Assignment 3 Answer Keys

1.

a) The ANOVA table is given as follows:

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>3</td>
<td>6026.83333</td>
<td>2008.94444</td>
<td>6.97</td>
<td>0.0022</td>
</tr>
<tr>
<td>Error</td>
<td>20</td>
<td>5767.00000</td>
<td>288.35000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>11793.83333</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At $\alpha = 5\%$, we conclude that there are treatment differences.

b) It is easy to verify that they are mutually orthogonal.

c)

```
proc glm data=one;
  class trt;
  model resp=trt;
  contrast 'C1' trt 1 1 -1 -1;
  contrast 'C2' trt 1 -1 1 -1;
  contrast 'C3' trt 1 -1 -1 1;
run;
```

Contrast  DF  Contrast SS  Mean Square  F Value  Pr > F
----------|-----|----------------|--------------|----------|--------|
C1        1   864.000000  864.000000  3.00     0.0989 |
C2        1   5162.666667 5162.666667 17.90    0.0004 |
C3        1   0.166667   0.166667   0.00     0.9811 |
Only C2 is significant. This implies that treatments difference is primarily due to the difference between low and high hormone levels.

2.

a) The ANOVA table is as follows,

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>3</td>
<td>20.25200000</td>
<td>6.75066667</td>
<td>17.55</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>Error</td>
<td>16</td>
<td>6.15600000</td>
<td>0.38475000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
<td>26.40800000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since P-value is less than 5%, we conclude that firing temperature affects the density of brick.

b) The orthogonal contrasts corresponding to orthogonal polynomials are given in the following SAS code

```
proc glm data=one;
  class trt;
  model resp=trt;
  contrast 'C1' trt -3 -1 1 3;
  contrast 'C2' trt 1 -1 -1 1;
  contrast 'C3' trt -1 3 -3 1;
run;
```

<table>
<thead>
<tr>
<th>Contrast</th>
<th>DF</th>
<th>Contrast SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>3.16840000</td>
<td>3.16840000</td>
<td>8.23</td>
<td>0.0111</td>
</tr>
<tr>
<td>C2</td>
<td>1</td>
<td>16.20000000</td>
<td>16.20000000</td>
<td>42.11</td>
<td>&lt;.0001</td>
</tr>
<tr>
<td>C3</td>
<td>1</td>
<td>0.88360000</td>
<td>0.88360000</td>
<td>2.30</td>
<td>0.1492</td>
</tr>
</tbody>
</table>
It is clear that linear and quadratic effects (contrasts) are significant.
c). omitted

3. a) The ANOVA table is

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Squares</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>3</td>
<td>489740.1875</td>
<td>163246.7292</td>
<td>12.73</td>
<td>0.0005</td>
</tr>
<tr>
<td>Error</td>
<td>12</td>
<td>153908.2500</td>
<td>12825.6875</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>643648.4375</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) The critical differences are as follows:
1. LSD: 174.48
2. Tukey: 237.74
3. Duncan: $(p=2,174.5)$, $(p=3,182.6)$, $(p=4,187.6)$
4. Scheffe: 259.13

c) The largest difference is required by Scheffe, and the smallest by LSD. Hence, Scheffe is the most conservative and LSA is the most powerful. In application, usually Tukey and Duncan will be preferred.

4. From Problem 1 in Assignment 2, $N = 65$, $a = 5$, $N - a = 60$, $MS_E = .1325$. Notice that $\mu_1$ is the control. Four pairs, $\mu_2 - \mu_1$, $\mu_3 - \mu_1$, $\mu_4 - \mu_1$ and $\mu_5 - \mu_1$ are compared. Let $m = 4$.

a) Using Bonferroni, the critical difference is

$$CD_b = t_{\alpha/2m}(N-a)\sqrt{MS_E(\frac{1}{n_i} - \frac{1}{n_1})} = t_{0.05/3}(60)\sqrt{.1325\left(\frac{1}{13} + \frac{1}{13}\right)} = 2.575\sqrt{.1325 \ast 2/13} = .3676$$
b) Using Dunnet, the critical difference is

\[ CD_d = d_a(a - 1, f)\sqrt{MS_E\left(\frac{1}{n_i} + \frac{1}{n_1}\right)} = d_{0.05}(4, 60)\sqrt{1.325 \times 2/13} = .358 \]

c) Notice that \( CD_b \) is bigger than \( CD_d \). It means that Bonferroni is more conservative than Dunnet.