \quad du$$

$$= \frac{\pi}{4}(R^2+d^2) + \frac{Rd}{2}\left(\frac{1}{2}\sqrt{4-\gamma^2} - \sqrt{1-\gamma^2}\right) - \frac{R^2}{2}\arccos\left(\frac{\gamma}{2}\right)$$

$$+ R^2\left(\arcsin\sqrt{\frac{2+\gamma}{4}} - \arcsin\sqrt{\frac{1+\gamma}{2}}\right).$$

4. Main Results. Using the results of section 3 we can write expressions for $P_A[Z=i]$, $P_B[Z=i]$

Case A

(i) $$0 < \gamma \leq 1$$

$$P_A[Z=0] = \frac{2}{\pi}\left\{(1-\gamma)[F(\theta_1|k) - F(\theta_2|k)] + (1+\gamma)[E(\theta_1|k) - E(\theta_2|k)]\right\}$$

$$P_A[Z=1] = \frac{2}{\pi}\arccos \frac{\gamma}{2}$$

$$- \frac{2}{\pi}\left\{(1-\gamma)[F(\theta_1|k) - F(\theta_2|k)] + (1+\gamma)[E(\theta_1|k) - E(\theta_2|k)]\right\}$$

$$+ \frac{2}{\pi}\left\{(1-\gamma)[F(\theta_2|k) - F(\theta_3|k)] + (1+\gamma)[E(\theta_2|k) - E(\theta_3|k)]\right\}$$

$$P_A[Z=2] = 1 - 2/\pi \arccos(\gamma/2)$$

$$- \frac{2}{\pi}\left\{(1-\gamma)[F(\theta_2|k) - F(\theta_3|k)] + (1+\gamma)[E(\theta_2|k) - E(\theta_3|k)]\right\}$$

(ii) $1 \le \gamma \le 2$

$$P_A[Z=0] = 0$$

$$P_A[Z=1] = \frac{2}{\pi} \arccos\left(\frac{\gamma}{2}\right)$$

$$- \frac{2}{\pi}\left\{(\gamma-1)[F(\theta_1|k) - F(\theta_3|k)] - (\gamma+1)[E(\theta_1|k) - E(\theta_3|k)]\right\}$$

$$P_A[Z=2] = 1 - \frac{2}{\pi} \arccos\left(\frac{\gamma}{2}\right)$$

$$+ \frac{2}{\pi}\left\{(\gamma-1)[F(\theta_1|k) - F(\theta_3|k)] - (\gamma+1)[E(\theta_1|k) - E(\theta_3|k)]\right\}$$

Since we can write k in terms of γ, viz $k^2 = 4\gamma|(1+\gamma)^2$, each of these expressions can be computed as functions of the single variable γ.

Case B

(i) $0 < \gamma \le 1$

$$P_B[Z=0] = 2 - \frac{2\gamma}{\pi}\sqrt{1-\gamma^2} - \frac{4}{\pi} \arcsin\sqrt{\frac{1+\gamma}{2}}$$

$$P_B[Z=1] = \frac{4}{\pi}\sqrt{1-\gamma^2} - \frac{\gamma}{4}\sqrt{4-\gamma^2} - 2 + \frac{2}{\pi} \arccos\left(\frac{\gamma}{2}\right)$$

$$+ 8/\pi \arcsin\sqrt{\frac{1+\gamma}{2}} - \frac{4}{\pi} \arcsin\sqrt{\frac{2+\gamma}{4}}$$

$$P_B[Z=2] = 1 + \frac{2\gamma}{\pi}\left(\frac{1}{2}\sqrt{4-\gamma^2} - \sqrt{1-\gamma^2}\right) - \frac{2}{\pi} \arccos\left(\frac{\gamma}{2}\right)$$

$$+ \frac{4}{\pi} \arcsin\sqrt{\frac{2+\gamma}{4}} - \frac{4}{\pi} \arcsin\sqrt{\frac{1+\gamma}{2}}$$

(ii) $1 \leq \gamma \leq 2$

$$P_B[Z=0] = 0$$

$$P_B[Z=1] = 2 - \frac{\gamma}{\pi}\sqrt{4-\gamma^2} + \frac{2}{\pi} \arccos\left(\frac{\gamma}{2}\right) - \frac{4}{\pi} \arcsin\sqrt{\frac{2+\gamma}{4}}$$

$$P_B[Z=2] = \frac{\gamma}{\pi}\sqrt{4-\gamma^2} - 1 + \frac{4}{\pi} \arcsin\sqrt{\frac{2+\gamma}{4}} - \frac{2}{\pi} \arccos\left(\frac{\gamma}{2}\right)$$

The form of the above results was chosen to make their evaluation by computer easy. In appendix 1 we give numerical results for values of γ between 0 and 2.

5. Appendix 1.

Case A

TABLE 2.

γ	$P_A[Z=0]$	$P_A[Z=1]$	$P_A[Z=2]$
.05	.96754373	.03244632	.00000995
.10	.93383333	.06608682	.00007986
.15	.89885809	.10087117	.00027074
.20	.86259978	.13675440	.00064582
.25	.82503203	.17369616	.00127181
.30	.78611941	.21166026	.00222033
.35	.74581616	.25061417	.00356967
.40	.70406431	.29052867	.00540703
.45	.66079114	.33137749	.00783138
.50	.61590559	.37313711	.01095730
.55	.56929310	.41578652	.01492038
.60	.52080808	.45930699	.01988494
.65	.47026235	.50368194	.02605571
.70	.41740686	.54889686	.03369629
.75	.36190060	.59493915	.04136026
.80	.30325380	.64179807	.05494812
.85	.24071124	.68946471	.06982405
.90	.17296773	.73793192	.08910035
.95	.09722563	.78719435	.11558002
1.00	0.00000000	.83724842	.16275158
1.05	0.00000000	.69569417	.30430583
1.10	0.00000000	.59982852	.40017148
1.15	0.00000000	.52150496	.47849504
1.20	0.00000000	.45455431	.54544569
1.25	0.00000000	.39600312	.60399688
1.30	0.00000000	.34408960	.65591040
1.35	0.00000000	.29765214	.70234786
1.40	0.00000000	.25587355	.74412645
1.45	0.00000000	.21815443	.78184557
1.50	0.00000000	.18404314	.81595686
1.55	0.00000000	.15319436	.84680564
1.60	0.00000000	.12534336	.87465664
1.65	0.00000000	.10029004	.89970996
1.70	0.00000000	.07788974	.92211026
1.75	0.00000000	.05804967	.94195033
1.80	0.00000000	.04073172	.95926828
1.85	0.00000000	.02596528	.97403472
1.90	0.00000000	.01388237	.98611763
1.95	0.00000000	.00482434	.99517566

Case B

TABLE 3.

γ	$P_A[Z=0]$	$P_A[Z=1]$	$P_A[Z=2]$
.05	.93636456	.06361554	.00001990
.10	.87288857	.12695197	.00015945
.15	.80973270	.18972786	.00053943
.20	.74706008	.25165699	.00128293
.25	.68503764	.31244571	.00251664
.30	.62383766	.37179008	.00437226
.35	.56363948	.42937235	.00698817
.40	.50463158	.48485701	.01051142
.45	.44701412	.53788569	.01510019
.50	.39100222	.58807085	.02092693
.55	.33683013	.63498739	.02818248
.60	.28475698	.67816137	.03708165
.65	.23507481	.71705399	.04787119
.70	.18812040	.75103816	.06084144
.75	.14429361	.77936186	.07634453
.80	.10408804	.80108707	.09482489
.85	.06814744	.81497710	.11687546
.90	.03738607	.81925609	.14335783
.95	.01332001	.81094666	.17573333
1.00	.00000000	.78200446	.21799555
1.05	0.00000000	.72734133	.27265867
1.10	0.00000000	.67366026	.32633974
1.15	0.00000000	.62102764	.37897236
1.20	0.00000000	.56951396	.43048604
1.25	0.00000000	.51919440	.48080560
1.30	0.00000000	.47014963	.52985037
1.35	0.00000000	.42246681	.57753319
1.40	0.00000000	.37624081	.62375919
1.45	0.00000000	.33157578	.66842422
1.50	0.00000000	.28858723	.71141277
1.55	0.00000000	.24740480	.75259520
1.60	0.00000000	.20817608	.79182392
1.65	0.00000000	.17107199	.82892801
1.70	0.00000000	.13629488	.86370512
1.75	0.00000000	.10409110	.89590890
1.80	0.00000000	.07477215	.92522785
1.85	0.00000000	.04875382	.95124618
1.90	0.00000000	.02664002	.97335998
1.95	0.00000000	.00945450	.99054550

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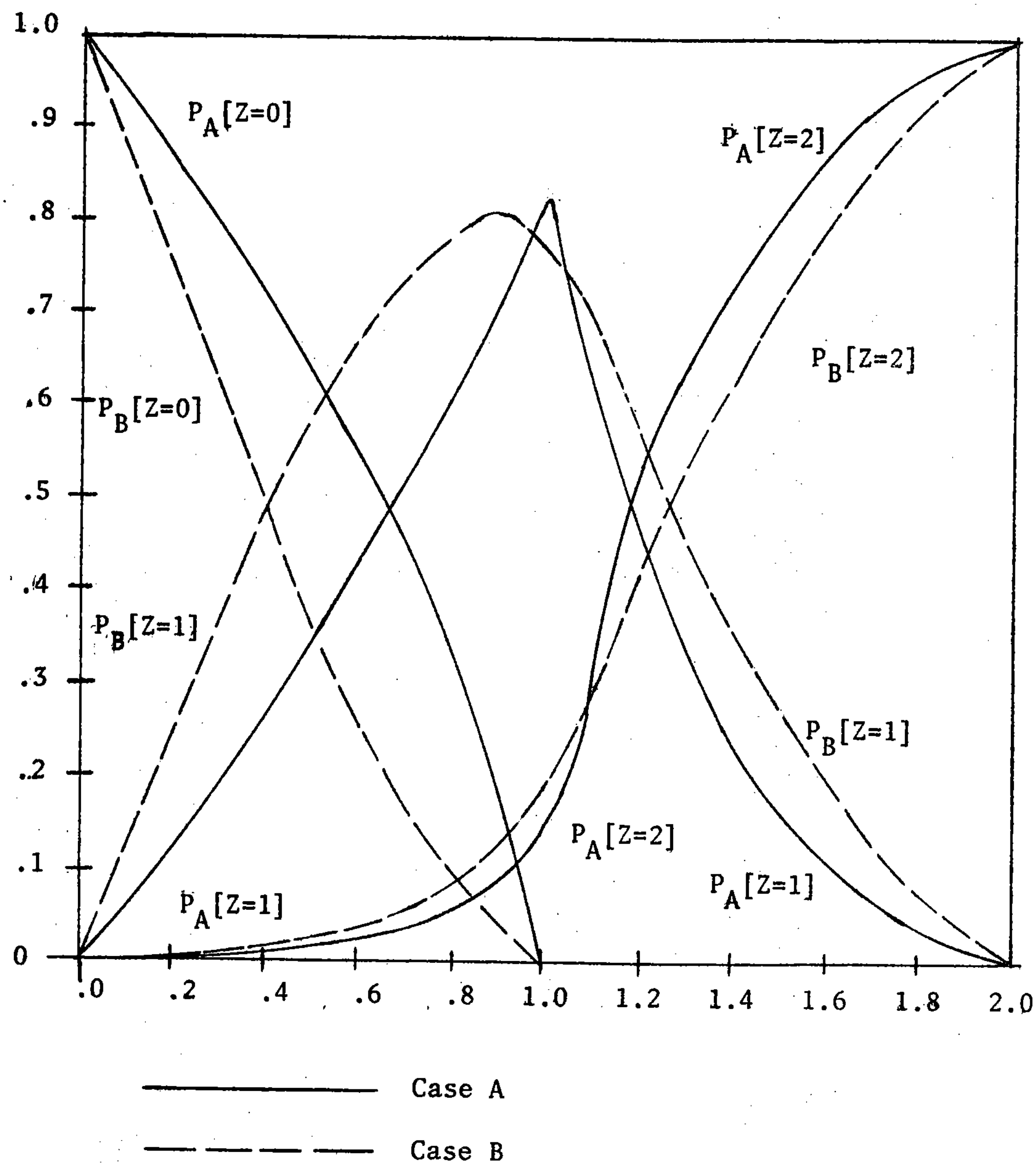
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P_A[Z=0]
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P_B[Z=0]
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P_A[Z=1]
P_A[Z=1]
Case A
Case B

GRAPH I

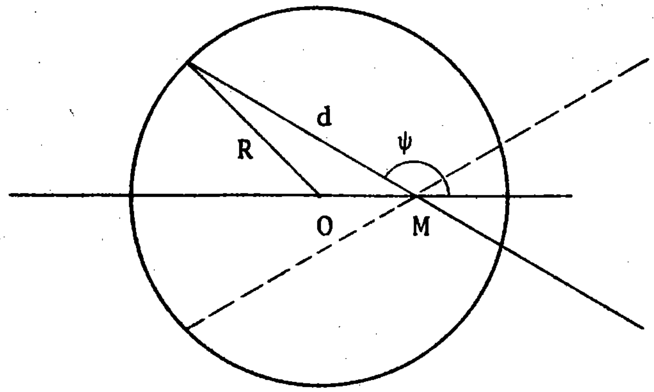

Fig. 1

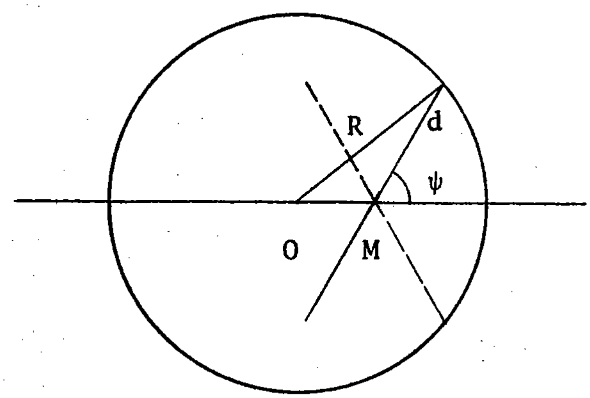

Fig. 2

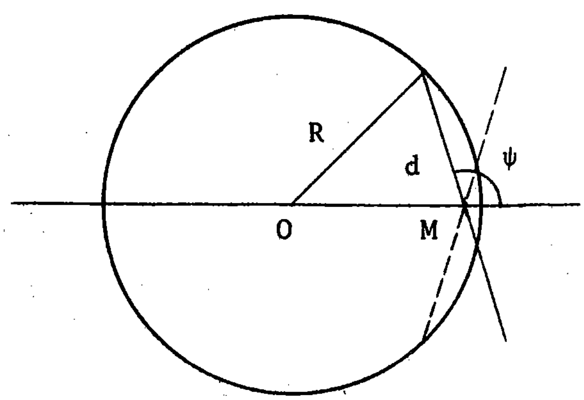

Fig. 3

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13. ABSTRACT

The distribution of the number of intersections of a needle and the circumference of a circle is studied under two distinct assumptions on the random manner in which the former is tossed on the circle.

Numerical results and graphs are also provided.

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