1. Hypotheses: \( H_0 : \mu = \mu_0 \) vs \( H_1 : \mu < \mu_0 \)

Since population standard deviation \( \sigma = 25 \) is known, we should use Z-test with test statistic

\[
Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}.
\]

We reject \( H_0 \) if \( Z < -Z_{0.05} = -1.645 \), which is equivalent to

\[
\bar{X} < \frac{-1.645 \times 25}{\sqrt{n}} + \mu_0
\]

So

\[
\beta = P(\text{fail to reject } H_0 | H_1)
\]

\[
= P \left( \frac{\bar{X} - \mu_1}{25/\sqrt{n}} \geq \frac{-1.645 \times 25}{\sqrt{n}} + \frac{\mu_0 - \mu_1}{25/\sqrt{n}} | H_1 \right)
\]

under \( H_1 : \bar{X} \sim N(\mu_1, \frac{25}{\sqrt{n}}) \)

We need at least 80% power for 15 beats/minute reduction, so

\[
\beta = P \left( Z \geq \frac{-1.645 \times 25 + 15}{\frac{25}{\sqrt{n}}} \right) \leq 0.2
\]

Hence,

\[
\frac{-1.645 \times 25 + 15}{\sqrt{n}} \geq 0.84
\]

\[
\frac{-1.645 \times 25 + 15}{\frac{25}{\sqrt{n}}} \geq 0.84 \times 25
\]

\[
\frac{25 \times (1.645 + 0.84)}{\sqrt{n}} \leq 15
\]

\[
n \geq \left[ \frac{25 \times (1.645 + 0.84)}{15} \right]^2 = 17.1534
\]

\[
\therefore \text{At least 18 patients must be studied.}
\]

2. \( \bar{y}_1 = 5.1, \ \bar{y}_2 = 6.2, \ \bar{y}_3 = 4.6, \ \ n = 4, \ \ a = 3, \ \ SS_E = 2.3, \ \ N = 12 \)
\[ \therefore \bar{y} = \frac{(5.1 + 6.2 + 4.6) \times 4}{12} = 5.3 \]

(a) ANOVA table:

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F-value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>2</td>
<td>5.36</td>
<td>2.68</td>
<td>10.4851</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>Error</td>
<td>9</td>
<td>2.3</td>
<td>0.2556</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>7.66</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ SS_T = \sum_{i=1}^{a} n_i (\bar{y}_{ii} - \bar{y})^2 = 4[(5.1 - 5.3)^2 + (6.2 - 5.3)^2 + (4.6 - 5.3)^2] = 5.36 \]

(b) \( H_0 : \mu_1 = \mu_2 = \mu_3 \) (or \( \tau_1 = \tau_2 = \tau_3 = 0 \))

\( H_a : \) At least one is different.

Based on F-test, reject \( H_0 \) at \( \alpha = 0.05 \)

(c) \( R^2 = \frac{5.36}{7.66} = 0.6997 \), which means 69.97% of the total variability can be explained by the model.

\[ CV = \frac{\sqrt{MSE}}{\bar{y}} \times 100\% = \frac{\sqrt{2556}}{5.3} \times 100\% = 9.539\% \]

(d) \( \hat{\tau}_1 = \bar{y}_1 - \bar{y} = 5.1 - 5.3 = -0.2 \)

\( \hat{\tau}_2 = \bar{y}_2 - \bar{y} = 6.2 - 5.3 = 0.9 \)

\( \hat{\tau}_3 = \bar{y}_3 - \bar{y} = 4.6 - 5.3 = -0.7 \)

(e1) \( \alpha' = \frac{\alpha}{2} = \frac{1 - 0.99}{2} = 0.005 \)

CI using Bonferroni:

\[ \sum C_{ij} \bar{y}_{i} \pm t_{\alpha/2m,(N-a)} \sqrt{MS_e \sum \frac{C_{ij}^2}{n_i}} \]

CI for \( \Gamma_1 \): \( (6.2 - 4.6) \pm t_{0.00259} \sqrt{0.2556 \left( \frac{4}{4} + \frac{1}{4} \right)} \)

\[ = 1.6 \pm 3.69 \times 4 \]

\[ = 208 \, 92 \]

CI for \( \Gamma_2 \): \( (6.2 + 4.6 - 2 \times 5.1) \pm t_{0.00259} \sqrt{0.2556 \left( \frac{4}{4} + \frac{1}{4} + \frac{1}{4} \right)} \)
\[ = 0.6 \pm 3.69 \times 0.6192 = 0.6 \pm 2.2848 \]
\[ = (-1.6848, 2.8848) \]

e2) CI using Scheffe: \( \sum c_i \bar{y}_i \pm \sqrt{(a-1)F_{a-1,N-a}} \sqrt{MS_E \sum c_i^2 / n_i} \)

CI for \( \Gamma_1 \): \( (6.2 - 4.6) \pm \sqrt{(3-1) \times F_{0.012.9}} \sqrt{0.2556 \left( \frac{1}{4} + \frac{1}{4} \right)} \)

\[ = 1.6 \pm \sqrt{2 \times 8.02} \times \pm 305 7 546 \ 410 5305 7 546 413 1; \]
\[ = 106 \ 63 18 \]

CI for \( \Gamma_2 \): \( (6.2 + 4.6 - 2 \times 5.1) \pm \sqrt{(3-1) \times F_{0.012.9}} \sqrt{0.2556 \left( \frac{4}{4} + \frac{1}{4} + \frac{1}{4} \right)} \)

\[ = \pm 06 \ 640.5049 9 204 647 9 \]
\[ = - (817 \ 899.9) \]

e3) Reject \( H_0 \) for \( \Gamma_1 \) but not \( \Gamma_2 \) using both methods.

Bonferroni is preferred since \( m=2 \), and Scheffe is too conservative when \( m \) is small.

3.

(a) ANOVA table:

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<th>Mean Square</th>
<th>F-value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4</td>
<td>4939.3</td>
<td>1234.8</td>
<td>5.1244</td>
<td>&lt; 0.01</td>
</tr>
<tr>
<td>Error</td>
<td>15</td>
<td>3614.5</td>
<td>240.9667</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>8553.8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) \( H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 \)

\( H_a : \) At least one \( \mu_i \) is different.

P-value < 0.01

Reject \( H_0 \).

(c) \( \hat{\sigma}^2 = 240.9667 \)

(d) d1)

<table>
<thead>
<tr>
<th>Contrast</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2</td>
<td>1</td>
<td>1817.160714</td>
<td>1817.160714</td>
<td>7.5411</td>
<td>0.01 &lt; P &lt; 0.025</td>
</tr>
</tbody>
</table>

d2) \( \hat{C}^2 = 2 \times 112.5 - 146.25 - 2 \times 149 - 158 + 2 \times 148.75 = -79.75 \)
\[
\hat{\beta}_2 = \frac{\hat{c}_2}{D_2} = \frac{-79.75}{2 + (-1)^2 + (-2)^2 + (-1)^2 + 2^2} = \frac{-79.75}{14} = -5.6964
\]

Test \( \beta_2 = 0 \), \( F_{20} = \frac{SS_{c2}}{MS_E} = \frac{1817.1607}{240.9667} = 7.5411 \)

P-value < 0.05
\[ \therefore \text{The quadratic effect is significant at } \alpha = 0.05. \]

d3) \( SS_{c3} + SS_{c4} = SS_F - SS_{c1} - SS_{c2} \)
\[
= 1817.1607 - 2839.225 = -4939.3
\]
\[
= 282.9143
\]
\[
F_{30} \leq \frac{282.9143/2}{240.9667} = 0.587
\]

Since \( F_{0.1,2,15} = 2.70 \), neither can be significant.