Order-restricted Bayesian Inference and Optimal Designs for the Simple Step-stress Accelerated Life Tests under Progressive Type-I Censoring

Crystal Wiedner & Dr. David Han
The University of Texas at San Antonio, TX 78249

Life Testing

We want to determine the life expectancy distribution of a solar lighting device. Can we do this?

Simple Step-Stress Accelerated Life Testing

Bayesian Framework

Likelihood Function

Assuming that a cumulative exposure model is appropriate and that the life distribution of a test unit is exponential at any level of stress, the likelihood function is:

\[ L(\lambda_1, \lambda_2) = \sum_{i=1}^{n} \lambda_1^y_i \lambda_2^{x_i} \exp(-\lambda_1 t_i - \lambda_2 s_i) \] (1)

Here, \( n \) is the number of units that failed at the respective stress level \( x \), i.e., the number of observed failures in the time interval \( (t_i, s_i) \), and \( U_i = \sum_{j=n}^{i} \lambda_1 (t_j - t_{j-1}) = N_i \lambda_1 (t_i - t_{i-1}) \). \( N_i \) is the number of units entering at the respective stress level which depend on the censoring proportion \( \pi \).

Solar Lighting Inference

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
<th>( \tau_1 )</th>
<th>( \tau_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>0.15</td>
<td>0.2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

The covariate was found to be 7.05666e-05.

Monte Carlo Simulations

Parameter and hyperparameter settings:

- \( \lambda_1 = 1.1052 \)
- \( \lambda_2 = 2.7183 \)
- \( \alpha_1 = \alpha_2 = 2 \)
- \( \gamma_1 = \gamma_2 = 0.0001 \)

The selection made for \( \lambda_1 \) and \( \lambda_2 \) were motivated by the desire to follow choices made for related frequentist work of Han and Bai (2020).

Using 1,000 simulations with \( n = 21 \) and then repeated for \( n = 41 \), estimators for the parameters of the posterior in Equation 2 were approximated by Monte Carlo simulations. Total test duration choices were \( \tau = 0.9 \) and \( \tau = 1.2 \) and \( \tau = 1.5 \).

Given the progressive Type-I censoring scheme, choices for the proportion of surviving units to censor after the first level were chosen as \( \tau = 0\% \), \( \tau = 10\% \) and \( \tau = 20\% \).

Simulation: \( n = 24 \), \( \alpha_1 = \alpha_2 = 2 \), \( \gamma_1 = \gamma_2 = \gamma \)

Design Optimization

Using equal step durations, the optimal value for the total test duration was obtained under the information-theoretic design criterion \( D \) as well as various criteria \( D/C/A/M/E \) based on the posterior variance-covariance matrix of \( \lambda \) (or \( \beta \) based on a linear link).

Design Utilities

- \( H \)-optimal design maximizes the expected information gain based on the posterior entropy.
- \( D \)-optimal design maximizes the expected determinant of the inverse of the posterior variance-covariance matrix.
- \( C \)-optimal design maximizes the expected reciprocal of the posterior variance at normal operating conditions.
- \( L \)-optimal design maximizes the expected reciprocal of the maximum posterior variance.
- \( E \)-optimal design maximizes the expected minimum eigenvalue of the inverse of the posterior variance-covariance matrix.

Conclusions

- Using a 3-parameter gamma distribution as a conditional prior, we have performed Bayesian estimation and design optimization for progressively Type-I censored simple SSALTs under continuous inspections assuming that the lifetimes are exponential and that a cumulative exposure model holds.

- This prior ensures that the failure rates increase as the stress level increases.

- This prior leads to a tractable joint posterior distribution, which is a mixture of gamma densities.

Future Directions

- Extending this framework to the interval monitoring setting
- Exploring different censoring schemes