1. (25 points) Short Answer Questions.

(a) (3 points) Fixing the significance level at \( \alpha = 5\% \), should we increase or decrease the sample size in order to decrease Type II error rate? What will happen to Type I error rate? What will happen to the power of the test?

\[ \alpha = 5\% \quad \text{Power} = 1 - \beta(\text{Type II error}) \]

If we decrease Type II error rate, power will increase.

In order to decrease Type II error rate, \( \beta(\text{Type II error}) = \beta = Pr(\text{accept } H_0 | H_1 \text{ is false}) \)

we need to increase the sample size.

but since we fix the significance level \( \alpha = 5\% \), Type I error rate will not change.

(b) (6 points) A sociologist is interested in studying the IQs of teachers for low income areas of a major city. Five schools were randomly chosen from low income areas and from each of these schools, five teachers were randomly chosen. The following table summarizes the mean IQs for each of these schools.

<table>
<thead>
<tr>
<th>School</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>97</td>
<td>94</td>
<td>109</td>
<td>98</td>
<td>103</td>
</tr>
</tbody>
</table>

MSE = 40. State an appropriate factor-effects ANOVA model (including assumptions and constraints).

\[ \alpha = 5 \quad n = 5 \]

Schools are random factor since we want to draw inference about population.

\[ \gamma_{ij} = \mu + \tau_i + \epsilon_{ij} \quad i = 1, 2, \ldots, 5 \quad j = 1, 2, \ldots, 5 \]

\( \mu \) - grand mean

\[ \tau_i \text{ iid } \mathcal{N}(0, \sigma^2_\tau) \]

\[ \epsilon_{ij} \text{ iid } \mathcal{N}(0, \sigma^2) \]

and \( \tau_i, \epsilon_{ij} \) are independent.

c) (6 points) Show estimates of model parameters in (b).

\[ \bar{Y}_1 = 97 \quad \bar{Y}_2 = 94 \quad \bar{Y}_3 = 109 \quad \bar{Y}_4 = 98 \quad \bar{Y}_5 = 103 \quad \bar{Y}_\cdot = 102.2 \]

\[ SS_{\text{Trt}} = \sum_{i=1}^{5} \frac{1}{n} (\bar{Y}_i - \bar{Y}_\cdot)^2 = 5 \left( (97-102.2)^2 + (94-102.2)^2 + (109-102.2)^2 + (98-102.2)^2 + (103-102.2)^2 \right) \]

\[ MS_{\text{Trt}} = \frac{SS_{\text{Trt}}}{df_{\text{Trt}}} = \frac{694}{4} = 173.5 \]

\[ MSE = 40 \]

\[ MS_{\text{Trt}} = \frac{SS_{\text{Trt}}}{df_{\text{Trt}}} = \frac{694}{4} = 173.5 - 40 = 133.5 \]

\[ \hat{\sigma}^2 = MSE = 40 \]

\[ \hat{\sigma}^2 = \frac{MS_{\text{Trt}} - MSE}{n} = \frac{133.5 - 40}{5} = 26.7 \]

\[ \hat{\mu} = \bar{Y}_\cdot = 102.2 \]
(d) (5 points) Calculate the power of the study in (b) (with \( \alpha = 0.05 \)). Only show formula with which you will obtain exact value of the power.

Test statistic we use is

\[
F_0 = \frac{MS_{\text{trt}}}{MS_E}
\]

\( H_0: \sigma_c^2 = 0 \)
\( H_1: \sigma_c^2 > 0 \)

Here, school is random factor.

\((na-a)MS_E/\sigma^2 \sim \chi^2_{na-a}\)
\(n=5, a=5\)

\((a-1)MS_{\text{trt}}/(\sigma^2 + n\sigma_c^2) \sim \chi^2_{a-1}\)

\(F_0 \sim F_{a-1, n-a}\)

where \(\chi^2 = 1 + n\sigma_c^2/\sigma^2\)
\(\lambda^2 = 1 + 5 \cdot \frac{26.7}{40} = 4.3375\)

\[
\text{Power} = \text{Prob} \left( F > F_{a-1, n-a \cdot 4.3375} \right)
\]
\(d_{n} = n-1=4\)
\(d_{a} = na-a = 20\)

(e) (5 points) An engineer is studying the mileage performance characteristics of four types of gasoline additives. In the road test he wishes to use four cars, each a different make. However, because of a time constraint, he can only study three types of gasoline additives with each car. Propose an experiment for this engineer, and determine the design parameters.

<table>
<thead>
<tr>
<th>car 1</th>
<th>car 2</th>
<th>car 3</th>
<th>car 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>gas A</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>gas B</td>
<td>B</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>gas C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>gas D</td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>

\[
Y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}
\]
\(i=1,2,\ldots,4\)
\(j=1,2,\ldots,4\)

\[
\Sigma \tau_i = \Sigma \beta_j = 0\]
\(\varepsilon_{ij} \sim N(0, \sigma^2)\)

Experiment should be BIBD: Balanced incomplete block design.

Consider car as block. 4 blocks, 4 treatments. \(b=4 \geq a=4\).

Each treatment contains 3 different treatments. \(k=3\)

Each treatment appears in 3 blocks. \(r=3\)

\(N = ar = bk = 12\)

Each pair of treatments appears together in 2 blocks. \(\lambda = 2\).

\(\lambda(a-1) = 6 = r(k-1)\)
2. (27 points) A chemist wishes to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one bolt to another, the chemist decides to select five bolts and applies all four chemicals in random order to each bolt. The resulting tensile strengths are summarized in the below, using MEANS statement in PROC GLM.

The GLM Procedure

<table>
<thead>
<tr>
<th>Level of chemical</th>
<th>N</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Mean</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>70.60000000</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>71.40000000</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>72.40000000</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>72.80000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level of bolt</th>
<th>N</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Mean</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>79.50000000</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>68.50000000</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>76.50000000</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>72.75000000</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>68.50000000</td>
</tr>
</tbody>
</table>

(a) (4 points) State what type of design had been possibly planned here. For the principles of experimental design employed in this design, explain what they intend to address.

Experiment design is used here is RCBD: randomized completely block design. Here we have 4 treatments (chemical agents) and 5 blocks (bolts). Since we do not care the inference about bolts, this is nuisance factor. We can decrease uncertainty by including this factor. We randomly apply all four chemicals to each bolt, in order to decrease the effect of this uncontrolled factors.

(b) (3 points) We ran the following codes in SAS.

```
proc glm data=exam2;
  class chemical bolt;
  model strength=chemical bolt;
  output out=exam2res p=pred r=res;
  data exam2new;
  set exam2res;
  q. dvi = (pred*pred);
proc glm data=exam2new;
  class chemical bolt;
  model strength=chemical bolt; dvi/ss3;
run; quit;
```
The SAS output is as follows,
Dependent Variable: strength

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>8</td>
<td>170.1581499</td>
<td>21.2697687</td>
<td>10.84</td>
<td>0.0603</td>
</tr>
<tr>
<td>Error</td>
<td>11</td>
<td>21.5918561</td>
<td>1.9628955</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>19</td>
<td>191.7500000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R-Square Coeff Var Root MSE strength Mean
0.997396 1.952860 1.401034 71.75000

Source   DF Type III SS Mean Square F Value Pr > F
chemical  3 0.09561028 0.0317010 0.02 0.9968
bolt      4 0.12306986 0.03076741 0.02 0.9894
dvi       1 0.20814992 0.20814992 0.11 0.7564

Interpret the above output with α = 0.01.

\[ Y_{ij} = \mu + r_i + b_j + e_{ij}, \quad \sum_{i=1}^{r} b_j = 0, \quad \sum_{j=1}^{b} e_{ij} = 0, \quad \text{N}(0, \sigma^2). \]

H0: \( r_i = b_j = 0 \) for all \( i, j \)
H1: not all of them are 0's

From the first output, we get \( F_6 = 10.84 \) p-value = 0.0003
thus we reject the null hypothesis and conclude that there are significant differences among different chemicals and bolts.

\[ R^2 = 0.997396 \Rightarrow \text{model is good.} \]

(c) (5 points) Based on your conclusion on (b), propose an appropriate factor-effects ANOVA model, and show the estimates of all parameters (including grand mean, all factor effects) in this model.

\[ Y_{ij} = \mu + r_i + b_j + e_{ij}, \quad \mu \text{-grand mean}, \quad r_i \text{-blocks}, \quad b_j \text{-treatments}. \]

\[ \hat{\mu} = \bar{Y} = \frac{\sum_{i=1}^{r} \sum_{j=1}^{b} Y_{ij}}{rb}, \quad \hat{r}_i = \frac{\sum_{j=1}^{b} Y_{ij}}{b}, \quad \hat{b}_j = \frac{\sum_{i=1}^{r} Y_{ij}}{r}. \]

\[ \hat{\mu} = 71.75, \quad \hat{r}_i = 1.75, \quad \hat{b}_j = -3.25, \quad \hat{e}_{ij} = 3.75. \]

\[ a = 4, \quad b = 5. \]

(d) (5 points) The (corrected) total variation of the data is 191.75. Construct the ANOVA table for this experiment, following the model in (c). And estimate the population variance.

\[ SS_{\text{Total}} = 191.75 \]

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F-value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocks</td>
<td>157</td>
<td>4</td>
<td>39.25</td>
<td>21.685</td>
<td>&lt;0.05</td>
</tr>
<tr>
<td>Trt</td>
<td>12.95</td>
<td>3</td>
<td>4.3167</td>
<td>2.316</td>
<td>&gt;0.05</td>
</tr>
<tr>
<td>Error</td>
<td>21.8</td>
<td>12</td>
<td>1.8167</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>191.75</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \sigma^2 = 1.8167 \]
(e) (2 points) State the hypotheses of the test on this design and draw conclusion at α=0.05.

\[ H_0: \tau_1 = \tau_2 = \tau_3 = \tau_4 = 0 \quad H_a: \text{not all } 0 \]

\[ F_0 = \frac{M_{S_{str}}}{M_{S_{E}}} = 2.376 < 3.49 \]

Thus we fail to reject the null hypothesis. Conclude there's no difference among

(f) (4 points) Calculate the power of this study (with α=0.05). Only show formula for chemical ageing with which you will obtain exact value of the power.

\[ F_0 = \frac{M_{S_{str}}}{M_{S_{E}}} \sim F(3, 12) \text{ under } H_0 \]

Non-central parameter \[ \delta = b \cdot \frac{\sum_{i=1}^{a} \tau_i^2}{\delta^2} = \frac{12.95}{\delta^2} = \frac{12.95}{1.8166} = 7.128 \]

\[ \Phi^2 = \frac{\delta}{\alpha} = \frac{7.128}{4} = 1.782 \]

Power = \[ 1 - \text{Prob} \left( F_{\delta_{1}, 4, 12}, 4, 12, \delta=7.128 \right) \]

(g) (4 points) Suppose the power in (f) is too low, and a new study will be planned to guarantee the power at least at 80%. State how you would like to plan the new study (how to calculate the sample size, and how to propose replications).

We can assume that we need replicates \( n \) for each treatment.

\[ N = abn \quad \delta = bn \cdot \frac{\sum_{i=1}^{a} \tau_i^2}{\delta^2} \quad \text{power} = 1 - \text{Prob} \left( F_{\delta_{1}, 4, abn-a, a-1, N-a, \delta} \right) \]

Control power at least 80%

We can run SAS from \( n=2 \) to \( n=10 \) (just for example). For each \( n \), we can calculate the power. Find the smallest \( n \) makes power > 8.

We do RCBD with replicates.

\[ N = ab, \quad \Rightarrow \text{decide } b \]

\[ \text{power} = 1 - \text{Prob} \left( F_{\delta_{1}, a-1, ab-1}, a-1, ab-1, \delta \right) \]

\[ \delta = b \cdot \frac{\sum_{i=1}^{a} \tau_i^2}{\delta^2} = b \times \frac{2.59}{1.8166} \]
3. (22 points) The effect of five different ingredients (A, B, C, D, E) on the reaction time of a chemical process is being studied. Each batch of new material is only large enough to permit five runs to be made. Furthermore, each run requires approximately 1.5 hours, so only five runs can be made in one day. The experimenter decides to run the experiment shown below with collected data.

<table>
<thead>
<tr>
<th>Batch</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A=8</td>
<td>B=7</td>
<td>D=1</td>
<td>C=7</td>
<td>E=3</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
<td>C=11</td>
<td>E=2</td>
<td>A=7</td>
<td>D=3</td>
<td>B=8</td>
<td>6.2</td>
</tr>
<tr>
<td>3</td>
<td>B=4</td>
<td>A=9</td>
<td>C=10</td>
<td>E=1</td>
<td>D=5</td>
<td>5.8</td>
</tr>
<tr>
<td>4</td>
<td>D=6</td>
<td>C=8</td>
<td>E=6</td>
<td>B=6</td>
<td>A=10</td>
<td>7.2</td>
</tr>
<tr>
<td>5</td>
<td>E=4</td>
<td>D=2</td>
<td>B=3</td>
<td>A=8</td>
<td>C=8</td>
<td>5</td>
</tr>
<tr>
<td>average</td>
<td>6.6</td>
<td>5.6</td>
<td>5.4</td>
<td>5</td>
<td>6.8</td>
<td>5.88</td>
</tr>
</tbody>
</table>

The grand mean is 5.88, and the level means for the four methods are

A: 8.4 B: 5.6 C: 8.8 D: 3.4 E: 3.2

Note that \( SST = \sum \sum \sum (Y_{ijk} - \bar{Y})^2 = 206.64 \).

(a) (3 points) What kind of design is used for the experiment? Describe the major advantages.

Latin square design is used here.

Major advantages: block on two nuisance factors, but use less experiments.

(b) (3 points) Let \( \tau_1, \tau_2, \tau_3, \tau_4, \tau_5 \) denote the effect of the four different methods (A, B, C, D, and E respectively). Calculate the point estimates of \( \tau_1, \tau_2, \tau_3, \tau_4, \tau_5 \).

\[
\hat{\tau}_j = Y_{.j} - \bar{Y} \quad j = A, B, \ldots, D
\]

\[
\hat{\tau}_A = Y_{A.} - \bar{Y} = 8.4 \quad \hat{\tau}_B = 5.6 \quad \hat{\tau}_C = 8.8 \quad \hat{\tau}_D = 3.4 \quad \hat{\tau}_E = 5.88
\]

(c) (4 points) Calculate \( SS_{Row}, SS_{Treatment}, SS_{Column}, SS_{E} \).

\[
p = 5
\]

\[
SS_{Row} = 5 \frac{\sum (\bar{Y}_{..j} - \bar{Y})^2}{p^2} = 5 \left[ (8.4 - 5.88)^2 + (5.6 - 5.88)^2 + (8.8 - 5.88)^2 + (3.4 - 5.88)^2 + (5.88 - 5.88)^2 \right] = 15.44
\]

\[
SS_{Total} = \frac{\sum \sum \sum Y_{ijk}^2 - \frac{1}{p} \sum \sum Y_{..i}^2}{p^2} = 206.64
\]

\[
SS_T = SS_{\tau_1} + SS_{\tau_2} + SS_{\tau_3} + SS_{\tau_4} + SS_{\tau_5} = 15.44
\]
(d) (4 points) Complete the following ANOVA table.

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rows</td>
<td>4</td>
<td>154.41</td>
<td>3.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment</td>
<td>4</td>
<td>141.44</td>
<td>35.36</td>
<td>11.29</td>
<td>&lt;0.05</td>
</tr>
<tr>
<td>Column</td>
<td>4</td>
<td>12.24</td>
<td>3.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>37.52</td>
<td>3.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>24</td>
<td>206.64</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(p=5)

(e) (2 points) State the conclusion on testing $H_0$: $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = 0$ versus $H_1$: at least one is not, using $\alpha = 0.05$.

$$F_0 = \frac{MS_{\text{treatment}}}{MSE} = 11.29 > F(0.05, 4, 12) = 3.26$$

P-value < 0.05 = α

thus we reject the null hypothesis and conclude that there is difference among treatments.

(f) (3 points) In the case that the current experiment lacks power to identify all the possible difference between different methods, describe how to have more experiments to increase the power.

we choose another 5 batches to do the same experiment, new 5 days.
which means new rows and new columns in additional squares.
this will increase the degree of freedom of error term, which will in turn
$$CD = \frac{q_{\alpha}(p, dfe)}{\sqrt{MSE}}$$
make CD be smaller.

(g) (3 points) Suppose that the engineer suspects that the workplaces used by the four operators may represent an additional source of variation. A fourth factor, workplace may be introduced to conduct another experiment. Describe the design principle for the new experiment.

Design: Graeco - Latin Squares.

Because now we have actually 3 nuisance factors: batch, day, workplace.
we can superimpose a square which is orthogonal to the original square.
which means, each pair of Greek letters like $A_1$, $B_8$, appear once.
use $a, b, g, d, q$ to denote 5 workplaces.
4. (26 points) An experiment was conducted to investigate the effects of the type of glass and the type of phosphor on the brightness of a television tube. The response variable is the current necessary (in microamps) to obtain a specified brightness level. The data are shown below.

<table>
<thead>
<tr>
<th>Glass Type</th>
<th>Phosphor Type</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>280,290,285</td>
<td>300,310,295</td>
<td>290,285,290</td>
<td></td>
</tr>
</tbody>
</table>

Here are the outputs from MEANS statement of PROC GLM in SAS:

<table>
<thead>
<tr>
<th>Glass Type</th>
<th>N</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>291.886667</td>
<td>9.0138762</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>235.0000000</td>
<td>11.4654392</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phosphor Type</th>
<th>N</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>260.0000000</td>
<td>27.7486739</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>273.3933333</td>
<td>32.5004080</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>256.8666667</td>
<td>34.8807482</td>
</tr>
</tbody>
</table>

(a) (4 points) Write the factor-effects ANOVA model for this experiment, explain the terms and specify assumptions and constraints.

Factorial design

\[ Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \]

\[ \tau_i = \text{glass type effect} \]

\[ \beta_j = \text{Phosphor type effect} \]

(b) (4 point) What are the estimates of effects for position 2, and for the following combination of position and temperature: (position, temperature) = (1,2)?

\[ \bar{Y}_{..} = 263.3333 \]

\[ \bar{Y}_{2..} = 235 \]

\[ \bar{Y}_{..2} = 263 \]

\[ \bar{Y}_{22} = 283.3333 \]

\[ \hat{\tau}_{2} = \bar{Y}_{2..} - \bar{Y}_{..} = 235 - 263.3333 = -28.3333 \]

\[ \hat{\beta}_{2} = \bar{Y}_{..2} - \bar{Y}_{2..} = 263 - 235 = 28 \]

\[ \hat{\tau\beta}_{12} = \bar{Y}_{12} - \bar{Y}_{1..} - \bar{Y}_{..2} + \bar{Y}_{..} = 3a.666667 - 291.66667 - 273.33333 + 263.33333 = 0 \]
SSA = b \sum_{i=1}^{b} (\bar{y}_{ri} - \bar{y}_{r..})^2

= q \left[ \sum_{i=1}^{b} (\bar{y}_{ri} - \bar{y}_{r..})^2 + (\bar{y}_{r..} - \bar{y}_{..})^2 \right]

\bar{y}_{..} = 263.333

(c) (4 points) Complete the following ANOVA table from SAS.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type I SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>5</td>
<td>15516.66667</td>
<td>3103.33333</td>
<td>58.89</td>
<td>&lt;0.0001</td>
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<td>Error</td>
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<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
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<td>166.66667</td>
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<td>glass*phosphor</td>
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<td>66.66667</td>
<td>1.26</td>
<td>&gt;0.05</td>
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</tbody>
</table>

(d) (6 points) Test if the main effects and interactions are significant at \( \alpha = 0.05 \), with hypotheses stated appropriately.

\( H_0: \beta_1 = \beta_2 = \ldots = \beta_3 = 0 \)  \( H_a: \) not all equal \( 0 \)

\[ F_{0} = \frac{73.789}{8.842} > \frac{F_{0.05, 2, 12}}{3.89} \]

\( H_0: \) not all \( \beta_j = 0 \), only \( \beta_j = 0 \)

\[ F_{0} = 1.26 < \frac{F_{0.05, 2, 12}}{18.64} \]

(e) (5 points) Use Bonferroni method to compare the following treatments: (1,2), (1,3), and (2,2) pairwisely, use \( \alpha = 0.05 \). Calculate the critical difference and report the results.

\[ \alpha = \frac{\alpha}{n} = \frac{0.05}{3} = 0.0166666 \]

\[ CD = t \sqrt{\frac{MSE}{n}} = 3 \]

\[ \sqrt{\frac{2}{n}} = 25.61 \]

\[ \bar{y}_{12} = 301.66667 > \bar{y}_{13} = 288.33333 > \bar{y}_{22} = 245.00000 \]

\[ \bar{y}_{12} - \bar{y}_{13} = 13.3333 < 25.61 \]  no difference between (1,2) and (1,3)

\[ \bar{y}_{12} - \bar{y}_{22} = 56.3333 > 25.61 \]  (1,2), (1,3) are different

(f) (3 points) Explain why we should start our investigation of factor effects by checking the interaction between glass type and phosphor type. (1,2), (1,2) are different. We should check the interaction term first because if interaction is significant, we do not need to check glass type and phosphor type, they both should be significant although test may show they are not.