Order Statistics Arising from Independent Binomial Populations*

by

Shanti S. Gupta and S. Panchapakesan

Purdue University

Department of Statistics

Division of Mathematical Sciences

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1. Introduction and Summary.

In practice problems do arise involving order statistics of discrete distributions. Siotani (1956), Khatri (1962) and Gupta (1966) have discussed the problem of order statistics from discrete distributions. This paper considers the order statistics arising from binomial distribution.

Let X_i $(i=1,2,\ldots,M)$ be the number of successes in N independent trials from a binomial distribution with p as the probability of success in each trial. Let X_i be arranged in order to give

$$X_{(1)} \leq X_{(2)} \leq \cdots\cdots\cdots \leq X_{(i)} \leq \cdots\cdot \leq X_{(M)} .$$

The discrete variable X_i or $X_{(i)}$ takes values $0,1,\ldots,N$. $(i=1,2,\ldots.,M)$. Tables are provided giving the cumulative distribution and the expected value and the variance of $X_{(1)}$ and $X_{(M)}$. Joint distribution of $X_{(i)}$ and $X_{(j)}(i<j)$ is obtained. The distribution of $X_{(j)} - X_{(i)}$ $(j > i)$ is also derived and special cases are studied. Some applications are described.

2. Moments of Order Statistics.

We adopt the following notations:

$$b(\alpha) \equiv b(\alpha;p,N) = \binom{N}{\alpha} p^{\alpha} (1-p)^{N-\alpha} , \quad \alpha=0,1,..,N .$$

$$B(s) = \sum_{\alpha=0}^{s} b(\alpha) .$$

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$$B(p,q) = \frac{\Gamma(p)\,\Gamma(q)}{\Gamma(p+q)}$$

$$I_p(a,b) = \frac{1}{B(a,b)} \int_0^p u^{a-1}\,(1-u)^{b-1}\,du$$

Let $p_i(x)$ be the probability that the ith order statistic $X_{(i)}$ is equal to x and let $P_i(x) = P\{X_{(i)} \leq x\}$ be the c.d.f. of $X_{(i)}$. Khatri (1962) has obtained the following results. (Note the changes in his notations):

$$p_i(x) = \sum_{k=0}^{i-1} \sum_{m=0}^{M-i} \frac{M!}{(i-1-k)!\,(k+m+1)!\,(M-i-m)!} \{B(x-1)\}^{i-1-k} \{b(x)\}^{k+m+1} \{1-B(x)\}^{M-i-m} \qquad (2.1)$$

where $B(x-1) = 0$ for $x = 0$. This can be rewritten as

$$p_i(x) = i\binom{M}{i} \int_{B(x-1)}^{B(x)} \omega^{i-1}(1-\omega)^{M-i}\,d\omega = I_{B(x)}(i,M-i+1) - I_{B(x-1)}(i,M-i+1). \qquad (2.2)$$

Further

$$P_i(x) = i\binom{M}{i} \int_0^{B(x)} \omega^{i-1}\,(1-\omega)^{M-i}\,d\omega = I_{B(x)}\,(i,M-i+1) \qquad (2.3)$$

$$E(X_{(i)}) = \sum_{x=0}^{N-1} [1-P_i(x)] \qquad (2.4)$$

$$E(X_{(i)}^2) = 2\sum_{x=0}^{N-1} x[1-P_i(x)] + \sum_{x=0}^{N-1} [1-P_i(x)] \qquad (2.5)$$

Incidentally we note that the approach of Khatri (1962) is to derive first the probability $p_i(x)$ using the usual combinatorial arguments and then reduce it after some simplifications to the integral form of (2.2) and then derive (2.3). But it can be easily seen that for both discrete and continuous random variables

$$P_i(x) = \sum_{t=i}^{M} \binom{M}{t} [B(x)]^t [1-B(x)]^{M-t} \tag{2.6}$$

where $B(x)$ is to be interpreted as the usual c.d.f. in the continuous case. Now (2.3) follows from (2.6), since

$$\sum_{t=i}^{M} \binom{M}{t} p^t (1-p)^{M-t} = \frac{1}{B(i,M-i+1)} \int_0^p \omega^{i-1} (1-\omega)^{M-i} \, d\omega \tag{2.7}$$

where $0 \leq p < 1$. (When $B(x) = 1$, (2.3) is obviously true). Now (2.2) follows at once from (2.3) by noting that

$$p_i(x) = P_i(x) - P_i(x-1) \quad .$$

For the special cases $i = 1$ and $i = M$, we obtain the following results from (2.3), (2.4) and (2.5).

$$p_1(x) = [1-B(x-1)]^M - [1-B(x)]^M \tag{2.8}$$

$$P_1(x) = 1 - [1-B(x)]^M \tag{2.9}$$

$$p_M(x) = [B(x)]^M - [B(x-1)]^M \tag{2.10}$$

$$P_M(x) = [B(x)]^M \tag{2.11}$$

$$E(X_{(1)}) = \sum_{x=0}^{N-1} [1-B(x)]^M \tag{2.12}$$

$$E(X_{(1)}^2) = 2\sum_{x=0}^{N-1} x\,[1-B(x)]^M + E(X_{(1)}) \tag{2.13}$$

$$E(X_{(M)}) = \sum_{x=0}^{N-1} [1-\{B(x)\}^M] \tag{2.14}$$

$$E(X_{(M)}^2) = 2\sum_{x=0}^{N-1} x\,[1-\{B(x)\}^M] + E\,(X_{(M)}) \tag{2.15}$$

3. Joint distribution of $X_{(i)}$ and $X_{(j)}$, $i < j$.

Let $p_{i,j}(x,y)(i < j)$ be the probability that $X_{(i)}$ is equal to x and $X_{(j)}$ is equal to y and let $P_{i,j}(x,y) = P(X_{(i)} \leq x\ ,\ X_{(j)} \leq y)$. If $x \geq y$,

$$P_{i,j}(x,y) = P\,\{X_{(j)} \leq y\}$$

$$= j\binom{M}{j}\int_0^{B(y)} u^{i-1}(1-u)^{M-j}du \tag{3.1}$$

If $x < y$, then a combinatorial argument leads to

$$P_{i,j}(x,y) = \sum_{s=i}^{M}\sum_{\substack{t=0\\ s+t\le M}}^{M-j} \frac{M!}{s!(M-s-t)!t!} \{B(x)\}^s \{B(y)-B(x)\}^{M-s-t} \{1-B(y)\}^t$$

$$= \sum_{s=i}^{j}\sum_{t=0}^{M-j} \frac{M!}{s!(M-s-t)!t!} \{B(x)\}^s \{B(y)-B(x)\}^{M-s-t} \{1-B(y)\}^t$$

$$+ (1-\delta_{jM}) \sum_{s=j+1}^{M}\sum_{t=0}^{M-s} \frac{M!}{s!(M-s-t)!t!} \{B(x)\}^s \{B(y)-B(x)\}^{M-s-t} \{1-B(y)\}^t$$

where δ_{jM} is the Kronecker delta.

By repeated application of the results

$$\sum_{t=a}^{n} \binom{n}{t} p^t(k-p)^{n-t} = \frac{1}{B(a,n-a+1)} \int_0^p u^{a-1}(1-u)^{n-a}\,du \qquad \text{and}$$

$$\sum_{t=0}^{b} \binom{n}{t} p^t(k-p)^{n-t} = k^n - \frac{1}{B(b+1,n-b)} \int_0^p u^b(1-u)^{n-b-1}\,du$$

where $0 \le p < 1$ and $p < k$; we obtain

$$P_{i,j}(x,y) = i \binom{M}{i} \int_0^{B(x)} u^{i-1}(1-u)^{M-i}\,du$$

$$- \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(y)}^{1} dv \int_0^{B(x)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}\,d\omega . \qquad (3.2)$$

Now we can write

$$p_{i,j}(x,y) = \begin{cases} 0 & , \text{ if } x > y \\ P_{i,j}(x,x) - P_{i,j}(x-1,x) & , \text{ if } x = y \\ P_{i,j}(x,y) - P_{i,j}(x-1,y) & \\ \qquad - P_{i,j}(x,y-1) + P_{i,j}(x-1,y-1), & \text{if } x < y . \end{cases} \tag{3.3}$$

Khatri (1962) has obtained the joint distribution directly in the form

$$p_{i,j}(x,y) = \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int\int \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j} d\omega dv \tag{3.4}$$

where the double integration is performed over the region given by

$$\begin{cases} v \geq \omega \\ P(x) \geq \omega \geq P(x-1) \\ P(y) \geq v \geq P(y-1) . \end{cases}$$

But Khatri's expression for the c.d.f $P_{i,j}(x,y)$, namely,

$$P_{i,j}(x,y) = \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(x)}^{B(y)} dv \int_{0}^{B(x)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j} \, d\omega$$

$$+ j \binom{M}{j} \int_{0}^{B(x)} v^{j-1} (1-v)^{M-j} \, dv \tag{3.5}$$

is valid only for $x \leq y$. Incidentally, from (3.2) and (3.5) we get the relation

$$\frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(x)}^{1} dv \int_{0}^{B(x)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j}\, d\omega$$

$$= i \binom{M}{i} \int_{0}^{B(x)} u^{i-1}(1-u)^{M-i}\, du - j \binom{M}{j} \int_{0}^{B(x)} u^{j-1}(1-u)^{M-j}\, du$$

$$= I_{B(x)}(i, M-i+1) - I_{B(x)}(j, M-j+1) \qquad (3.6)$$

$$E(X_{(i)}X_{(j)}) = \sum_{x=0}^{N} \sum_{y=0}^{N} xy\, p_{i,j}(x,y)$$

$$= \sum_{x=0}^{N} x^2\, p_{i,j}(x,x) + \sum_{x=0}^{N-1} \sum_{y=x+1}^{N} xyp_{i,j}(x,y)$$

So

$$\mathrm{Cov}(X_{(i)},X_{(j)}) = \sum_{x=0}^{N} x^2\, P_{i,j}(x,x) + \sum_{x=0}^{N-1} \sum_{y=x+1}^{N} xyp_{i,j}(x,y)$$

$$- \left\{\sum_{x=0}^{N-1} [1-P_i(x)]\right\}\left\{\sum_{x=0}^{N-1} [1-P_j(x)]\right\} \qquad (3.7)$$

An explicit expression for $\mathrm{Cov}\ (X_{(i)}, X_{(j)})$ is very complicated. However, for the special case where $i = 1$ and $j = M$, we obtain by usual algebraic simplifications,

$$\mathrm{Cov}(X_{(1)},X_{(M)}) = NE(X_{(1)}) - (1-\delta_{N1}) \sum_{y=1}^{N-1} \sum_{x=0}^{y-1} [B(y)-B(x)]^M - E(X_{(1)})\, E(X_{(M)}), \qquad (3.8)$$

where $E(X_{(1)})$ and $E(X_{(M)})$ are given by (2.12) and (2.14) and δ_{N1} is the Kronecker delta.

4. Distribution of $Y_{i,j} = X_{(j)} - X_{(i)}$ $(j > i)$.

$Y_{i,j}$ represents a generalized range and can take values $0, 1, \ldots, N$. For $r \geq 0$,

$$P(Y_{i,j} = r) = \sum_{k=0}^{N-r} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \iint_A \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j} dv d\omega \tag{4.1}$$

where A is the region given by

$$\begin{cases} v \geq \omega \\ B(k) \geq \omega \geq B(k-1) \\ B(k+r) \geq v \geq B(k+r-1) \end{cases}$$

This can be rewritten as

$$P\{Y_{i,j} = r\} = \begin{cases} \displaystyle\sum_{k=0}^{N} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{\omega}^{B(k)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j} dv, r=0 \\ \displaystyle\sum_{k=0}^{N-r} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{B(k+r-1)}^{B(k+r)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j} dv, r>0 \end{cases} \tag{4.2}$$

So $E(Y_{i,j})$

$$= \sum_{r=1}^{N} r \sum_{k=0}^{N-r} \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{B(k+r-1)}^{B(k+r)} \omega^{i-1}(v-\omega)^{j-i-1}(1-v)^{M-j} dv \tag{4.3}$$

But we also know that

$$E(Y_{i,j}) = E(X_{(j)}) - E(X_{(i)})$$

$$= \sum_{x=0}^{N-1} i \binom{M}{i} \int_0^{B(x)} \omega^{i-1}(1-\omega)^{M-i}\, d\omega - \sum_{x=0}^{N-1} j \binom{M}{j} \int_0^{B(x)} \omega^{j-1}(1-\omega)^{M-j}\, d\omega$$

$$= \sum_{x=0}^{N-1} [I_{B(x)}(i,M-i+1) - I_{B(x)}(j,M-j+1)] \tag{4.4}$$

(4.3) and (4.4) lead to the relation

$$\sum_{r=1}^{N} \sum_{k=0}^{N-r} r \frac{M!}{(i-1)!(j-i-1)!(M-j)!} \int_{B(k-1)}^{B(k)} d\omega \int_{B(k+r-1)}^{B(k+r)} \omega^{i-1} (v-\omega)^{j-i-1}(1-v)^{M-j} dv$$

$$= \sum_{x=0}^{N-1} [I_{B(x)} (i,M-i+1) - I_{B(x)} (j,M-j+1)] \tag{4.5}$$

An explicit expression for variance of $Y_{i,j}$ is complicated. However, for the special case $i = 1$ and $j = M$, we obtain

$$P\{Y_{1,M} = r\} = \begin{cases} \sum_{k=0}^{N} [B(k) - B(k-1)]^M = \sum_{k=0}^{N} \{b(k)\}^M & \text{if } r = 0 \\ \sum_{k=0}^{N-r} [\{B(k+r)-B(k-1)\}^M - \{B(k+r)-B(k)\}^M \\ \quad -\{B(k+r-1) - B(k-1)\}^M + \{B(k+r-1)-B(k)\}^M], & \text{if } r > 0 \end{cases} \tag{4.6}$$

This has been obtained by Siotani (1956) with different notations from the joint distribution of $X_{(1)}$ and $X_{(M)}$.

Another special case, that can be of interest, is when $j = i + 1$. Then we have

$$P(Y_{i,i+1}=r)=\begin{cases}\sum_{k=0}^{N}[I_{B(k)}(i,M-i+1)-I_{B(k-1)}(i,M-i+1)-\binom{M}{i}[\{B(k)\}^{i}-\{B(k+1)\}^{i}][1-B(k)]^{M-i}] & r=0\\ \sum_{k=0}^{N-r}\binom{M}{i}[\{B(k)\}^{i}-\{B(k-1)\}^{i}][\{1-B(k+r-1)\}^{M-i}-\{1-B(k+r)\}^{M-i}], & r>0\end{cases} \quad (4.7)$$

In particular if $i = M-1$, we obtain

$$P(Y_{M-1,M}=r)=\begin{cases}\sum_{k=0}^{N}[\{B(k)\}^{M}-\{B(k-1)\}^{M-1}\{MB(k)-(M-1)B(k-1)\}], & r=0\\ \sum_{k=0}^{N-r}M[\{B(k)\}^{M-1}-\{B(k-1)\}^{M-1}]\,b(k+r), & r>0\end{cases} \quad (4.8)$$

5. Asymptotic Results.

Let $Z_i = \frac{X_i - Np}{\sqrt{Np(1-p)}}$. Then, for large N, $P\{X_i \le x\} \approx P\{Z_i \le z\}$, where Z_i is a normal variate with mean zero and variance unity and $z = \frac{x-Np}{\sqrt{Np(1-p)}}$.

As the transformation from X to Z preserves order, we have

$$P\{X_{(i)} \le x\} \approx P\{Z_{(i)} \le z\}$$

$$= i\binom{M}{i}\int_0^{\Phi(z)} u^{i-1}(1-u)^{M-i}du\,, \quad (5.1)$$

where

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{t^2}{2}} dt \quad .$$

Further we can write

$$E(X_{(i)}^r) \approx E([Z_{(i)} \sqrt{Np(1-p)} + Np]^r)$$

$$= \sum_{\alpha=0}^{r} \binom{r}{\alpha} (Np)^{r-\frac{\alpha}{2}} (1-p)^{\frac{\alpha}{2}} E(Z_{(i)}^{\alpha}) \tag{5.2}$$

In particular

$$E\, X_{(i)} \approx Np + \sqrt{Np(1-p)} \;\; E\, Z_{(i)} \tag{5.3}$$

$$V(X_{(i)}) \approx Np(1-p)\; V(Z_{(i)}) \tag{5.4}$$

6. Applications and Description of the Tables.

The binomial model is of interest in some statistical inference problems. For example, in a life test experiment, truncated at a fixed time, the number of failures is a binomial random variable. Hence if we assume the c.d.f. of the life distribution to be $p_i = F(t,\theta_i)$ and we are interested in testing hypotheses about p_i or, alternatively, about θ_i, then the distribution of the ordered number of failures becomes relevant especially if one is interested in a ranking or selection problems. More specifically, we give the following examples.

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Errata Sheet

page	line	for	read
12	4(bottom)	$p_{[M]} \geq p_o$	$p_{(M)} \leq p_o$
	2(bottom)	$x_{(M)} \leq x_o(\alpha,N,M)$	$X_{(1)} > x_o(\alpha,N,M)$
13	1	for the inequality (6.2)	$\sup\limits_{0<p_{(M)}\leq p_o<1} P\{x_{(1)}>x_o(\alpha,N,M)\} \leq \alpha$
	3	$p_{[M]} = p_o$	$p_{(M)} = p_o$
	5	$p_{[1]} \leq p_o' \ldots x_{(1)} \geq x_o'(\alpha,N,M)$	$p_{(1)} \geq p_o' \ldots X_{(M)} < x_o'(\alpha,N,M)$
	7	$p_{[3]} \geq .4$	$p_{(3)} \leq .4$
	8	$x(.05,5,3,) = 1$	$x_o(.05,5,3) = 2$
	9	$\ldots = .038) \ldots p_{[3]} \geq .4$	$\ldots = .032) \ldots p_{(3)} \leq .4$

1. Sobel and Huyett (1957), Gupta and Sobel (1960) and Gupta (1966) have discussed the problems of selection and ranking for the parameters of several binomial populations. The problems discussed deal with selecting a subset or selecting a single population using the indifference zone approach. For both formulations, the probability of a correct selection depends on the distribution of X_{max} or $X_{max} - X$ where X_{max} is the largest of a set of independent and identical binomial random variables and X is another binomial random variable distributed independently of X_{max}. For subset selection problems we are interested in evaluating the probability

$$P\{X_{max} - X \le d\} = \sum_{x=0}^{N} \binom{N}{x} p^x (1-p)^{N-x} \{ \sum_{\alpha=0}^{x+d} \binom{N}{\alpha} p^\alpha (1-p)^{N-\alpha} \}^{M-1} .$$

$$= \sum_{x=0}^{N} b(x) \{B(x+d)\}^{M-1} . \tag{6.1}$$

Thus, the tables of the present paper are useful in evaluating the above probability. The expected size of the selected subset is given by a similar expression (see (6.3) of Gupta and Sobel (1960)) which can be computed with the help of the tables in the present paper.

2. Siotani (1956) has considered tests of hypotheses of the type $p_1 = p_2 = \dots = p_M = p$ and suggested the use of the range for this case. He has constructed tables for the distribution of the range. If we are interested in testing $H: p_{[M]} \ge p_0$, then a quick test for this is as follows:

Reject H if $x_{(M)} \le x_0(\alpha, N, M)$ where α is the level of significance. To construct this test, we wish to obtain $x_0(\alpha, N, M)$ satisfying

$$\sup_{1>p_{[M]}\geq p_0>0} P\{X_{(M)} \leq x_0\ (\alpha,N,M)\} \leq \alpha \quad . \tag{6.2}$$

It can be shown that the probability on the left hand side of (6.2) is maximized when $p_{[M]} = p_0$. Hence the necessary constants $x_0(\alpha,N,M)$ can be obtained from the tables of this paper. A similar test for the hypothesis $H: p_{[1]} \leq p_0'$ can be constructed using the rejection region $x_{(1)} \geq x_0'(\alpha,N,M)$.

Illustration. As an example consider the case of $M = 3$ populations with $N = 5$ ind. trials with $x_1 = 2$, $x_2 = 3$, $x_3 = 0$. To test $H: p_{[3]} \geq .4$, at $\alpha = .05$ we find from the tables that $x(.05,5,3) = 1$, (actual level of significance = .038). Thus the hypothesis that $p_{[3]} \geq .4$ is accepted.

3. The usual tests for outliers suggest the use of statistics $X_{(M)}-X_{(M-i)}$, $1 \leq i \leq M - 1$. The results in Section 4, deal with the distribution of statistics of this type. For the particular case $X_{(M)} - X_{(M-1)}$, a simpler form of the distribution is given by (4.8).

4. Description of the Tables.

The tables at the end give the c.d.f. and the first two moments of the largest and smallest of M independent and identical binomial random variables, each denoting the number of successes in N independent trials with p as the associated parameter. The range of values of common p is: $p = .05(.05).50$; the values of N are : $N = 1(1)20$; the values of M are: $M=1(1)10$. These tables were computed in 1960 by Miss Ann Elmer of Bell Telephone Laboratories while one of the authors was a member of the technical staff of Bell Laboratories.

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13. ABSTRACT

This paper deals with order statistics from the binomial distribution. Let X_i $(i=1,2,\ldots,M)$ be the number of successes in N independent trials from a binomial distribution with p as the probability of success, in each trial. Let X_i be arranged in order to give $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(M)}$. The discrete variable X_i or $X_{(i)}$ take values $0,1,\ldots,N$ $(i=1,\ldots,M)$. Tables are provided giving the cumulative distribution and the expected value and variance of $X_{(1)}$ and $X_{(M)}$. Joint distribution of $X_{(i)}$ and $X_{(j)}$ is obtained. The distribution of $X_{(j)} - X_{(i)}$ $(j > i)$ is also derived and special cases are studied.

Some applications are described.

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